# Non-collocated Fuzzy Logic and Input Shaping Control Strategy for Elastic Joint Manipulator: Vibration Suppression and Time Response Analysis

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*Abstract-* **Conventional model-based control strategies are very complex and difficult to synthesize due to high complexity of the dynamics of robots manipulator considering joint elasticity. This paper presents investigations into the development of hybrid control schemes for trajectory tracking and vibration control of a flexible joint manipulator. To study the effectiveness of the controllers, initially a collocated proportional-derivative (PD)-type Fuzzy Logic Controller (FLC) is developed for tip angular position control of a flexible joint manipulator. This is then extended to incorporate a non-collocated Fuzzy Logic Controller and input shaping scheme for vibration reduction of the flexible joint system. The positive zero-vibration-derivative-derivative (ZVDD) shaper is designed based on the properties of the system. Simulation results of the response of the flexible joint manipulator with the controllers are presented in time and frequency domains. The performances of the hybrid control schemes are examined in terms of input tracking capability, level of vibration reduction and time response specifications. Finally, a comparative assessment of the control techniques is presented and discussed.**

*Keywords-Elastic joint, vibration control, input shaping, fuzzy logic control*

## I. INTRODUCTION

Nowadays, elastic joint manipulators have received a thorough attention due to their light weight, high manoeuvrability, flexibility, high power efficiency, and large number of applications. However, controlling such systems still faces numerous challenges that need to be addressed. The control issue of the flexible joint is to design the controller so that link of robot can track a prescribed trajectory precisely with minimum vibration to the link. In order to achieve these objectives, various methods using different technique have been proposed. Yim [1], Oh and Lee [2] proposed adaptive output-feedback controller based on a backstepping design. This technique is proposed to deal with parametric uncertainty in flexible joint. The relevant work also been done by Ghorbel et al. [3]. Lin and Yuan [4] and Spong et al. [5] introduced non linear control approach using namely feedback linearization technique and the integral manifold technique respectively. A robust control design was reported by Tomei [6] by using simple PD control and Yeon and Park [7] by applying robust $H_\infty$ control. Among the proposed techniques, the conventional feedback control design handled by pole placement method and LQR method also have been widely used due to its simplicity implementation. Particularly in LQR method, the values of $Q$ and $R$ matrices are pre-specified to determine optimal feedback control gain via Riccati equation [8].

On another aspect, an acceptable system performance with reduced vibration that accounts for system changes can be achieved by developing a hybrid control scheme that caters for rigid body motion and vibration of the system independently. This can be realized by utilizing control strategies consisting of either non-collocated with collocated feedback controllers or feed-forward with feedback controllers. In both cases, the former can be used for vibration suppression and the latter for input tracking of a flexible manipulator. A hybrid collocated and non-collocated controller has widely been proposed for control of a flexible structure [9, 10, 11]. The works have shown that the control structure gives a satisfactory system response with significant vibration reduction as compared to a response with a collocated controller. A feedback control with a feed-forward control to regulate the position of a flexible structure has previously been proposed [12, 13]. A control law partitioning scheme which uses end-point sensing device has also been reported [14]. The scheme uses end-point position signal in an outer loop controller to control the flexible modes, whereas the inner loop controls the rigid body motion independent of the flexible dynamics of the manipulator.

This paper presents investigation into the development of control schemes for trajectory tracking of tip angular position and vibration control of flexible joint manipulator. Control strategies based on collocated PD-type FLC with non-collocated Fuzzy and the combination of collocated PD-type FLC with input shaping are investigated. For non-collocated control, a deflection angle feedback through a fuzzy logic control configuration whereas input shaping is utilised as a feed-forward scheme for reducing a deflection effect. A simulation environment is developed within Simulink and Matlab for evaluation of performance of the control schemes. The rest of the paper is structured as follows: Section 2 provides a brief description of the single-link flexible joint manipulator system considered in this study. Section 3 describes the modelling of the system derived using Euler-Lagrange formulation. The composite collocated PD-type FLC with Fuzzy Logic Control and input shaping scheme are described in Section 4. Simulation results and comparative assessment are presented in Section 5 and the paper is concluded in Section 6.

## II. THE FLEXIBLE JOINT MANIPULATOR SYSTEM

The flexible joint manipulator system considered in this work is shown in Fig. 1, where $\theta$, is the tip angular position and $\alpha$ is the deflection angle of the flexible joint. The base of the flexible joint manipulator which determines the tip angular position of the flexible link is driven by servomotor, while the flexible link will response based on base movement. The deflection of link will be determined by the flexibility of the spring as their intrinsic physical characteristics.
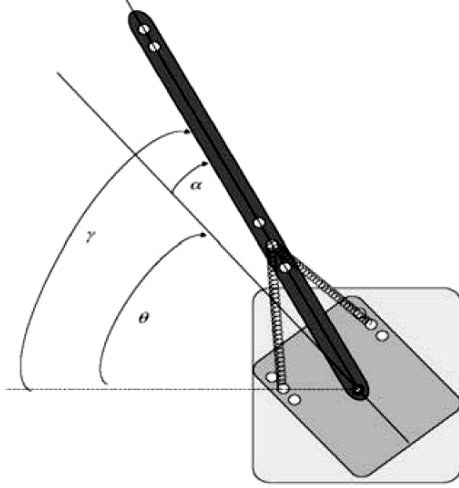
Fig.1. Description of the flexible joint manipulator system.

| Symbol | Quantity | Value |
|---|---|---|
| $R_m$ | Armature Resistance (Ohm) | 2.6 |
| $K_m$ | Motor Back-EMF Constant (V.s/rad) | 0.00767 |
| $K_t$ | Motor Torque Constant (N.m/A) | 0.00767 |
| $J_{link}$ | Total Arm Inertia (kg.m$^2$) | 0.0035 |
| $J_{eq}$ | Equivalent Inertia (kg.m$^2$) | 0.0026 |
| $K_g$ | High gear ratio | 14:5 |
| $K_{stiff}$ | Joint Stiffness | 1.2485 |
| $B_{eq}$ | Equivalent Viscous Damping (N.m.s/rad) | 0.004 |
| $\eta_g$ | Gearbox Efficiency | 0.9 |
| $\eta_m$ | Motor Efficiency | 0.69 |

## III. MODELLING OF THE FLEXIBLE JOINT MANIPULATOR

This section provides a brief description on the modelling of the flexible joint manipulator system, as a basis of a simulation environment for development and assessment of the hybrid Fuzzy Logic control techniques. The Euler-Lagrange formulation is considered in characterizing the dynamic behaviour of the system.

The linear model of the uncontrolled system can be represented in a state-space form [15] as shown in equation (1), that is

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{1}$$

with the vector $x = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix}^T$ and the matrices $A$, $B$ and $C$ are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{K_{stiff}}{J_{eq}} & \dfrac{-\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \\ 0 & \dfrac{-K_{stiff}(J_{eq} + J_{arm})}{J_{eq} J_{arm}} & \dfrac{\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & \dfrac{\eta_m \eta_g K_t K_g}{J_{eq} R_m} & \dfrac{-\eta_m \eta_g K_t K_g}{J_{eq} R_m} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

In equation (1), the input $u$ is the input voltage of the servomotor, $V_m$ which determines the flexible joint manipulator base movement. In this study, the values of the parameters are defined in Table 1.

## IV. CONTROL ALGORITHM

In this section, control schemes for rigid body motion control and vibration suppression of a flexible joint manipulator are proposed. Initially, a collocated PD-type FLC controller is designed. Then a non-collocated fuzzy logic control and input shaping scheme are incorporated in the closed-loop system for control of vibration of the system.

### A. PD-type fuzzy logic controller (PD-FLC)

A PD-type fuzzy logic controller (FLC) utilizing tip angular position error and derivative of tip angular position error is developed to control the rigid body motion of the system. The hybrid fuzzy control system proposed in this work is shown in Fig. 2, where $r(t)$, $\theta(t)$ and $\alpha(t)$ are the desired angle, tip angular position and deflection angle of the flexible joint manipulator, whereas $k_1$, $k_2$ and $k_3$ are scaling factors for two inputs and one output of the fuzzy logic controller used with the normalised universe of discourse for the fuzzy membership functions.
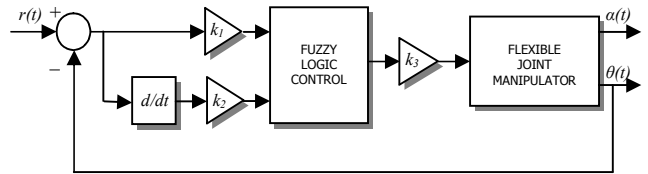


Fig. 2 PD-type fuzzy logic control structure.

For PD-type FLC, triangular membership functions are chosen for tip angle error, tip angle error rate, and input voltage with 50% overlap. Normalized universes of discourse are used for both tip angle error and its error rate and input voltage. Scaling factors $k_1$ and $k_2$ are chosen in such a way as to convert the two inputs within the universe of discourse and activate the rule base effectively, whereas $k_3$ is selected such that it activates the system to generate the desired output. Initially all these scaling factors are chosen based on trial and error. To construct a rule base, the tip angle error, tip angle error rate, and input voltage are partitioned into five primary fuzzy sets as

Tip angle error $E$ = {NM NS ZE PS PM},
Tip angle error rate $V$ = {NM NS ZE PS PM},
Voltage $U$ = {NM NS ZE PS PM},

where $E$, $V$, and $U$ are the universes of discourse for tip angle error, tip angel error rate and input voltage, respectively. The $n$th rule of the rule base for the FLC, with angle error and angle error rate as inputs, is given by

$R_n$: IF( $e$ is $E_i$) AND ( $\dot{e}$ is $V_j$) THEN ($u$ is $U_k$),

where , $R_n$, $n$=1, 2,…$N_{max}$, is the $n$th fuzzy rule, $E_i$, $V_j$, and $U_k$, for $i, j, k$ = 1,2,…,5, are the primary fuzzy sets.

A PD-type FLC was designed with 11 rules as a closed loop component of the control strategy for maintaining the angular position of flexible joint manipulator. The rule base was extracted based on underdamped system response and is shown in Table 2. The three scaling factors, $k_1$, $k_2$ and $k_3$ were chosen heuristically to achieve a satisfactory set of time domain parameters. These values were recorded as, $k_1$ = 0.552, $k_2$ = 0.073 and $k_3$ = -985.

TABLE II
LINGUISTIC RULES OF THE PD-TYPE FUZZY LOGIC CONTROLLER

| No. | Rules |
|---|---|
| 1. | If ( $e$ is NM) and ( $\dot{e}$ is ZE) then ($u$ is PM) |
| 2. | If ( $e$ is NS) and ( $\dot{e}$ is ZE) then ($u$ is PS) |
| 3. | If ( $e$ is NS) and ( $\dot{e}$ is PS) then ($u$ is ZE) |
| 4. | If ( $e$ is ZE) and ( $\dot{e}$ is NM) then ($u$ is PM) |
| 5. | If ( $e$ is ZE) and ( $\dot{e}$ is NS) then ($u$ is PS) |
| 6. | If ( $e$ is ZE) and ( $\dot{e}$ is ZE) then ($u$ is ZE) |
| 7. | If ( $e$ is ZE) and ( $\dot{e}$ is PS) then ($u$ is NS) |
| 8. | If ( $e$ is ZE) and ( $\dot{e}$ is PM) then ($u$ is NM) |
| 9. | If ( $e$ is PS) and ( $\dot{e}$ is NS) then ($u$ is ZE) |
| 10. | If ( $e$ is PS) and ( $\dot{e}$ is ZE) then ($u$ is NS) |
| 11. | If ( $e$ is PM) and ( $\dot{e}$ is ZE) then ($u$ is NM) |

*B. PD-type Fuzzy Logic with non-collocated Fuzzy Logic Controller (PD-FLC-FLC)*

A combination of PD-type fuzzy logic and non-collocated fuzzy logic control scheme for control of tip angular position and vibration suppression of the system respectively is presented in this section. The use of a non-collocated control system, where the deflection angle of the flexible joint manipulator is controlled by measuring its angle, can be applied to improve the overall performance, as more reliable output measurement is obtained. The control structure comprises two feedback loops: (1) The hub angle as input to PD-type FLC for tip angular position control. (2) The deflection angle as input to a separate non-collocated control law for vibration control. These two loops are then summed together to give a torque input to the system. A block diagram of the composite fuzzy logic control scheme is shown in Fig. 3.
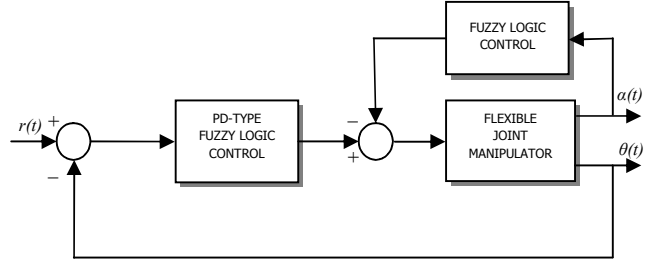


Fig. 3 PD-type FLC with non-collocated fuzzy logic control structure.

For tip angular position control, the PD-type FLC strategy developed in the previous section is adopted whereas for the vibration control loop, the deflection angle feedback through a non-collocated fuzzy logic control scheme is utilised. In designing the non-collocated fuzzy logic control, a basic triangle and trapezoidal forms are chosen for input and output membership functions. Fig. 4 shows the membership functions of the fuzzy logic controller for vibration control. It consists of Negative Big (NB), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Big (PB) as shown in the diagram. The universes of discourses of deflection angle, deflection angle rate and input voltage are from 1.2 to -1.2 rad, -0.015 to 0.015 rad/s and -447 to 447 V respectively.
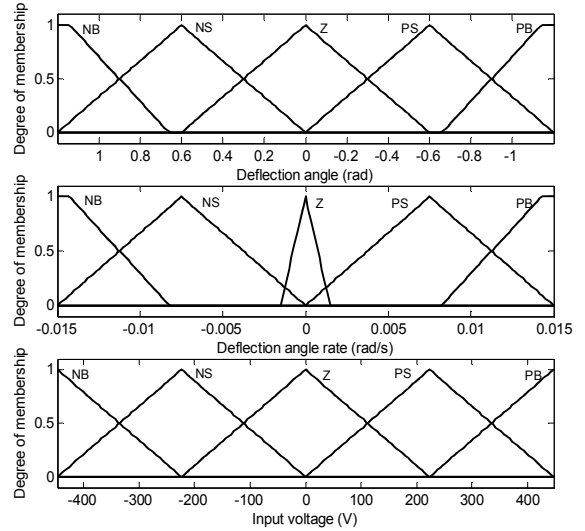


Fig. 4 Membership functions of inputs and output signals.

Table 3 lists the generated linguistic rules for vibration control. The rules are designed based on the condition of the deflection angle and the deflection angle rate as illustrated in Fig. 5. Consider the joint of the manipulator rotates to anti-clockwise direction and the link deflects on clockwise direction. As illustrated in Fig. 5(a), at this condition intuitively the torque should be applied to clockwise direction in order to compensate the deflection. In this case the relation between input voltage and torque per inertia is shown in equation (3),

$$\tau = \frac{V_m \eta_m \eta_g K_t K_g}{R_m} \qquad (3)$$

Meanwhile, if the joint rotates to clockwise direction as shown in Fig. 5(b) and the link deflects to anti-clockwise direction, the torque should be imposed to anti-clockwise direction to suppress the deflection motion. In the case there is no deflection, no torque should be applied. Furthermore, the proposed fuzzy logic control adopts well-known Mamdani min-max inference and centre of area (COA) methods.
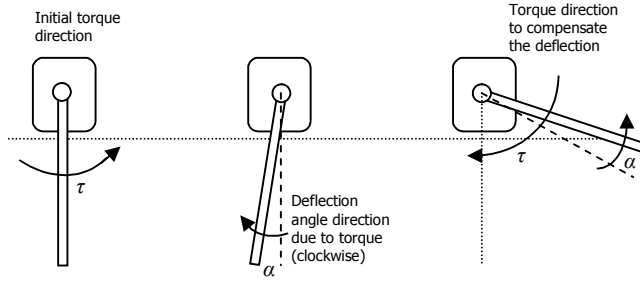
(a) Direction of torque to reduce deflection angle when link moves anti-clockwise
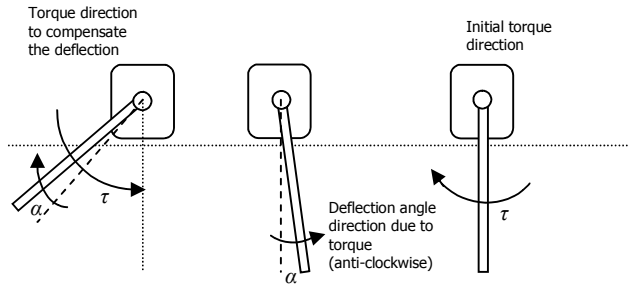
(b) Direction of torque to reduce deflection angle when link moves clockwise

Fig. 5 Rules generation based on the motion condition.

TABLE III
FUZZY RULES FOR VIBRATION CONTROL

| Deflection angle | Deflection angle rate | $\dot{\alpha}$ | | | | |
|---|---|---|---|---|---|---|
| | | PB | PS | Z | NS | NB |
| $\alpha$ | PB | PB | PB | PB | NB | NB |
| | PS | PB | PS | PS | NS | NB |
| | Z | PB | PS | Z | NS | NB |
| | NS | PB | PS | NS | NS | NB |
| | NB | PB | PB | NB | NB | NB |

### C. PD-type Fuzzy Logic with input shaping (PD-FLC-IS)

A control structure for control of rigid body motion and deflection angle reduction of the flexible joint manipulator based on PD-type FLC and input shaping scheme is proposed in this section. The positive input shapers are proposed and designed based on the properties of the system. In this study, the input shaping control scheme is developed using a Zero-Vibration-Derivative-Derivative (ZVDD) input shaping technique [13]. Previous experimental study with a flexible manipulator has shown that significant vibration reduction and robustness is achieved using a ZVDD technique [16]. A block diagram of the PD-type FLC with input shaping control technique is shown in Fig. 6.

The input shaping method involves convolving a desired command with a sequence of impulses known as input shaper. The design objectives are to determine the amplitude and time location of the impulses based on the natural frequencies and damping ratios of the system. The positive input shapers have been used in most input shaping schemes. The requirement of positive amplitude for the impulses is to avoid the problem of large amplitude impulses. In this case, each individual impulse must be less than one to satisfy the unity magnitude constraint. In addition, the robustness of the input shaper to errors in natural frequencies of the system can be increased by solving the derivatives of the system vibration equation. This yields a positive ZVDD shaper with parameter as

$$t_1 = 0, \; t_2 = \frac{\pi}{\omega_d}, \; t_3 = \frac{2\pi}{\omega_d}, \; t_4 = \frac{3\pi}{\omega_d}$$

$$A_1 = \frac{1}{1 + 3H + 3H^2 + H^3}, \; A_2 = \frac{3H}{1 + 3H + 3H^2 + H^3}$$

$$A_3 = \frac{3H^2}{1 + 3H + 3H^2 + H^3}, \; A_4 = \frac{H^3}{1 + 3H + 3H^2 + H^3} \qquad (4)$$

where

$$H = e^{-\zeta\pi / \sqrt{1-\zeta^2}}, \; \omega_d = \omega_n \sqrt{1-\zeta^2}$$

$\omega_n$ and $\zeta$ representing the natural frequency and damping ratio respectively. For the impulses, $t_j$ and $A_j$ are the time location and amplitude of impulse $j$ respectively.
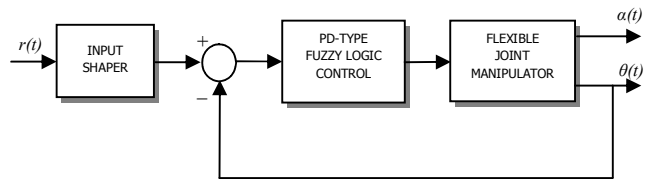
Fig. 6 PD-type FLC with input shaping control structure.

## V. IMPLEMENTATION AND RESULTS

In this section, the proposed control schemes are implemented and tested within the simulation environment of the flexible joint manipulator and the corresponding results are presented. The manipulator is required to follow a trajectory of 50º. System responses namely the tip angular position and deflection angle are observed. To investigate the vibration of the system in the frequency domain, power spectral density (PSD) of the deflection angle response is obtained. The performances of the control schemes are assessed in terms of vibration suppression, trajectory tracking and time response specifications. Finally, a comparative assessment of the performance of the control schemes is presented and discussed.

Figs. 7-9 show the responses of the flexible joint manipulator to the reference input trajectory using PD-FLC in time-domain and frequency domain (PSD). These results were considered as the system response under rigid body motion control and will be used to evaluate the performance of the non-collocated fuzzy logic control and input shaping scheme. The steady-state tip angular trajectory of 50º for the flexible joint manipulator was achieved within the rise and settling times and overshoot of 0.222 s, 0.565 s and 1.78 % respectively. It is noted that the manipulator reaches the required position within 1 s, with little overshoot. However, a noticeable amount of vibration occurs during movement of the manipulator. It is noted from the deflection angle response that the vibration of the system settles within 3 s with a maximum residual of $\pm 15$º. Moreover, from the PSD of the deflection angle response the vibrations at the flexible joint are dominated by the first three vibration modes, which are obtained as 2.94 Hz, 8.83 Hz and 14.52 Hz with magnitude of 32.38 dB, -12.08 dB and -26.52 dB respectively.

The tip angular position, deflection angle and power spectral density responses of the flexible joint manipulator using PD-FLC-FLC and PD-FLC-IS are shown in Figs. 7-9 respectively. It is noted that the proposed control schemes are capable of reducing the system vibration while maintaining the trajectory tracking performance of the manipulator. Similar tip angular position, deflection angle and power spectral density of deflection angle responses were observed as compared to the PD-FLC. With PD-FLC-FLC, the steady-state tip angular trajectory of 50º for the flexible joint manipulator was achieved within the rise and settling times and overshoot of 0.215 s, 0.613 s and 9.68 % respectively. While, with PD-FLC-IS, the manipulator reached the rise, settling times and overshoot of 0.469 s, 0.857 s and 0.16 % respectively. It is noted that, the settling time of PD-FLC-FLC is much faster as compared to the case of PD-FLC-IS. However, the percentage overshoot of PD-FLC-IS is much lower than the case of PD-FLC-FLC. Besides, the results also demonstrate a significant amount of deflection angle reduction at the tip angle of the manipulator with both controllers. The vibration of the system using PD-FLC-FLC settles within 1 s which is much faster as compared to PD-FLC-IS. However, in terms of maximum residual of deflection angle, the PD-FLC-IS and PD-FLC-FLC produces a four-fold and two-fold improvement respectively as compared

to PD-FLC scheme. Moreover, from the PSD of the deflection angle response, the magnitudes of vibrations using PD-FLC-FLC and PD-FLC-IS were reduced to [1.87, -41.95, -48.25] dB and [17.59, -25.28, -36.69] dB respectively for the first three modes of vibration.

Table 4 summarises the magnitude of vibration of deflection angle and specifications of tip angular position response for both control schemes. It is noted that high performance in the reduction of vibration of the system is achieved using PD-FLC-FLC as compared to PD-FLC-IS. This is observed and compared to the PD-FLC at the first three modes of vibration. In addition, as demonstrated in the tip angular trajectory response, a slightly faster response with higher overshoot is obtained using PD-FLC-FLC as compared to the PD-FLC-IS. Comparisons of the specifications of the tip angular trajectory responses are summarised in Table 4. Besides, as demonstrated in the tip angular trajectory response with PD-FLC-FLC, the minimum phase behaviour of the manipulator is unaffected. Nevertheless, the implementation of PD-FLC-IS is much easier as compared to PD-FLC-FLC as a large amount of design effort is required in order to determine the best range of membership functions parameters.
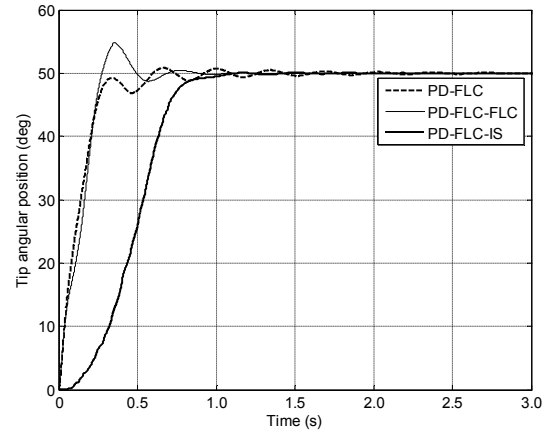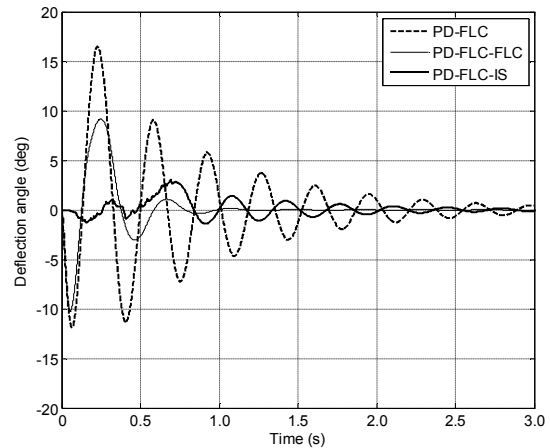


Fig. 7 Tip angular position response.



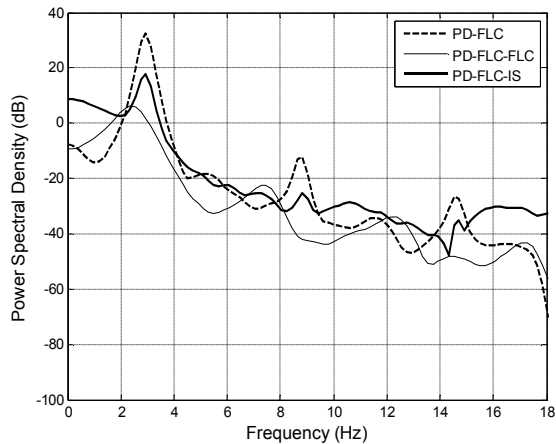Fig. 8 Deflection angle response.

Fig. 9 PSD of deflection angle.

TABLE IV
MAGNITUDE OF VIBRATION AND SPECIFICATIONS OF TIP ANGULAR POSITION

| Controller | | PD-FLC | PD-FLC-FLC | PF-FLC-IS |
|---|---|---|---|---|
| Magnitude of vibration (dB) | Mode 1 | 32.38 | 1.87 | 17.59 |
| | Mode 2 | -12.08 | -41.95 | -25.28 |
| | Mode 3 | -26.52 | -48.25 | -36.69 |
| Specifications of tip angular position response | Settling time (s) | 0.565 | 0.613 | 0.857 |
| | Rise time (s) | 0.222 | 0.215 | 0.469 |
| | Overshoot (%) | 1.78 | 9.68 | 0.16 |

## VI. CONCLUSIONS

The development of techniques for trajectory tracking and vibration suppression of a flexible joint manipulator has been presented. The control schemes have been developed based on collocated PD-type FLC with non-collocated fuzzy logic control and PD-type FLC with input shaping scheme. The proposed control schemes have been implemented and tested within simulation environment of a flexible joint manipulator. The performances of the control schemes have been evaluated in terms of input tracking capability and vibration suppression at the resonance mode of the manipulator. Acceptable performance in input tracking and vibration control has been achieved with proposed control strategies. A comparative assessment of the control schemes has shown that the PD-FLC-FLC performs better than the PD-FLC-IS in respect of deflection angle reduction of the elastic joint. Moreover, in term of speed of responses, PD-FLC-FLC results in a faster settling time response with high overshoot as compared to PD-FLC-IS. The work thus developed and reported in this paper forms the basis of design and development of hybrid control schemes for input tracking and vibration suppression of multi-link flexible manipulator systems and can be extended to and adopted in practical applications.

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