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ORIGINAL ARTICLE

Stochastic numerical treatment for solving Falkner–Skan equations using feedforward neural networks

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Abstract In this article, the artificial intelligence techniques have been used for the solution of Falkner–Skan (FS) equations based on neural networks optimized with three methods including active set technique, sequential quadratic programming and genetic algorithms (GA) hybridization. Log-sigmoid activation function is used in artificial neural network architecture. The proposed techniques are applied to a number of cases for Falkner–Skan problems, and results were compared with GA hybrid results in all cases and were found accurate. The level of accuracy is examined through statistical analyses based on a sufficiently large number of independent runs.

Keywords Falkner–Skan · ANN · Log-sigmoid · Boundary value problems

1 Introduction

Falkner–Skan nonlinear equation is generalized form of Blasius boundary layer problem with two boundary layers [1]. The solutions of this equation are always of great interest for engineers and scientists and have been analyzed by a number of researchers starting from work done by Falkner and Skan [1] for boundary layer flow in a viscous fluid. Later, Hartree [2] studied this equation numerically. In recent years, Harris et al. [3] studied the unsteady heat flow with constant wall temperature. Agarwal and O'Regan [4] studied infinite interval problems that were arising in nonlinear mechanics and non-Newtonian flow, whereas Pantokartoras [5] solved the Falkner–Skan flow with variable viscosity and constant wall temperature. The solution of Falkner–Skan equations by improved spectral homotopy analysis method was analyzed by Sandile et al. [6], in 2011. Many other powerful mathematical techniques have been used for obtaining the appropriate solution of these equations and can be seen in articles [7–11].

Many researchers have devoted their attentions to study the existence of solution of the Falkner–Skan (FS) equation and its uniqueness applied in different fields; names of few researches are listed here for the deep interest of reader Weyl [12], Rosenhead [13], Tam [14] and Hartman [15]. Moreover, some of authors work has described the ranges of validity of FS-equation boundary layer parameters as well as similarity variable. Further, Yang [16] describes a nonexistence numerical solutions through the application of upper and lower bounds on it (i.e., the non-dimensional wall shear stress). However, regardless of the number of attempts regarding the solution of FS-equation, this two-point boundary value problem even though lacks a general closed-form FS-equation solution.

In recent years, a lot of attention has been devoted to the study of artificial neural networks (ANNs) to investigate the problems arising in diverse fields of applied science and technology [17]. Arqub et al. recently applied continuous genetic algorithm for finding the numerical solution of systems of second-order BVP [18, 19]. Well-established strength of neural networks as a universal function

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