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RESEARCH ARTICLE

Aligned magnetic field on dusty Casson fluid over a stretching sheet with Newtonian heating

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Abstract

Boundary layer flow and heat transfer on Casson fluid with dust particle over a stretching sheet is numerically investigated. The influences of aligned magnetic field together with Newtonian heating are considered in this problem. The governing equations are first transformed into ordinary differential equations using the appropriate similarity transformation variables. The numerical computation using Runge-Kutta Fehlberg (RKF45) method is employed to generate the results. Several physical parameters for both phases (fluid and particle) such as aligned angle, magnetic field parameter, Casson parameter, fluid particle interaction parameter, Prandtl number and conjugate parameter are investigated and analysed. The results in term of distribution velocity and temperature are presented graphically. The finding revealed that a rise in aligned angle and magnetic field parameter led to decrease the velocity profile and increase the temperature profile for both phases.

***Keywords*:** Aligned magnetic field, two-phase flow, Casson fluid, stretching sheet, Newtonian Heating

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INTRODUCTION

The multiphase flow is the interaction of several phase (solid, liquid and gas). The flow of solid-liquid is one of the simplest multiphase flow which is normally encountered in the flow of mud, flow of corpuscles in plasma and sedimentation. The two-phase flow is studied in such a way that the governing equations is separate accordingly. The propulsion and combustion in rockets and transport of powder materials suspension in pipe are some of engineering applications of fluid flow with particle suspension (Datta and Mishra, 1982). Gopinath, 2008 revealed the flow in magnetohydodynamic (MHD) generator is affected by the aggregation of slag and seeded particle in the channel.

The growth of particle such as ash or soot on the wall of certain engineering machines have led to a reforming the interest in the study of fluid flow embeded with particle suspension. The concentration of dust and a relaxation time of dust particle are the major influence of dust particle on gases which has pionered discussed by (Saffman, 1962). The boundary layer theory of dusty gas past an infinite plate was investigated by (Chakrabarti, 1974). The hydrodyamic of dusty fluid flow over a stretching with heat transfer and variable fluid properties effect was carried out by (Vajravelu *et al.*, 2013). In additional to the work of (Ramesh *et al.*, 2015) on the convective surface of dusty fluid over a stretching sheet, (Isa *et al.*, 2016) has taking challenges to study the stagnation point flow by considering the magnetic field effect towards the fluid. Meanwhile, the mixture of water and oil with metal particles on the natural convection flow over a heated vertical surface has been conducted by (Siddiqa *et al.*, 2015).

Recent development on the industrial applications have heightened the need for non-Newtonian fluid due to some restriction in properties of Newtonian fluid. Processing of plastic foam and food stuff, production of plastic sheets and extrusion of molten polymers are several applications of non-Newtonian fluid flow. A number of studies have attempted to develop the non-Newtonian fluid model with various of physical effect (Aurangzaib *et al.*, 2013). One of the non-Newtonian fluid model that possess the interactive behaviour of solid and liquid phases is recognized as Casson fluid. The human blood contained protein, fibrinogen and globulin in an aqueous base plasma can be clasify as Casson fluid (Mukhopadhyay *et al.*, 2013). A more recent analysis in the influence of aligned magnetic field on the boundary layer flow of Casson fluid and viscous fluid over a stretching sheet has been highlighted as in (Hakeem *et al.*, 2016; Kalaivanan *et al.*, 2015; Nur Syamilah *et al.*, 2016). In another study, (Sandeep *et al.*, 2015) examined the mixture of conducting dust particle in a nanofluids (dusty nanofluid) with the present of magnetic field over a permeable stretching/shrinking surface.

The fluid filled with the dust particle has been one of the major interesting research subject due to the various applications, therefore this paper attempts to provide a more detailed investigation regarding the effects of dust particle in a Casson fluid flow over a stretching sheet. The aligned magnetic field with thermal boundary condition, Newtonian heating (NH) is also considered. The governing equations of two-phase model are first transformed into ordinary differential equations using a similarity transformation before numerical computation can be carried out . The influence of several physical

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parameters on velocity as well as temperature of fluid are analysed through table and graph.

PROBLEM FORMULATION

The steady, incompressible two dimensional boundary layer flow of dusty Casson fluid over a stretching sheet with a stretching linear velocity $u_w(x) = ax$ is considered. An aligned magnetic field with an acute angle α_1 is applied to the flow as illustrates in Figure 1. The assumptions where dust particles are in spherical shape, uniform size and number density are taken as constant throughout the flow is deliberated.

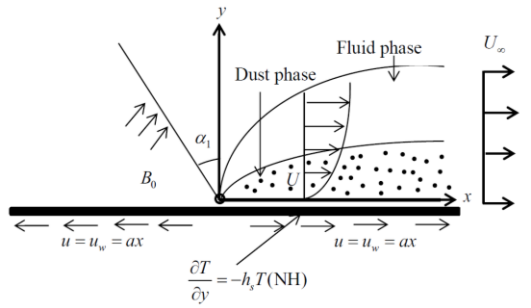

Fig. 1 Flow configuration.

Under the boundary layer and Boussinesq approximations, the momentum and energy equations for two-phase flow can be written as:

Fluid phase:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\left(1+\frac{1}{A}\right)\left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{\rho_p}{\tau_v}(u_p - u) - \sigma u {B_0}^2 \sin^2\alpha_1, \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{\rho_p c_s}{\gamma_T}(T_p - T), \tag{3}$$

Dust phase:

$$\frac{\partial}{\partial x}(u_p) + \frac{\partial}{\partial y}(v_p) = 0 \tag{4}$$

$$\rho_p\left(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \frac{\rho_p}{\tau_v}(u - u_p), \tag{5}$$

$$\rho_p c_s\left(u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right) = -\frac{\rho_p c_s}{\gamma_T}(T_p - T), \tag{6}$$

where (u,v) and (u_p, v_p) are the velocities components of the fluid and particle phase along x and y axes, respectively. μ is the coefficient of viscosity of the fluid, ρ and ρ_p are the density of fluid and dust phase, α_1 is the aligned angle, $\tau_v = 1/k$ is the relaxation time of particles phase, k is the Stoke's resistance (drag force), c_p and c_s are specific heat of fluid and dust particle, T and T_p are the temperature of fluid and particle phase, γ_T is the thermal relaxation time, B_0 is the magnetic-field strength, $A = \mu_B\sqrt{2\pi_c}\,/\,p_y$ is the non-Newtonian (Casson) parameter and h_s is heat transfer parameter.

Here, a is the positive constant and the boundary conditions for the present problem are

$$u = u_w(x) = ax,\ \ v = 0,\ \ \frac{\partial T}{\partial y} = -h_s T \quad \text{at } y = 0,$$

$$u \to 0,\ u_p \to 0,\ v_p \to v,\ T \to T_\infty,\ T_p \to T_\infty \quad \text{as } y \to 0 \tag{7}$$

The similarity transformations (8) have been introduced to the governing equations (1) to (6) to transform them into dimensionless equations,

$$\begin{aligned} &u = axf'(\eta),\ \ v = -\sqrt{av}\,f(\eta),\ \ \eta = \sqrt{\frac{a}{v}}y,\ \ \theta(\eta) = \frac{T - T_\infty}{T_\infty} \\ &u_p = axF'(\eta),\ \ v_p = -\sqrt{av}F(\eta),\ \ \theta_p(\eta) = \frac{T_p - T_\infty}{T_\infty}, \end{aligned} \tag{8}$$

where ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

Now, the transformed ordinary differential equations (1)-(6) can be expressed as

$$\left(1+\frac{1}{A}\right)f'''(\eta) + f(\eta)f''(\eta) - \left(f'(\eta)\right)^2 + \beta N\left(F'(\eta) - f'(\eta)\right) - M\sin^2\alpha_1 f'(\eta) = 0, \tag{9}$$

$$\theta''(\eta) + \Pr f(\eta)\theta'(\eta) + \frac{2}{3}\beta N\left(\theta_p(\eta) - \theta(\eta)\right) = 0, \tag{10}$$

$$\left(F'(\eta)\right)^2 - F(\eta)F''(\eta) + \beta\left(F'(\eta) - f'(\eta)\right)] = 0, \tag{11}$$

$$\theta_p'(\eta)F(\eta) + \frac{2}{3}\frac{\beta}{\Pr\lambda}\left(\theta(\eta) - \theta_p(\eta)\right) = 0 \tag{12}$$

and boundary conditions (7) reduces to

$$\begin{aligned} &f(0) = 0,\ \ f'(0) = 1,\ \ \theta'(0) = -\gamma\left(1+\theta(0)\right) \text{ as } \eta = 0 \\ &f'(\eta) \to 0, F(\eta) \to 0, F(\eta) \to f(\eta), \theta(\eta) \to 0, \theta_p(\eta) \to 0 \text{ as } \eta \to \infty \end{aligned} \tag{13}$$

where a prime denotes differentiation with respect to η. $N = \rho_p / \rho$ is the mass concentration of particle phase, $M = \sigma B_0^2 / \rho a$ is the magnetic field parameter, $\beta = 1/a\tau_v$ is the fluid-particle interaction parameter, $\Pr = \mu c_p / k$ is the Prandtl number, $\lambda = c_s / c_p$ is the specific heat ratio of mixture and $\gamma = -h_s\left(v/a\right)^{1/2}$ is the conjugate parameter for NH.

According to (Andersson *et al.*, 1992), the exact solution for Eq. (9) by considering only magnetic field is expressed as

$$f(\eta) = \frac{1}{m}\left(1 - e^{-m\eta}\right), \tag{14}$$

where $m = \sqrt{1+M^2}$. Using (14), then the exact expression for temperature profile is

$$\theta(\eta) = C_1\int_\eta^\infty e^{-\Pr\int_\eta^\infty f\,d\eta}\,d\eta, \tag{15}$$

where $C_1 = \dfrac{\gamma(1-\theta(0))}{e^{-\Pr\int_\eta^\infty f\,d\eta}}$. The non-dimensional quantities of physical interest of the the skin friction coefficient C_f and the local Nusselt number Nu_x are defined by

$$C_f\,\mathrm{Re}_x^{1/2} = f''(0),\ \ Nu_x\,\mathrm{Re}_x^{-1/2} = \gamma\left(1+\frac{1}{\theta(0)}\right), \tag{16}$$

where $\mathrm{Re}_x = (ax^2/v)$ is the Reynolds number.

NUMERICAL PROCEDURE

The analytical solutions is difficult to obtained for the complexity of the present governing equations. For that, the numerical solution of Runge-Kutta Fehlberg (RKF45) method on Maple software is employed to solve the system of ordinary equations (9) - (12) with boundary conditions (13) in order to evaluate the flow properties and

heat transfer for both fluid and dust phases. The maximum finite boundary layer thickness $\eta_\infty = 5$ and 8 is computed to satisfy the boundary conditions asymptotically for both phases.

RESULTS AND DISCUSSION

The governing equations of dusty Casson fluid model comprises of two part which are fluid and dust phases. The flow and heat transfer of the present problem are analysed graphically through several physical parameters, namely, particle-fluid interaction parameter β, Casson parameter A, aligned angle α_1, magnetic field parameter M, Prandtl number $\Pr$ and conjugate parameter, γ. The values of skin friction coefficeint and Nusselt number are also tabulated in Table 3. To determine the correctness of the present results, comparison of skin friction coefficient $f''(0)$ and surface temperature $\theta(0)$ with existed work are made for some limitating cases in the absence of fluid particle interaction parameter and Casson parameter (see Table 1 and 2). It was found, the analytic solutions of (Andersson *et al.*, 1992) and Eq. (15) as well as numerical solution presented in (Salleh *et al.*, 2010) was consistent with the present findings. The numerical results of velocity $(f'(\eta), F'(\eta))$ and temperature profiles $(\theta(\eta), \theta_p(\eta))$ for fluid and dust phases, respectively are computed for several fixed pertinent parameter which are $\alpha_1 = \pi/6, M = 2, \beta = 1, \gamma = 0.5, A = 1, N = 12.6, \Pr = 16.7$ and $\lambda = 0.25$

Fig. 2 and Fig. 3 show the effect of Casson parameter, A, for the fluid flow with and without dust particle. It should be noted that, the present problem turns to the those classical problem of single phase flow at $\beta = 0$. From Fig. 2, the decreasing pattern of fluid velocity is observed with increasing on values of A, for both Casson $(\beta = 0)$ and dusty Casson fluid $(\beta = 1)$. Also, the fluid velocity is promptly achieve the asymptotic behaviour when $\beta = 1$ compared to $\beta = 0$. It is worth to mention that the flow of fluid tend to decelerate in the present of dust particles. In Fig. 3, the decreasing trend is noticable for both phases as A increase. Physically, fluid begins to behave like a rigid body for bigger value of A since the plastic dynamic viscosity of the fluid (μ_B) is also higher at which there are high tendency of flow to decelerate. Fig. 4 and Fig. 5 illustrate the effects of magnetic field, M and aligned angle, α_1 on the velocity profile respectively. It is noticed from both figures that the increasing M and α_1 reduces the velocity profile for fluid and dust phases. The increasing aligned angle enhances the development of magnetic field which in turns generate the Lorentz force that act opposite to the flow direction and slower the mobility of the fluid flow. One of the most significant findings to emerge from this study is that the aligned angle determines the position of magnetic field effect as α_1 can be varies from $0° - 90°$ and if $\alpha_1 = 0$ reflect to the absence of magnetic field effect. Meanwhile, at $\alpha_1 = \pi/2$ the present problem is identical to the transverse magnetic field flow. The velocity of fluid literally causes the dust phase velocity, one can observed that both phases decreases with increasing value of M and α_1.

Fig. 6 and Fig. 7 demonstrate the effect of fluid-particle interaction parameter, β on the velocity and temperature profiles for both phases. The decreasing trend is observed for fluid phase and dust phase showed increasing trend as β increases. Theoretically, increasing in β reduces the relaxation time of particle velocity and enhances the drag force (Stokes's law). In the physical of view, for a large value of β causes the growth of the drag force generated by dust particle that behaves opposite to the moving fluid. Actually, this forces continously decrease the velocity of fluid so that both velocities of dust and fluid phase become equlibrium. The dust particle is the source of absorbing the heat from the fluid, thus the decreasing temperature profile of fluid phase is expected.

Fig. 8 and Fig. 9 captured the effects of conjugate parameter, γ and Prandtl number, $\Pr$ on temperature profile for both phases. A rise in γ increases the heat transfer strength which led to enhance the capability to transfer the heat which resulted to the increases in temperature of fluid. Meanwhile, increasing on values of $\Pr$ decreases the temperature for both phases due to the fact that larger $\Pr$ imply to the lower thermal diffusivity and thus weaken the thermal boundary layer thickness. From the mathematical understanding as denoted by Eqs. (10) - (13), both parameters have no significant effect on flow properties and those parameters can also be used as the control agent in rate of cooling for certain devices.

Table 3 provides the values of skin friction and Nusselt number for different values of magnetic parameter, aligned angle and Casson parameter. The increasing magnitude of skin friction coefficient and decreasing of Nusselt number has been perceived in increasing those parameters. The results presented in Table 3 is reserved for the reference for future study.

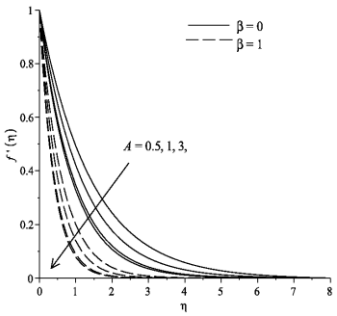

Fig. 2 Influence of A for velocity profile with and without dust particle.

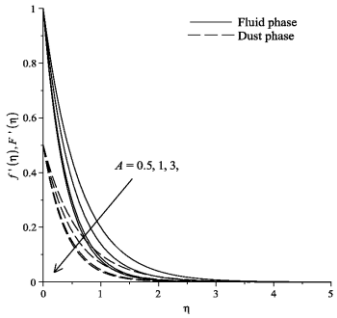

Fig. 3 Influence of A on velocity profile.

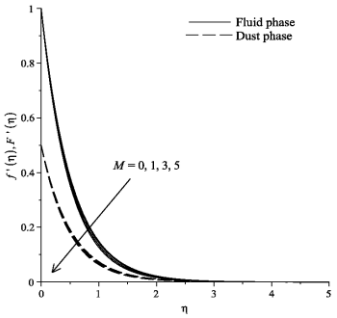

Fig. 4 Influence of M on velocity profile.

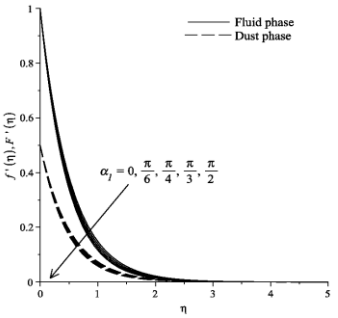

Fig. 5 Influence of α_1 on velocity profile.

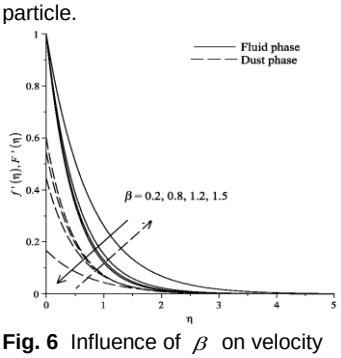

Fig. 6 Influence of β on velocity profile.

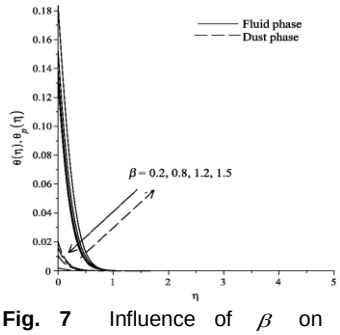

Fig. 7 Influence of β on temperature profile.

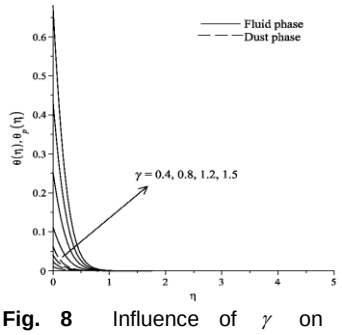

Fig. 8 Influence of γ on temperature profile

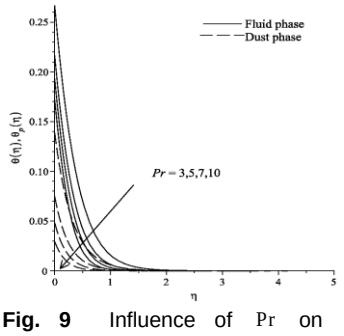

Fig. 9 Influence of $\Pr$ on temperature profile.

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Table 1 Comparision results for $f''(0)$, when $\beta = N = 0, A, \lambda \to \infty$.

M	(Andersson *et al.*, 1992)	Present
0.0	-1.0000	-1.0000
0.2	-1.0954	-1.0954
0.5	-1.2247	-1.2247
1.0	-1.4142	-1.4141
1.2	-1.4832	-1.4832
1.5	-1.5811	-1.5811
2.0	-1.7321	-1.7320

Table 2 Comparision results for $\theta(0)$, when $M = \beta = N = 0, \lambda \to \infty$ and $\gamma = 1$.

Pr	(Salleh *et al.*, 2010)	Exact Eq. (15)	Present
3	6.0258	6.0516	6.0513
5	1.7659	1.7604	1.7604
7	1.1351	1.1168	1.1168
10	0.7653	0.7645	0.7645
100	0.1612	0.14781	0.1478

Table 3 Computation of $f''(0)$ and $Nu_x \mathrm{Re}_x^{-1/2}$ for various values of M, α_1 and A .

M	α_1	A	$f''(0)$	$Nu_x \mathrm{Re}_x^{-1/2}$
0.5	$\pi/6$	1	-1.75006	2.64540
1	$\pi/6$	1	-1.76781	2.64037
1.5	$\pi/6$	1	-1.78539	2.63541
2	$\pi/6$	1	-1.80280	2.63050
2	$\pi/6$	1	-1.80280	2.63050
2	$\pi/4$	1	-1.87084	2.61145
2	$\pi/3$	1	-1.93650	2.59325
2	$\pi/2$	1	-2.00000	2.57582
2	$\pi/6$	1	-1.80280	2.63050
2	$\pi/6$	3	-2.20794	2.51980
2	$\pi/6$	6	-2.36039	2.47975
2	$\pi/6$	10	-2.43087	2.46151

CONCLUSION

The present study was investigate to determine the effect of dust particle suspension in a fluid. The pigmentation and accumulation of dust particle may obstruct the performance of the certain machine and the numerical analyses might help the engineers to overcome this problem. The velocity of fluid and dust phase is decrease with increasing magnetic field parameter, Casson parameter and aligned angle. For fluid phase, the velocity and temperature is decreased in the increasing of fluid particle interaction parameter. In contrary, the opposite trend is observed for dust phase. The influence of increasing the values of magnetic field parameter, aligned angle and Casson parameter are to increase the magnitude of skin friction and decrease the Nusselt number. The bigger on skin friction coefficient produces more drag force along the sheet and become one of the factors that influence the wall design of certain device.

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