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Thermal radiation effects on stagnation point flow past a stretching/ shrinking sheet in a Maxwell fluid with slip condition

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Abstract. In this study, the numerical solution of the thermal radiation effects on a stagnation point flow past a stretching/shrinking sheet in a Maxwell fluid with slip condition is considered. The transformed boundary layer equations are solved numerically using the Runge-Kutta-Fehlberg (RKF) method. Numerical solutions are obtained for the skin friction coefficient and the wall temperature as well as the temperature and the velocity profiles. The features of the flow and the heat transfer characteristics for various values of Prandtl number, stretching/shrinking parameter, thermal radiation parameter, Maxwell parameter, dimensionless velocity slip parameter and thermal slip parameter are analyzed and discussed.

1. Introduction

Understanding the behaviour of convective boundary layer flow on a stretching sheet is important in industrial manufacturing processes. This includes both metal and polymer sheets such as the cooling of metallic plate, blowing glass and spinning fibre for paper production. Previous study by Crane [1], proposed a mathematical solution for two-dimensional flow of stretching surface in a inert fluid. Other researchers deliberated several aspects in this problem and attained similar results. Another research paper published by Gupta and Gupta [2] had investigated the mass transfer and the heat for thin layer of viscous fluid over stretching sheet with a moving stream of suction or a blowing. Further improvement was made by analysed the performance of laminar boundary layer flow over a continuous and linearly stretching sheet and its effect on the heat transfer [3].

The stagnation point of boundary layer flow over shrinking surface was introduced by Wang [4] and Salleh et al. [5]. The stagnation point is a region where maximum pressure, highest rates of heat transfer and mass loss were encountered [4]. Problem associated with boundary layer flows on stagnation point and stretching surface has fascinated many researchers. This type of problem was extended to other type of industrial fluid like a viscoelastic fluid, micropolar fluid, nanofluid and Maxwell fluid [6]–[10].

In aforementioned studies, the flow field obeys the no slip conditions. Under this conditions, the flowing fluid that was adjoin with a solid form is static in dependence to the body at the contact surface [11]. According to Bhattacharyya et al. [12], no slip assumptions are not applicable for all cases of fluid flow. Under certain circumstances, this conditions may be replaced with partial or slip condition. Martin and Boyd [13] analyzed the effect of heat transfer and momentum on slip flow in the laminar boundary layer. Meanwhile, Aman et al. [14] examined the effects of slip condition in a mixed convection boundary layer flow that are close to the stagnation-point on a vertical surface. On the other hand, Sahoo [15] considered the effect of partial slip condition on stretching sheet embedded in non-Newtonian fluid. In continuation to this, Raisi and Ghasemi [16] applied both slip and no-slip conditions on the forced convection on laminar nanofluid in a micro channel to elucidate the numerical aspect. Recently, Nandy and Mahapatra [17] observed the effects on MHD stagnation flow over a stretching surface in nanofluid and micropolar fluid when heat generation/absorption and slip were applied respectively. Therefore, the objective of this research paper is to discuss the effects of thermal radiation on stagnation point flow over a stretching/ shrinking sheet in Maxwell fluid under the slip condition. The results obtained under special cases are then compared with those in the literatures to support the validity of our result.

2. The Problem Formulation And Mathematical Equations

Consider a fixed two-dimensional stagnation point flow of an upper-convected Maxwell fluid flowing through a stretching/shrinking sheet. It is assumed that $u_w(x) = ax$ is the stretching/shrinking velocity, $u_e(x) = cx$ is the external velocity, T is the temperature of the fluid, T_w and T_∞ is the wall and ambient temperature, respectively while $q_r = -\dfrac{4\sigma*}{3k*}\dfrac{\partial T^4}{\partial y}$ is a radiative heat flux where $\sigma*$ is Stefan Boltzman constant and $k*$ represent the absorption coefficient. The physical model and coordinate system for this problem is presented in figure 1.
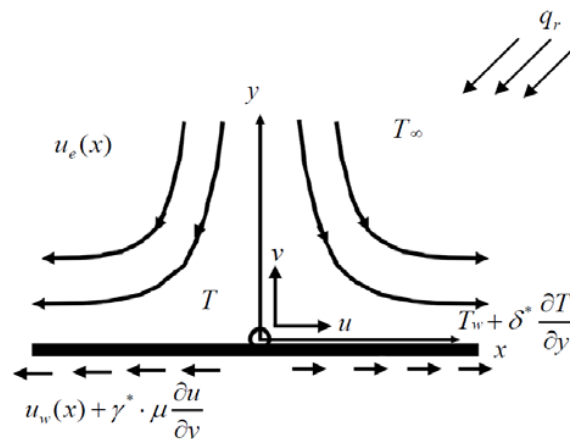

Figure 1. Physical model and coordinate system.

The central equations are describes as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x\partial y}\right) = u_e\frac{\partial u_e}{\partial x} + v\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Subject to the boundary conditions:

$$u = u_w(x) + \gamma *.\mu\frac{\partial u}{\partial y}, \qquad v = 0, \qquad T = T_w(x) + \delta *.\frac{\partial T}{\partial y} \qquad at \qquad y = 0 \tag{4}$$

$$u = u_e(x), \qquad T \to T_\infty, \qquad C \to C_\infty \qquad as \qquad y \to \infty$$

where u and v are the components for velocity along the x and y axes, ρ is the fluid density, $\gamma*$ is the parameter for dimensional velocity under slip condition, $\delta*$ is the parameter for dimensional thermal of the fluid under slip condition, λ is the parameter for relaxation time of the fluid, v is the kinematic viscosity of the fluid, C_p is the heat value specifically at a constant pressure and k is the thermal conductivity of the fluid.

In transforming the non-linear partial differential equations, the similarity variables are follows:

$$\eta = \left(\frac{a}{v}\right)^{1/2}y, \quad \psi = (av)^{1/2}xf(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where η is dimensionless variables. The stream function, ψ for similarity solution is describe in numerical as

$$u = \frac{\partial\psi}{\partial y} \text{ and } v = -\frac{\partial\psi}{\partial x} \tag{6}$$

This satisfies (1).

Then, substitute equations (5) and (6) into (2) and (3), which the following nonlinear ordinary differential equations was acquired

$$f''' + ff'' + \varepsilon^2 - f'^2 + \beta\left(2ff'f'' - f^2f'''\right) = 0 \tag{7}$$

$$\frac{1}{\Pr}\left(1 + \frac{4N_R}{3}\right)\theta'' + \theta'f = 0 \tag{8}$$

and the boundary conditions (4) becomes

$$f(0) = 0, \quad f'(0) = 1 + \gamma f''(0), \quad \theta(0) = 1 + \delta\,\theta'(0),$$
$$f'(\eta) \to \varepsilon, \qquad \theta(\eta) \to 0 \qquad as \qquad \eta \to \infty \tag{9}$$

Noticed that primes denote differentiation with respect to η. $\beta = \lambda a$ is the Maxwell parameter, $N_R = \frac{4\sigma *T_\infty^3}{k*k}$ is the thermal radiation parameter, $\Pr = \frac{v\rho c_p}{k}$ is the Prandtl number, $\gamma = \gamma*\left(\frac{a}{v}\right)^{\frac{1}{2}}$ is the dimensionless velocity slip parameter, $\delta = \delta*\left(\frac{a}{v}\right)^{\frac{1}{2}}$ is thermal slip parameter and $\varepsilon = \frac{c}{a}$ is a stretching/ shrinking parameter. Notice that $\varepsilon > 0$ is for stretching case while $\varepsilon < 0$ is for shrinking case.

3. Numerical Solution

Equations (7) and (8) were issued to the boundary conditions and (9) were deciphered numerically using Runge-Kutta-Fehlberg (RKF) method. The non-linear equations (7) and (8) are converted into a system of first order differential equations as follows:

$$\frac{df_0}{d\eta} = f_1$$

$$\frac{df_1}{d\eta} = f_2$$

$$\frac{df_2}{d\eta} = \frac{(f_1)^2 - \varepsilon^2 - f_0 f_2 - 2\beta f_0 f_1 f_2}{1 - \beta f_0^2} \qquad (10)$$

$$\frac{d\theta_0}{d\eta} = \theta_1$$

$$\frac{d\theta_1}{d\eta} = \frac{\Pr[-f_0 \theta_1]}{\left(1 + \frac{4}{3} N_R\right)}$$

Subsequently the boundary conditions (9) take the form,

$$f_0(0) = 0, \qquad f_1(0) = 1 + \gamma\, f_2(0), \qquad f_1(\infty) = \varepsilon$$
$$\theta_0(0) = 1 + \delta\, \theta_1(0), \qquad \theta_0(\infty) = 0 \qquad (11)$$

where $f_0 = f(\eta)$ and $\theta_0 = \theta(\eta)$. Aforementioned boundary value problem is first transformed into an preliminary value problem by properly conjecturing the missing slopes $f_2(0)$ and $\theta_1(0)$. The subsequent IVP is deciphered by shooting method for a set of parameters present in the central equations with an identified value of $f_1(0)$ and $\theta_0(0)$. The concurrent principle is mostly governed by an excellent deduction of the preliminary conditions in the shooting approached. The iterative course is resolved until the relative distinct between the recent iterative values of $f_2(0)$ matches with the preceding iterative value of $f_2(0)$ up to a tolerance of 10^{-7} [18].

4. Results and discussion

Equations (7) and (8) that are assigned to the boundary conditions (9) were accomplished numerically by means of the RKF approach which was programmed in MAPLE software byconsidering six parameters. The parameter includes the Prandtl number Pr, the stretching/shrinking parameter ε, the thermal radiation parameter N_R, the Maxwell parameter β, the dimensionless velocity slip parameter γ and the thermal slip parameter δ. With the intention of validating the efficiency of the applied approach, the assessment values have been made. The outcome from this mathematical evaluation was collated with the previous studies done by Abel et al. [8], Sadeghy et al. [16] and Mamaloukas et al. [19], as shown in table 1. Whereas, in table 2 the result was compared with Mohamed et al. [10]. Based from this, the mathematical evaluation has been satisfactory solution and that they are in good agreement.

Table 1. The comparison of the skin friction coefficient $f''(0)$ for a different values of Maxwell parameter β when $\text{Pr} = 7$, $\delta = \gamma = N_R = 0$ and in a stretching sheet $\varepsilon = 0.2$.

β	$f''(0)$			
	Sadeghy et al. [16]	Mamaloukas et al. [19]	Abel et al. [8]	Present results
0	-1.0000	-0.999962	-0.999962	-0.988935
0.2	-1.0549	-	-1.051948	-1.059283
0.4	-1.10084	-1.101850	-1.101850	-1.091750
0.6	-1.0015016	-	-1.150163	-1.124796
0.8	-1.19872	-1.196692	-1.196692	-1.158325
1.2	-	-1.285257	-1.285257	-1.226443

Table 2. The comparison of the wall temperature $\theta(0)$ for a different values of Prandtl number Pr when $\beta = \gamma = N_R = 0$ and $\delta = \varepsilon = 1$.

Pr	$\theta(0)$	
	Mohamed et al. [10]	Present results
0.1	0.79852	0.798522
0.72	0.59629	0.596293
1	0.55621	0.556209
7	0.32144	0.321440
10	0.28384	0.283838
100	0.11137	0.111373

Table 3. Values of the skin friction coefficient $f''(0)$ and the wall temperature $\theta(0)$ for different values of the Maxwell parameter β when $\text{Pr} = 7$, $\gamma = \delta = N_R = 0.5$ with $\varepsilon = 0.1$ (stretching).

β	$f''(0)$	$\theta(0)$
0.0	-0.579190	0.622767
0.1	-0.585010	0.624086
0.2	-0.590722	0.625385
0.4	-0.601819	0.628504
0.6	-0.612477	0.630729

The value of the skin friction coefficient $f''(0)$ and the wall temperature $\theta(0)$ for various values of β is presented in table 3. The increase of β has reduced the value of $f''(0)$ and while the $\theta(0)$ increases.

Table 4. Values of the skin friction coefficient $f''(0)$ the wall temperature $\theta(0)$ for a different values of the stretching parameter ε when $\text{Pr} = 7, \beta = 0.1, \gamma = \delta = 0.5$ and $N_R = 0.5$.

ε	$f''(0)$	$\theta(0)$
0.0	-0.600741	0.629045
0.2	-0.548045	0.617273
0.4	-0.452464	0.600865
0.6	-0.324809	0.583402
0.8	-0.172358	0.566334

Table 5. Values of the skin friction coefficient $f''(0)$ the wall temperature $\theta(0)$ for a different values of the shrinking parameter ε when Pr = 7, $\beta = 0.1$, $\gamma = \delta = 0.5$ and $N_R = 0.5$.

| ε	$f''(0)$	$\theta(0)$
-0.2	-0.598274	0.630156
-0.4	-0.538626	0.618946
-0.6	-0.434423	0.601758
-0.8	-0.299016	0.583572

In considering the stretching/shrinking parameter ε, from tables 4 and 5, as stretching parameter ε increases, the skin friction coefficient $f''(0)$ also increases while the wall temperature $\theta(0)$ decreases.
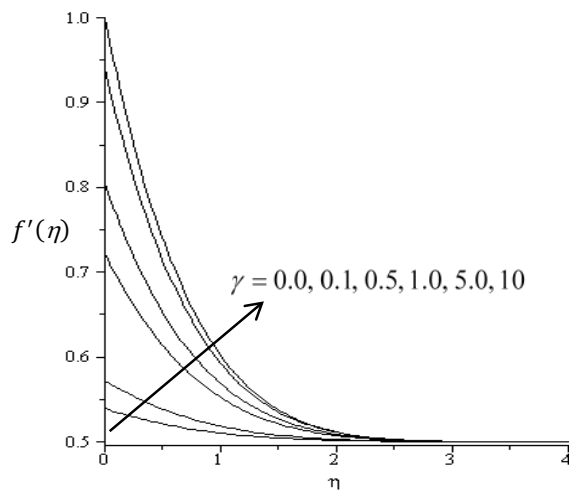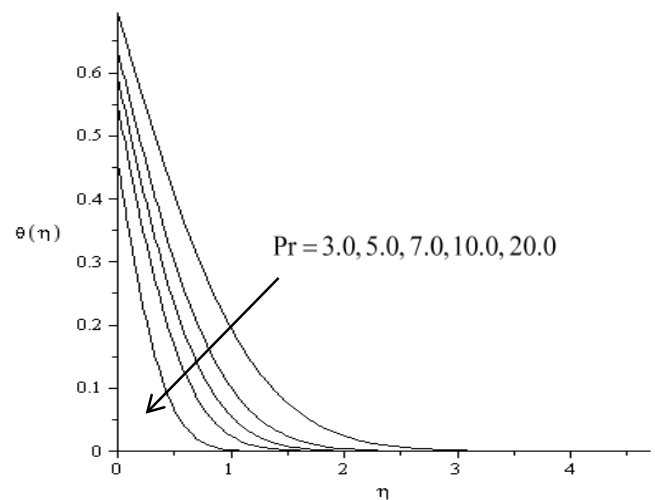

Figure 2. The effect of dimensionless velocity slips parameter γ on the velocity profile $f'(\eta)$.
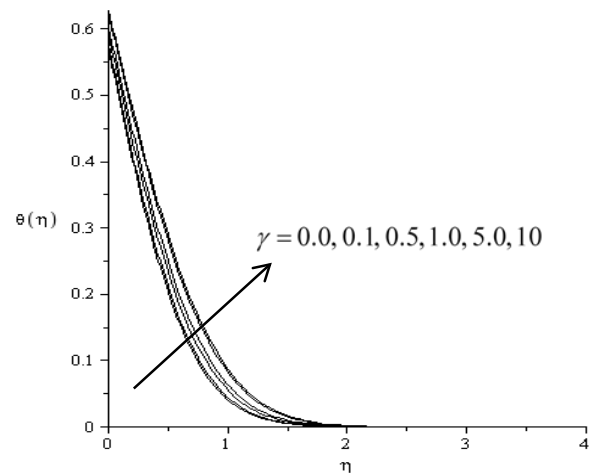
Figure 3. The effect of dimensionless velocity slips parameter γ on the temperature profile $\theta(0)$.
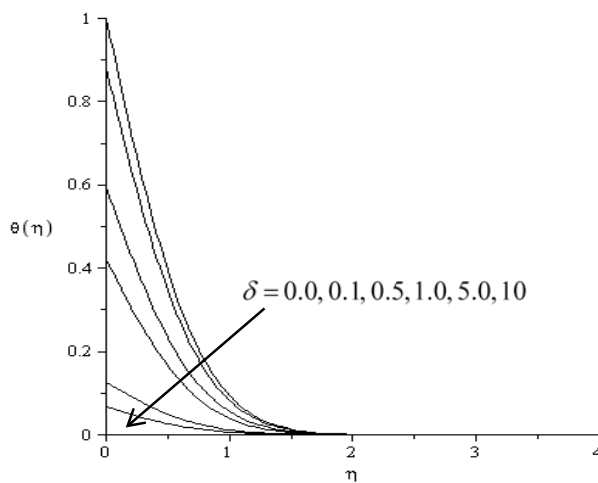
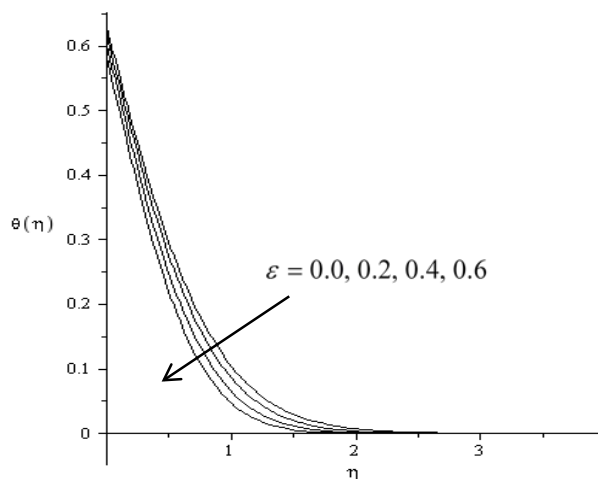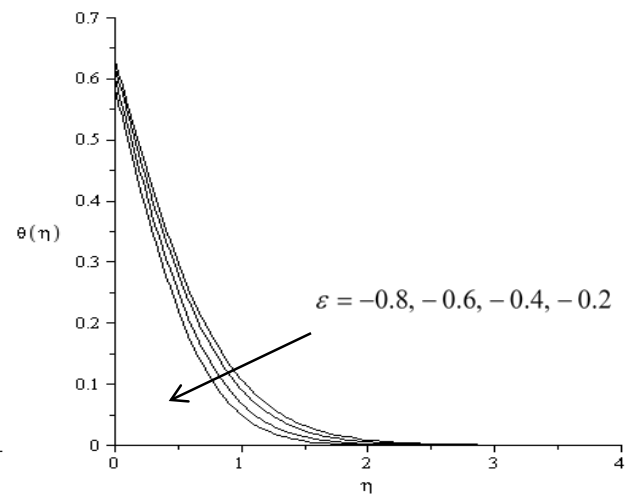**Figure 4.** The effect of thermal slip parameter δ on the temperature profile $\theta(\eta)$.

Figure 5. The effect of Prandtl parameter Pr on the temperature profile $\theta(\eta)$.

Figures 2 and 3 illustrate the velocity profile $f'(\eta)$ and temperature profile $\theta(0)$ for several ranges of γ values. It is found that the increase of γ enhanced the temperature and the velocity. The results suggest that the manifestation of velocity slip effect thickening both thermal and velocity boundary layer thicknesses. Meanwhile, in figure 4, the presence of thermal slip parameter δ was inversely proportional to the temperature. As δ increases, the temperature and its boundary layer thickness decline. figure 5 displays the temperature profiles $\theta(\eta)$ for several values of Pr. The result conclude that as Pr rises, the thermal boundary layer thickness would also decreases which declines the values of the wall temperature in the boundary layer. Prandtl number influences the value of fluid thermal conductive. As Prandtl number become small, the fluid thermal conductive become higher. Substantially when the Pr value upsurges, the value of thermal diffusivity will fall. This situation leads to the reducing of energy ability that lessens the thermal boundary layer.

Figures 6 and 7 present the temperature profile $\theta(\eta)$ for various values of stretching/ shrinking sheet parameter ε in case when $\mathrm{Pr} = 7$, $\delta = \gamma = N_R = 0.5$ and $\beta = 0.1$ respectively. The graphs describe the relationship between the temperature profile and the thermal boundary layer thickness. As the value for temperature profile reduces, the thermal boundary layer thickness decreases. The consequence of the stretching/ shrinking parameter is to reduce the thickness of the boundary layer. Similar trends occur in shrinking case. Figure 8 present the temperature profile $\theta(\eta)$ for numerous values of thermal radiation parameter N_R. Based from the graph, there is proportional relation between thermal radiation parameter N_R and temperature profile $\theta(\eta)$. Physically, presence of thermal radiation transmit amount of heat onto fluid flow. This situation contributes to the rise of temperature in the boundary layer.
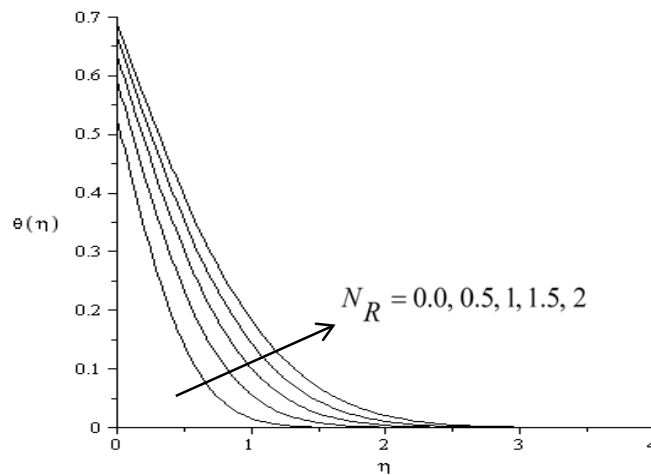

Figure 6. The effect of stretching parameter ε on the temperature profile $\theta(\eta)$.

Figure 7. The effect of shrinking parameter ε on the temperature profile $\theta(\eta)$.

Figure 8. The effect of thermal radiation parameter
N_R on the temperature profile $\theta(\eta)$.

5. Conclusions

The problem of the thermal radiation effects on stagnation point flow past a stretching/ shrinking sheet in Maxwell fluid with slip condition has been considered in this paper. The Runge-Kutta-Fehlberg (RKF) approach is applied to untangle the underlying problem numerically. It was prove how the Prandtl number Pr, the stretching/shrinking parameter ε, thermal radiation parameter N_R, the Maxwell parameter β, the dimensionless velocity slip parameter γ and thermal slip parameter δ affect the skin friction coefficient $f''(0)$, the wall temperature $\theta(0)$, the velocity $f'(\eta)$ and the temperature $\theta(\eta)$ profile. Overall a rises of Prandtl number, stretching/shrinking parameter and thermal slip parameter result to the decreasing in the wall temperature and thermal boundary layer thickness. Next, a rises of Maxwell parameter contribute to the increasing values of skin friction coefficient, while the temperature decreasing the skin friction coefficient is not affected by the Prandtl number, thermal slip parameter and the thermal radiation parameter. Therefore, the presence of thermal radiation has transmitted amount of heat onto fluid flow which therefore rises the temperature in the boundary layer.

References

[1]   Crane L J 1970 Flow past a stretching plate *Z. Angew. Math. Phys*. **21** pp 645–647
[2]   Gupta A S and Gupta P S 1977 Heat and mass transfer on a stretching sheet with suction or
         blowing *Can. J. Chem. Eng.* **55** pp 744–746
[3]   Chen M and Char C K 1988 Heat transfer of a continuous, stretching surface with suction or
         blowing *J. Math. Anal. Appl.* **135** pp 568–580
[4]   Wang C Y 2008 Stagnation flow towards a shrinking sheet *Int. J. Non. Linear. Mech.* **43(5)** pp
         377–382
[5]   Salleh M Z, Nazar R and Pop I 2009 Forced Convection Boundary Layer Flow At a Forward
         Stagnation Point with Newtonian Heating *Chem. Eng. Commun.* **196** pp 987–996

[6]   Mahmoud S E and Waheed M A A 2012 MHD flow and heat transfer of a micropolar fluid
        over a stretching surface with heat generation (absorption) and slip velocity *J. Egypt. Math.*
        Soc. **20(1)** pp 20–27
[7]   Bachok N, Ishak A and Pop I 2012 Boundary layer stagnation-point flow and heat transfer over
        an exponentially stretching /shrinking sheet in a nanofluid *Int. J. Heat Mass Transf.* **55(25)**
        pp 8122–8128
[8]   Subhas Abel M, Tawade J V and Nandeppanavar M M 2011 MHD flow and heat transfer for
        the upper-convected Maxwell fluid over a stretching sheet *Meccanica* **47(2)** pp 385–393
[9]   Sadeghy S M, Hajibeygi K and Taghavi H 2006 Stagnation point flow of upper-convected
        Maxwell fluids *Int J Non-Linear Mech.* **41** pp 1242–1247
[10]   Mohamed M K A, Salleh M Z, Ishak A and Pop I 2015 Stagnation point flow and heat transfer
        over a stretching/shrinking sheet in a viscoelastic fluid with convective boundary condition
        and partial slip velocity *Eur. Phys. J. Plus* **130(8)** p 171
[11]   Prabhakara S and Deshpande M D 2004 The no-slip boundary condition in fluid mechanics
        Resonance **9(5)** pp 61–71
[12]   Bhattacharyya G C, Mukhopadhyay K and Layek S 2011 Slip effects on boundary layer
        stagnation-point flow and heat transfer towards a shrinking sheet *Int. J. Heat Mass Transf.*
        54(1–3) pp 308–313
[13]   Michael I D B and Martin J 2006 Momentum and heat transfer in a laminar boundary layer
        with slip flow *J. Thermophys. heat Transf.* **20(4)** pp 710–719
[14]   Aman I P F and Ishak A 2011 Mixed convection boundary layer flow near stagnation-point on
        vertical surface with slip *Appl. Math. Mech.* **32(12)** pp 1599–1606.
[15]   Sahoo B 2010 Flow and heat transfer of a non-Newtonian fluid past a stretching sheet with
        partial slip *Commun. Nonlinear Sci. Numer. Simul.* **15(3)** pp 602–615.
[16]   Raisi S A A and Ghasemi B 2011 A numerical study on the forced convection of laminar
        nanofluid in a microchannel with both slip and no-slip conditions *Numer. Heat Transf. Part*
        A Appl. **59(2)** pp 114–129.
[17]   Nandy S K and Mahapatra T R 2013 Effects of slip and heat generation/absorption on MHD
        stagnation flow of nanofluid past a stretching/shrinking surface with convective boundary
        conditions *Int. J. Heat Mass Transf.* **64** pp 1091–1100.
[18]   Khairul M, Mohamed A, Salleh M Z, Nazar R and Ishak A 2013 Numerical investigation of
        stagnation point flow over a stretching sheet with convective boundary conditions pp 1–10.
[19]   Mamaloukas U S, Abel C, Tawade M S and Mahabaleswar J V 2011 On effects os a transverce
        magnetic field on an UCM fluid over a stretching sheet *Int. J. Pure Appl. Math.* **66(1)** pp 1–
        9.