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# Mathematical modelling of wave impact on floating breakwater

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**Abstract.** The impact of breaking wave on shoreline can be lessened or prevented by placing some kind of protection before the wave to reduce the speed of the wave before attacking the shoreline. Such protection can be in the form of a breakwater which is a structure designed to help reducing the wave intensity in whether in inshore waters or relatively shallow water. Thus, a mathematical model of Pressure Impulse,  $P$  is used to model the effect of waves exerted on a wall of a breakwater. A two-dimensional field of equations is derived for  $P$  which are applicable in three regions of breakwater problems by expressing this in terms of eigenfunctions that satisfy the boundary conditions apart from that the impact region and the matching of the three regions (before the breakwater, under the breakwater and after the breakwater). As in Cooker, we found that the equations of  $P$  in region 1 and region 3 are same as Cooker only that equation in region 3 has to include a secular term.

## 1. Introduction

This paper discusses a mathematical model of the large brief pressure brought by breaking waves against the coastal structures. Of the vast open ocean, waves are among the most familiar features often seen and heard. Waves roll into the beach at any given time and they are of all sizes and shapes. Thus, if they are not stopped by anything, waves can travel across entire ocean basins causing erosions and as well as overtopping that can damage our land properties. The most familiar ocean waves are called wind-driven waves which are caused by the wind. Others are called tsunamis and tidal waves caused by underwater disturbances and the gravitational pull respectively.

In order to prevent the beach erosions and land properties from being damaged by ocean waves, protections such as seawalls and breakwaters were built and many researchers and experimenters were driven to study the wave impact on coastal structures. Rouville et al. [1] were amongst the earliest but produced very little data while Bagnold and his colleagues [2] who formed a committee did a laboratory test to investigate the nature of the shock pressure exerted on the vertical wall. The researches in this area then evolved theoretically and experimentally, both at model and full scale and generally confirmed Bagnold's observations. Munireddy and Neelamani [3] have modified Goda's



formula to estimate the shoreward pressures on the seawall in the presence of the offshore breakwater. He carries out a statistical analysis for different relative breakwater heights and found out that the increment in the relative breakwater heights reduces the value of wave pressures significantly.

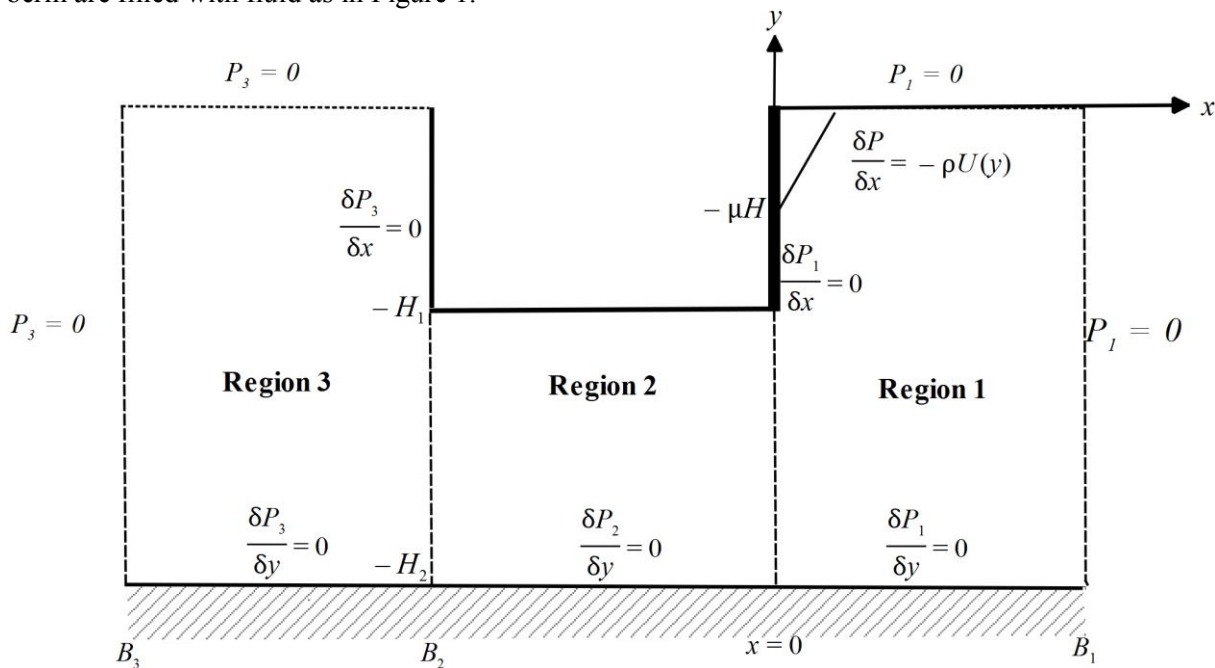
In this presented paper, our work solely aims for the theoretical studies of wave impact in coastal structures which is floating breakwater, a structure to prevent waves of smaller heights and period. Thus, we looked into the research of Cooker and Peregrine [4,5] who proposed using the pressure impulse  $P(x,y)$  that is the time integral of the pressure over the duration of the impact as in equation (1) to study the impact.

$$P(x, y) = \int_{t_b}^{t_a} p(x, y, t) dt \tag{1}$$

This results as [4, 5] give a simplified, but much more stable, model of wave impact on the coastal structures. This presented paper continues the work of Noar and Greenhow [6,7] who extended Cooker’s model with a ditch or berm into breakwater.

**2. Mathematical modelling**

A floating breakwater is a structure that floats in the water and functions to reduce the extreme wave impact before it hits anything behind the structure given that a shoreline. As to extending the work of Md Noar and Greenhow [6,7], we modeled the breakwater also with a berm but with both sides of the berm are filled with fluid as in Figure 1.



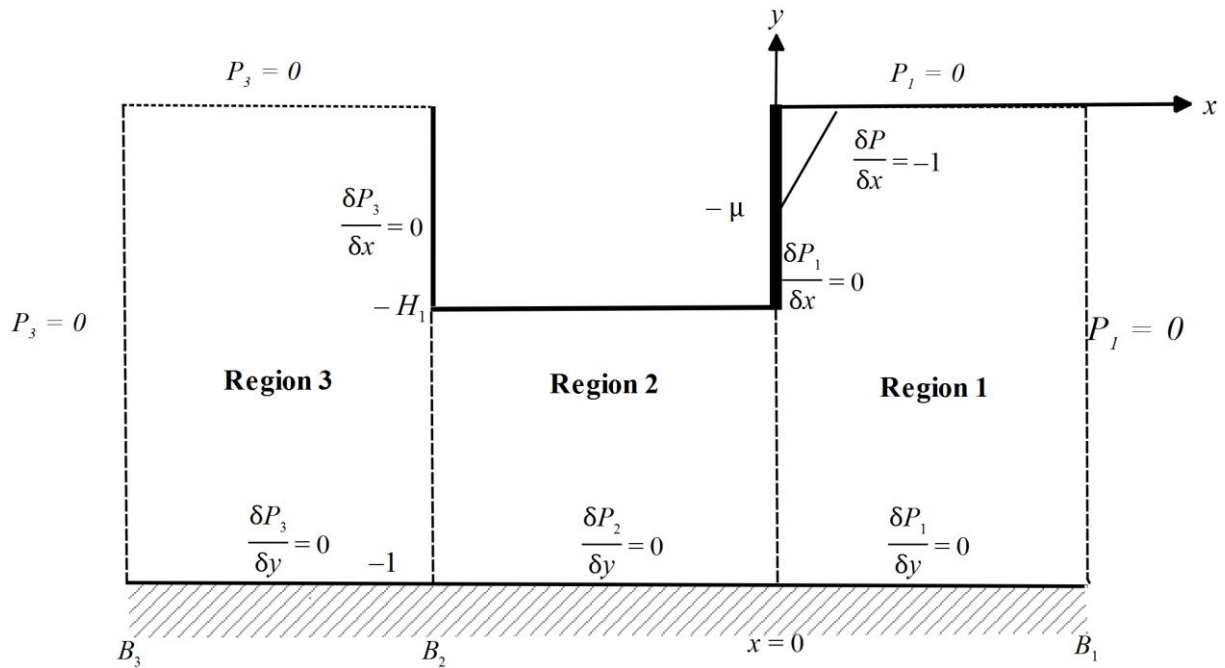
**Figure 1.** Boundary conditions for the pressure impulse of the wave impact on a floating breakwater.

This breakwater problem is split into three regions. Region 1 is filled by water and where the wave is coming from.  $H_1$  is the height of the breakwater and  $H_2$  is the distance down from the breakwater structure to the seabed at  $x=0$ . Region 2 is where the breakwater structure is and filled with water underneath it. The height  $H_2 - H_1$  is the distance down the base of the breakwater structure to the

seabed. Region 3 is similar to Region 1 but without the impact region since it is the back of the breakwater. We also let  $P_1$ ,  $P_2$  and  $P_3$  be the solutions in the respective regions.

The boundary conditions to be applied to Laplace’s equation are found to be as in Figure 1. Since the pressure at the free surface is a constant and taken to be the zero reference pressure, therefore the boundary condition at the free surface is  $P_1=0$  and  $P_2=0$ . At a rigid or solid boundary which is in contact with the liquid during the impact, the normal velocity of the breaking wave before and after the impact remains unchanged which gives  $\frac{\partial P}{\partial n}=0$  in the respective regions where  $n$  denotes the components normal the solid boundary region. And also, we have  $P_1=P_2$  and  $P_2=P_3$  under the condition of when liquid meets another liquid in order for the pressure impulse to be continuous.

Since our calculations involved some measured units, so we choose to simplify the problems by nondimensionalising the boundary conditions which is giving us a dimensionless boundary-value problem but rather than introducing new variables, we stick to the variables that naturally appear in the problem as in Figure 2.



**Figure 2.** Dimensionless boundary –value problem for the pressure impulse of the wave impact on a floating breakwater.

Thus, since we require the pressure along the line  $x=0$  and  $x=B_2$ , hence for the pressure impulse to be continuous, at  $x=0$ , along the impact region, the breakwater and the boundary between Region 1 and 2, we need the following equations :

$$\frac{\partial P_1}{\partial x} = \begin{cases} -1 & , -\mu \leq y \leq 0 \\ 0 & , -H_1 \leq y \leq -\mu \\ \frac{\partial P_2}{\partial x} & , -1 \leq y \leq -H_1 \end{cases} \quad (2)$$

and

$$P_1 = P_2 \quad \text{for } -1 \leq y \leq -H_1 \quad (3)$$

While along  $x = B_2$ , along the breakwater and the boundary between Region 2 and 3, we require the following :

$$\frac{\partial P_3}{\partial x} = \begin{cases} 0 & , -H_1 \leq y \leq 0 \\ \frac{\partial P_2}{\partial x} & , -1 \leq y \leq -H_1 \end{cases} \quad (4)$$

and

$$P_2 = P_3 \quad \text{for } -1 \leq y \leq -H_1 \quad (5)$$

### 3. Results and discussions

Cooker's modified model has the solution of Laplace's equation in the Fourier series form for the boundary-value problem which use hyperbolic terms as in equation (6).

$$P(x, y; \mu) = \sum_{n=0}^{\infty} a_n \sin\left(\frac{\lambda_n y}{H}\right) \frac{\sinh\left(\frac{\lambda_n (x - B)}{H}\right)}{\cosh\left(\frac{\lambda_n B}{H}\right)} \quad (6)$$

$$\text{with } a_n = \int_{-\mu H}^0 \frac{-2\rho U}{\lambda_n} \sin\left(\frac{\lambda_n y}{H}\right) \text{ and } \lambda_n = \left(n + \frac{1}{2}\right)\pi$$

Using the solution in equation (6), the solutions for the boundary-value problems of Figure 2 denoted as  $P_1$  for Region 1,  $P_2$  for Region 2,  $P_3$  for Region 3 are found to be given by the following eigenfunctions as in equation (7), equation (8) and equation (9) respectively.

$$P_1(x, y; \mu) = \sum_{n=1}^{\infty} a_n \frac{\sin\left(\frac{\lambda_n y}{H_2}\right)}{\cosh\left(\frac{\lambda_n B_1}{H_2}\right)} \sinh\left(\frac{\lambda_n (x - B_1)}{H_2}\right) \quad (7)$$

$$P_2(x, y; \mu) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{\lambda_n (2y + H_1 + H_2)}{H_2 - H_1}\right) \sinh\left(\frac{2\lambda_n x}{H_2 - H_1}\right) + \sum_{n=1}^{\infty} c_n \cos\left(\frac{\gamma_n (2y + H_1 + H_2)}{H_2 - H_1}\right) \sinh\left(\frac{2\gamma_n x}{H_2 - H_1}\right) + \frac{b_0 x}{H_2 - H_1} \quad (8)$$

$$P_3(x, y; \mu) = \sum_{n=1}^{\infty} d_n \frac{\sin\left(\frac{\lambda_n y}{H_2}\right)}{\cosh\left(\frac{\lambda_n B_3}{H_2}\right)} \sinh\left(\frac{\lambda_n (x - B_3)}{H_2}\right) \quad (9)$$

where  $\mu$  is the dimensionless constant to indicate how much of a wall is hit and is taken to be within  $0 \leq \mu \leq 1$ .  $H_2$  is the total water depth at time of impact, from seabed to the top of the wave with  $H_2 > 0$ .  $B_2$  is the width of the breakwater,  $B_1$  and  $B_3$  are the distances along the seabed from the breakwater. To be noted that  $\lambda_n = \left(n + \frac{1}{2}\right)\pi$  and  $\gamma_n = n\pi$ .

After taking the derivatives partially up to second order of the solutions, the divergence of equations (7), (8) and (9) gives :

$$\nabla^2 P_1 = 0 \quad (10)$$

$$\nabla^2 P_2 = 0 \quad (11)$$

and

$$\nabla^2 P_3 = 0 \quad (12)$$

The equations (10), (11) and (12) shows the result that all the equations (7), (8) and (9) satisfy all the boundary problems shown in Figure 2 except on to the impact region of the breakwater and the matching lines at  $x=0$  and  $x=B_2$ . In order to get the results, equations (7), (8) and (9) will be transformed to a matrix system of equations and will be solved numerically by using MATLAB. This will be done in our future works.

### Acknowledgements

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