

An Adjustable Approach to Interval-Valued Intuitionistic Fuzzy Soft Sets Based Decision Making

Hongwu Qin, Xiuqin Ma, Tutut Herawan, and Jasni Mohamad Zain

Faculty of Computer Systems and Software Engineering
Universiti Malaysia Pahang
Lebuh Raya Tun Razak, Gambang 26300, Kuantan, Malaysia
qhwump@gmail.com, xueener@yahoo.com.cn,
tutut@ump.edu.my, jasni@ump.edu.my

Abstract. Research on soft set based decision making has received much attention in recent years. Feng et al. presented an adjustable approach to fuzzy soft sets based decision making by using level soft set, and subsequently extended the approach to interval-valued fuzzy soft set based decision making. Jiang et al. generalize the approach to solve intuitionistic fuzzy soft sets based decision making. In this paper, we further generalize the approaches introduced by Feng et al. and Jiang et al. Using reduct intuitionistic fuzzy soft sets and level soft sets of intuitionistic fuzzy soft sets, an adjustable approach to interval-valued intuitionistic fuzzy soft set based decision making is presented. Some illustrative example is employed to show the feasibility of our approach in practical applications.

Keywords: Soft sets, Interval-valued intuitionistic fuzzy soft sets, Decision making, Level soft sets.

1 Introduction

In 1999, Molodtsov [1] proposed soft set theory as a new mathematical tool for dealing with vagueness and uncertainties. At present, work on the soft set theory is progressing rapidly and many important theoretic results have been achieved [2-8].

The research on fuzzy soft set has also received much attention since its introduction by Maji et al. [9]. Majumdar and Samanta [10] have further generalized the concept of fuzzy soft sets. Maji et al. [11-13] extended soft sets to intuitionistic fuzzy soft sets. Yang et al. [14] proposed the concept of the interval-valued fuzzy soft sets. Jiang et al. [15] proposed a more general soft set model called interval-valued intuitionistic fuzzy soft set by combining the interval-valued intuitionistic fuzzy sets and soft sets.

At the same time, there has been some progress concerning practical applications of soft set theory, especially the use of soft sets in decision making [16-21]. Maji et al. [16] first applied soft sets to solve the decision making problem. Roy and Maji [17] presented a novel method to cope with fuzzy soft sets based decision making problems. Kong et al. [18] pointed out that the Roy-Maji method [17] was incorrect and they presented a revised algorithm. Feng et al. [19] discussed the validity of the

Roy-Maji method [17] and presented an adjustable approach to fuzzy soft sets based decision making by means of level soft sets. Yang et al. [14] applied the interval-valued fuzzy soft sets to analyze a decision making problem. The method they used is based on fuzzy choice value. Feng et al. [20] gave deeper insights into decision making involving interval-valued fuzzy soft sets. They analyzed the inherent drawbacks of fuzzy choice value based method and proposed a flexible scheme by using reduct fuzzy soft sets and level soft sets. Similarly, Jiang et al. [21] presented an adjustable approach to intuitionistic fuzzy soft sets based decision making by using level soft sets of intuitionistic fuzzy soft sets.

In this paper, we further generalize the approaches introduced by Feng et al. [20] and Jiang et al. [21]. Concretely, we define the notion of reduct intuitionistic fuzzy soft sets and present an adjustable approach to interval-valued intuitionistic fuzzy soft set based decision making by using reduct intuitionistic fuzzy soft sets and level soft sets of intuitionistic fuzzy soft sets. Firstly, by computing the reduct intuitionistic fuzzy soft set, an interval-valued intuitionistic fuzzy soft set is converted into an intuitionistic fuzzy soft set, and then the intuitionistic fuzzy soft set is further converted into a crisp soft set by using level soft sets of intuitionistic fuzzy soft sets. Finally, decision making is performed on the crisp soft set.

The rest of this paper is organized as follows. The following section briefly reviews some basic notions of soft sets. Section 3.1 defines the concept of reduct intuitionistic fuzzy soft sets. Section 3.2 recalls the level soft sets. We present our algorithm to interval-valued intuitionistic fuzzy soft set based decision making problems and illustrate example in Section 3.3. Finally, conclusions are given in Section 4.

2 Preliminaries

In this section, we recall the basic notions of soft sets, interval-valued fuzzy soft sets and interval-valued intuitionistic fuzzy soft sets.

Let U be an initial universe of objects, E be the set of parameters in relation to objects in U , $\mathcal{P}(U)$ denote the power set of U and $A \subseteq E$. The definition of soft set is given as follows.

Definition 2.1 ([1]). A pair (F, A) is called a *soft set* over U , where F is a mapping given by

$$F : A \rightarrow \mathcal{P}(U).$$

From definition, a soft set (F, A) over the universe U is a parameterized family of subsets of the universe U , which gives an approximate description of the objects in U . Before introduce the notion of the interval-valued fuzzy soft sets, let us give the concept of the interval-valued fuzzy sets [22].

An interval-valued fuzzy set Y on an universe U is a mapping such that $Y : U \rightarrow \text{Int}([0,1])$, where $\text{Int}([0,1])$ stands for the set of all closed subintervals of $[0, 1]$, the set of all interval-valued fuzzy sets on U is denoted by $\mathcal{S}(U)$. Suppose that

$Y \in \mathcal{S}(U), \forall x \in U, Y(x) = [Y^-(x), Y^+(x)]$ is called the degree of membership an element x to Y . $Y^-(x)$ and $Y^+(x)$ are referred to as the lower and upper degrees of membership x to Y , where $0 \leq Y^-(x) \leq Y^+(x) \leq 1$.

Definition 2.2 ([14]). Let U be an initial universe and E be a set of parameters, $A \subseteq E$, a pair (F, A) is called an *interval-valued fuzzy soft set* over U , where F is a mapping given by

$$F : A \rightarrow \mathcal{S}(U).$$

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U . $\forall \varepsilon \in A, F(\varepsilon)$ is referred as the interval fuzzy value set of paramete ε . Clearly, $F(\varepsilon)$ can be written as $F(\varepsilon) = \{ \langle x, F(\varepsilon)^-(x), F(\varepsilon)^+(x) \rangle : x \in U \}$, where $F(\varepsilon)^-(x)$ and $F(\varepsilon)^+(x)$ be the lower and upper degrees of membership of x to $F(\varepsilon)$ respectively.

Finally, we will introduce the concepts of interval-valued intuitionistic fuzzy set [23] and interval-valued intuitionistic fuzzy soft set.

An interval-valued intuitionistic fuzzy set on a universe Y is an object of the form $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in Y \}$, $\mu_A(x) : Y \rightarrow \text{Int}([0,1])$ and $\gamma_A(x) : Y \rightarrow \text{Int}([0,1])$ satisfy the following condition: $\forall x \in Y, \sup \mu_A(x) + \sup \gamma_A(x) \leq 1$.

Definition 2.3 ([15]). Let U be a universe set, E be a set of parameters, $\mathcal{I}(U)$ denotes the set of all interval-valued intuitionistic fuzzy sets of U and $A \subseteq E$. A pair (F, A) is called an *interval-valued intuitionistic fuzzy soft set* over U , where F is a mapping given by

$$F : A \rightarrow \mathcal{I}(U).$$

$\forall \varepsilon \in A, F(\varepsilon)$ is an interval-valued intuitionistic fuzzy set of U . $F(\varepsilon)$ can be written as: $F(\varepsilon) = \{ \langle x, \mu_{F(\varepsilon)}(x), \gamma_{F(\varepsilon)}(x) \rangle : x \in U \}$, where $\mu_{F(\varepsilon)}(x)$ is the interval-valued fuzzy membership degree, $\gamma_{F(\varepsilon)}(x)$ is the interval-valued fuzzy non-membership degree. For illustration, we consider the following house example.

Example 1. Consider an interval-valued intuitionistic fuzzy soft set (F, A) which describes the ‘‘attractiveness of houses’’ that Mr. X is considering for purchase. Suppose that there are six houses under consideration, namely the universes $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, and the parameter set $A = \{e_1, e_2, e_3, e_4, e_5\}$, where e_i stand for ‘‘beautiful’’, ‘‘large’’, ‘‘cheap’’, ‘‘modern’’ and ‘‘in green surroundings’’ respectively. The tabular representation of (F, A) is shown in Table 1. Obviously, we can see that the precise evaluation for each object on each parameter is unknown while the lower and upper limits of such an evaluation are given. For example, we cannot present the precise membership degree and non-membership degree of how beautiful house h_1 is,

however, house h_1 is at least beautiful on the membership degree of 0.7 and it is at most beautiful on the membership degree of 0.8; house h_1 is not at least beautiful on the non-membership degree of 0.1 and it is not at most beautiful on the non-membership degree of 0.2.

Table 1. Interval-valued intuitionistic fuzzy soft set (F, A)

U	e_1	e_2	e_3	e_4	e_5
h_1	[0.7,0.8], [0.1,0.2]	[0.82,0.84], [0.05,0.15]	[0.52,0.72], [0.18,0.25]	[0.55,0.6], [0.3,0.35]	[0.7,0.8], [0.1,0.2]
h_2	[0.85,0.9], [0.05,0.1]	[0.7,0.74], [0.17,0.25]	[0.7,0.75], [0.1,0.23]	[0.7,0.75], [0.15,0.25]	[0.75,0.9], [0.05,0.1]
h_3	[0.5,0.7], [0.2,0.3]	[0.86,0.9], [0.04,0.1]	[0.6,0.7], [0.2,0.28]	[0.2,0.3], [0.5,0.6]	[0.65,0.8], [0.15,0.2]
h_4	[0.4,0.6], [0.3,0.4]	[0.52,0.64], [0.23,0.35]	[0.72,0.78], [0.11,0.21]	[0.3,0.5], [0.4,0.5]	[0.8,0.9], [0.05,0.1]
h_5	[0.6,0.8], [0.15,0.2]	[0.3,0.35], [0.5,0.65]	[0.58,0.68], [0.18,0.3]	[0.68,0.77], [0.1,0.2]	[0.72,0.85], [0.1,0.15]
h_6	[0.3,0.5], [0.3,0.45]	[0.5,0.68], [0.25,0.3]	[0.33,0.43], [0.5,0.55]	[0.62,0.65], [0.15,0.35]	[0.84,0.93], [0.04,0.07]

3 An Adjustable Approach to Interval-Valued Intuitionistic Fuzzy Soft Set Based Decision Making

In this section we present an adjustable approach to interval-valued intuitionistic fuzzy soft set based decision making problems by combining the reduct intuitionistic fuzzy soft sets and level soft sets of intuitionistic fuzzy soft sets. First we define the concept of reduct intuitionistic fuzzy soft sets, and then recall the level soft sets, finally present our algorithm and illustrate example.

3.1 Reduct Intuitionistic Fuzzy Soft Sets

The concept of reduct fuzzy soft set is proposed in [20]. By adjusting the value of opinion weighting vector, an interval-valued fuzzy soft set can be converted into a fuzzy soft set, which makes the making decision based on interval-valued fuzzy soft set much easier.

Similarly, we can introduce the idea to making decision based on interval-valued intuitionistic fuzzy soft set, that is, convert both interval-valued membership degree and interval-valued non-membership degree into one fuzzy value. As a result, an interval-valued intuitionistic fuzzy soft set will be transformed to an intuitionistic fuzzy soft set, which will facilitate the making decision based on interval-valued intuitionistic fuzzy soft set. We define the notion of reduct intuitionistic fuzzy soft set as follows to illustrate the idea.

Let U be a universe set, E be a set of parameters and $A \subseteq E$. Let (F, A) be an interval-valued intuitionistic fuzzy soft set over U such that $\forall \mathcal{E} \in A, F(\mathcal{E})$ is an interval-valued intuitionistic fuzzy set with $F(\mathcal{E}) = \{ \langle x, \mu_{F(\mathcal{E})}(x), \gamma_{F(\mathcal{E})}(x) \rangle : x \in U \}, \forall x \in U$.

Definition 3.1. Let $\alpha, \beta, \phi, \varphi \in [0,1], \alpha + \beta = 1, \phi + \varphi = 1$. The vector $W = (\alpha, \beta, \phi, \varphi)$ is called an opinion weighting vector. The intuitionistic fuzzy soft set (F_W, A) over U such that

$$F_W(\mathcal{E}) = \{ (x, \alpha \mu_{F(\mathcal{E})}^-(x) + \beta \mu_{F(\mathcal{E})}^+(x), \phi \gamma_{F(\mathcal{E})}^-(x) + \varphi \gamma_{F(\mathcal{E})}^+(x)) : x \in U \}, \forall \mathcal{E} \in A,$$

is called the *weighted reduct intuitionistic fuzzy soft set* of the interval-valued intuitionistic fuzzy soft set (F, A) with respect to the opinion weighting vector W .

By adjusting the value of α, β, ϕ and φ , an interval-valued intuitionistic fuzzy soft set can be converted into any reduct intuitionistic fuzzy soft set decision maker desired. Specially, let $\alpha = 1, \beta = 0, \phi = 0$ and $\varphi = 1$, we have the pessimistic-pessimistic reduct intuitionistic fuzzy soft set (PPRIFS), denoted by (F_{-+}, A) and defined by

$$F_{-+}(\mathcal{E}) = \{ (x, \mu_{F(\mathcal{E})}^-(x), \gamma_{F(\mathcal{E})}^+(x)) : x \in U \}, \forall \mathcal{E} \in A.$$

Let $\alpha = 0, \beta = 1, \phi = 1$ and $\varphi = 0$, we have the optimistic-optimistic reduct intuitionistic fuzzy soft set (OORIFS), denoted by (F_{+-}, A) and defined by

$$F_{+-}(\mathcal{E}) = \{ (x, \mu_{F(\mathcal{E})}^+(x), \gamma_{F(\mathcal{E})}^-(x)) : x \in U \}, \forall \mathcal{E} \in A.$$

Let $\alpha = 0.5, \beta = 0.5, \phi = 0.5$ and $\varphi = 0.5$, we have the neutral-neutral reduct intuitionistic fuzzy soft set (NNRIFS), denoted by (F_{NN}, A) and defined by

$$F_{NN}(\mathcal{E}) = \{ (x, (\mu_{F(\mathcal{E})}^-(x) + \mu_{F(\mathcal{E})}^+(x))/2, (\gamma_{F(\mathcal{E})}^-(x) + \gamma_{F(\mathcal{E})}^+(x))/2) : x \in U \}, \forall \mathcal{E} \in A.$$

Example 2. Compute the PPRIFS (F_{-+}, A) , OORIFS (F_{+-}, A) and NNRIFS (F_{NN}, A) of the interval-valued intuitionistic fuzzy soft set (F, A) shown in Table 1. The results are shown in Table 2, 3 and 4 respectively.

Table 2. Pessimistic-pessimistic reduct intuitionistic fuzzy soft set of (F, A)

U	e_1	e_2	e_3	e_4	e_5
h_1	[0.7, 0.2]	[0.82, 0.15]	[0.52, 0.25]	[0.55, 0.35]	[0.7, 0.2]
h_2	[0.85, 0.1]	[0.7, 0.25]	[0.7, 0.23]	[0.7, 0.25]	[0.75, 0.1]
h_3	[0.5, 0.3]	[0.86, 0.1]	[0.6, 0.28]	[0.2, 0.6]	[0.65, 0.2]
h_4	[0.4, 0.4]	[0.52, 0.35]	[0.72, 0.21]	[0.3, 0.5]	[0.8, 0.1]
h_5	[0.6, 0.2]	[0.3, 0.65]	[0.58, 0.3]	[0.68, 0.2]	[0.72, 0.15]
h_6	[0.3, 0.45]	[0.5, 0.3]	[0.33, 0.55]	[0.62, 0.35]	[0.84, 0.07]

Table 3. Optimistic-optimistic reduct intuitionistic fuzzy soft set of (F, A)

U	e_1	e_2	e_3	e_4	e_5
h_1	[0.8, 0.1]	[0.84, 0.05]	[0.72, 0.18]	[0.6, 0.3]	[0.8,0.1]
h_2	[0.9, 0.05]	[0.74, 0.17]	[0.75, 0.1]	[0.75, 0.15]	[0.9, 0.05]
h_3	[0.7, 0.2]	[0.9, 0.04]	[0.7, 0.2]	[0.3, 0.5]	[0.8, 0.15]
h_4	[0.6, 0.3]	[0.64,0.23]	[0.78,0.11]	[0.5, 0.4]	[0.9, 0.05]
h_5	[0.8, 0.15]	[0.35, 0.5]	[0.68, 0.18]	[0.77, 0.1]	[0.85, 0.1]
h_6	[0.5, 0.3]	[0.68, 0.25]	[0.43, 0.5]	[0.65, 0.15]	[0.93, 0.04]

Table 4. Neutral-neutral reduct intuitionistic fuzzy soft set of (F, A)

U	e_1	e_2	e_3	e_4	e_5
h_1	[0.75, 0.15]	[0.83, 0.1]	[0.62, 0.22]	[0.58, 0.33]	[0.75,0.15]
h_2	[0.88, 0.08]	[0.72, 0.21]	[0.73, 0.17]	[0.73, 0.2]	[0.83, 0.08]
h_3	[0.6, 0.25]	[0.88, 0.07]	[0.65, 0.24]	[0.25, 0.55]	[0.73, 0.18]
h_4	[0.5, 0.35]	[0.58,0.29]	[0.75,0.16]	[0.4, 0.45]	[0.85, 0.08]
h_5	[0.7, 0.18]	[0.33, 0.58]	[0.63, 0.24]	[0.73, 0.15]	[0.79, 0.13]
h_6	[0.4, 0.38]	[0.59, 0.28]	[0.38, 0.53]	[0.64, 0.25]	[0.89, 0.06]

3.2 Level Soft Sets

Feng et al. [19] initiated the concept of level soft set to solve fuzzy soft set based decision making problem. Subsequently, the same author applied level soft set to solve interval-valued fuzzy soft sets based decision making problem [20]. Jiang et al. [21] further generalize the approach introduced in [19] by applying level soft set to solve intuitionistic fuzzy soft sets based decision making. Level soft set of intuitionistic fuzzy soft set is defined as follows.

Definition 3.2 ([21]). Let $\varpi = (F, A)$ be an intuitionistic fuzzy soft set over U , where $A \subseteq E$ and E is a set of parameters. Let $\lambda : A \rightarrow [0,1] \times [0,1]$ be an intuitionistic fuzzy set in A which is called a threshold intuitionistic fuzzy set. The level soft set of ϖ with respect to λ is a crisp soft set $L(\varpi; \lambda) = (F_\lambda, A)$ defined by

$$F_\lambda(\varepsilon) = L(F(\varepsilon); \lambda(\varepsilon)) = \{x \in U \mid \mu_{F(\varepsilon)}(x) \geq \mu_\lambda(\varepsilon) \text{ and } \gamma_{F(\varepsilon)}(x) \leq \gamma_\lambda(\varepsilon)\} \quad \forall \varepsilon \in A.$$

According to the definition, four types of special level soft set are also defined in [21], which are called Mid-level soft set $L(\varpi, mid_\varpi)$, Top-Bottom-level soft set $L(\varpi, topbottom_\varpi)$, Top-Top-level soft set $L(\varpi, toptop_\varpi)$ and Bottom-bottom-level soft set $L(\varpi, bottombottom_\varpi)$.

3.3 Our Algorithm for Decision Making Based on Interval-Valued Intuitionistic Fuzzy Soft Sets

In this section we present our algorithm for decision making based on interval-valued intuitionistic fuzzy soft sets. By considering appropriate reduct intuitionistic fuzzy

soft sets and level soft sets of intuitionistic fuzzy soft sets, interval-valued intuitionistic fuzzy soft sets based decision making can be converted into only crisp soft sets based decision making. Firstly, by computing the reduct intuitionistic fuzzy soft set, an interval-valued intuitionistic fuzzy soft set is converted into an intuitionistic fuzzy soft set, and then the intuitionistic fuzzy soft set is further converted into a crisp soft set by using level soft sets of intuitionistic fuzzy soft sets. Finally, decision making is performed on the crisp soft set. The details of our algorithm are listed below.

Algorithm 1.

1. Input the interval-valued intuitionistic fuzzy soft set (F, A) .
2. Input an opinion weighting vector $W = (\alpha, \beta, \phi, \varphi)$ and compute the weighted reduct intuitionistic fuzzy soft set $\overline{\omega} = (F_W, A)$ of the interval-valued intuitionistic fuzzy soft set (F, A) with respect to the opinion weighting vector W (or choose $\overline{\omega} = \text{PPRIFS}(F_{-+}, A)$, $\text{OORIFS}(F_{+-}, A)$ or $\text{NNRIFS}(F_{NN}, A)$ of (F, A)).
3. Input a threshold intuitionistic fuzzy set $\lambda : A \rightarrow [0,1] \times [0,1]$ (or give a threshold value pair $(s, t) \in [0,1] \times [0,1]$); or choose the mid-level decision rule; or choose the top-bottom-level decision rule; or choose the top-top-level decision rule; or choose the bottom-bottom-level decision rule) for decision making.
4. Compute the level soft set $L(\overline{\omega}; \lambda)$ with regard to the threshold intuitionistic fuzzy set λ (or the (s, t) -level soft set $L(\overline{\omega}; s, t)$; or the mid-level soft set $L(\overline{\omega}, \text{mid}_{\overline{\omega}})$; or the top-bottom-level soft set $L(\overline{\omega}, \text{topbottom}_{\overline{\omega}})$; or the top-top-level soft set $L(\overline{\omega}, \text{toptop}_{\overline{\omega}})$; or the bottom-bottom-level soft set $L(\overline{\omega}, \text{bottombottom}_{\overline{\omega}})$).
5. Present the level soft set $L(\overline{\omega}; \lambda)$ (or $L(\overline{\omega}; s, t)$; or $L(\overline{\omega}, \text{mid}_{\overline{\omega}})$; or $L(\overline{\omega}, \text{topbottom}_{\overline{\omega}})$; or $L(\overline{\omega}, \text{toptop}_{\overline{\omega}})$; or $L(\overline{\omega}, \text{bottombottom}_{\overline{\omega}})$) in tabular form and compute the choice value c_i of $o_i, \forall i$.
6. The optimal decision is to select o_k if $c_k = \max_i c_i$.
7. If k has more than one value then any one of o_k may be chosen.

There are two remarks here.

Firstly, reader is referred to [16] for more details regarding the method of computing the choice value in the fifth step of the above algorithm,

Secondly, in the last step of Algorithm 1, one may go back to the step 2 or step 3 to modify opinion weighting vector or the threshold so as to adjust the final optimal decision when there are too many “optimal choices” to be chosen.

The advantages of Algorithm 1 are mainly twofold.

Firstly, we need not treat interval-valued intuitionistic fuzzy soft sets directly in decision making but only deal with the related reduct intuitionistic soft sets and finally the crisp level soft sets after choosing certain opinion weighting vectors and thresholds. This makes our algorithm simpler and easier for application in practical problems.

Secondly, there are a large variety of opinion weighting vectors and thresholds that can be used to find the optimal choices, hence our algorithm has great flexibility and

adjustable capability. Table 5 gives some typical schemes that arise from Algorithm 1 by combining reduct intuitionistic soft set PPRIFS (F_{\rightarrow}, A) and several typical level soft sets. As pointed out in [19], many decision making problems are essentially humanistic and subjective in nature; hence there actually does not exist a unique or uniform criterion for decision making in an imprecise environment. This adjustable feature makes Algorithm 1 not only efficient but more appropriate for many practical applications.

To illustrate the basic idea of Algorithm 1, let us consider the following example.

Example 3. Let us reconsider the decision making problem based on the interval-valued intuitionistic fuzzy soft sets (F, A) as in Table 1.

If we select the first scheme ‘‘Pes-Mid’’ in Table 5 to solve the problem, at first we compute the reduct intuitionistic fuzzy soft set PPRIFS $\varpi = (F_{\rightarrow}, A)$ as in Table 2 and then use the mid-level decision rule on $\varpi = (F_{\rightarrow}, A)$ and obtain the mid-level soft set $L(\varpi; mid)$ with choice values as in Table 6.

From Table 6, it is clear that the maximum choice value is $c_2 = 5$, so the optimal decision is to select h_2 .

Table 5. Typical schemes for interval-valued intuitionistic fuzzy soft set based decision making

Scheme	Reduct intuitionistic fuzzy soft set	Level soft set
Pes-Topbot	PPRIFS $\varpi = (F_{\rightarrow}, A)$	$L(\varpi; topbottom)$
Pes-Toptop	PPRIFS $\varpi = (F_{\rightarrow}, A)$	$L(\varpi; toptop)$
Pes-Mid	PPRIFS $\varpi = (F_{\rightarrow}, A)$	$L(\varpi; mid)$
Pes-Botbot	PPRIFS $\varpi = (F_{\rightarrow}, A)$	$L(\varpi; bottombottom)$

Table 6. Tabular representation of the mid-level soft set $L(\varpi; mid)$ with choice values

U	e_1	e_2	e_3	e_4	e_5	Choice value(c_i)
h_1	1	1	0	1	0	$c_1 = 3$
h_2	1	1	1	1	1	$c_2 = 5$
h_3	0	1	1	0	0	$c_3 = 2$
h_4	0	0	1	0	1	$c_4 = 2$
h_5	1	0	1	1	0	$c_5 = 3$
h_6	0	0	0	1	1	$c_6 = 2$

4 Conclusion

In this paper, we present an adjustable approach to interval-valued intuitionistic fuzzy soft set based decision making by using reduct intuitionistic fuzzy soft sets and level

soft sets of intuitionistic fuzzy soft sets, which further generalize the approaches introduced by Feng et al. [20] and Jiang et al. [21]. An interval-valued intuitionistic fuzzy soft set based decision making problem is converted into a crisp soft set based decision making problem after choosing certain opinion weighting vectors and thresholds. This makes our algorithm simpler and easier for application in practical problems. In addition, a large variety of opinion weighting vectors and thresholds that can be used to find the optimal alternatives make our algorithm more flexible and adjustable.

Acknowledgments. This work was supported by PRGS under the Grant No. GRS100323, Universiti Malaysia Pahang, Malaysia.

References

1. Molodtsov, D.: Soft set theory_First results. *Computers and Mathematics with Applications* 37, 19–31 (1999)
2. Maji, P.K., Biswas, R., Roy, A.R.: Soft set theory. *Computers and Mathematics with Applications* 45, 555–562 (2003)
3. Aktas, H., Cagman, N.: Soft sets and soft groups. *Information Sciences* 177, 2726–2735 (2007)
4. Acar, U., Koyuncu, F., Tanay, B.: Soft sets and soft rings. *Computers and Mathematics with Applications* 59, 3458–3463 (2010)
5. Jun, Y.B.: Soft BCK/BCI-algebras. *Computers and Mathematics with Applications* 56, 1408–1413 (2008)
6. Feng, F., Jun, Y.B., Zhao, X.: Soft semirings. *Computers and Mathematics with Applications* 56, 2621–2628 (2008)
7. Xiao, Z., Gong, K., Xia, S., Zou, Y.: Exclusive disjunctive soft sets. *Computers and Mathematics with Applications* 59, 2128–2137 (2010)
8. Herawan, T., Mat Deris, M.: A direct proof of every rough set is a soft set. In: *Proceeding of the Third Asia International Conference on Modeling and Simulation, AMS 2009, Bali, Indonesia*, pp. 119–124. IEEE Press, Los Alamitos (2009)
9. Maji, P.K., Biswas, R., Roy, A.R.: Fuzzy soft sets. *Journal of Fuzzy Mathematics* 9, 589–602 (2001)
10. Majumdar, P., Samanta, S.K.: Generalized fuzzy soft sets. *Computers and Mathematics with Applications* 59, 1425–1432 (2010)
11. Maji, P.K.: More on intuitionistic fuzzy soft sets. In: Sakai, H., Chakraborty, M.K., Hassani, A.E., Ślęzak, D., Zhu, W. (eds.) *RSFDGrC 2009. LNCS*, vol. 5908, pp. 231–240. Springer, Heidelberg (2009)
12. Maji, P.K., Biswas, R., Roy, A.R.: Intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics* 9, 677–692 (2001)
13. Maji, P.K., Roy, A.R., Biswas, R.: On intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics* 12, 669–683 (2004)
14. Yang, X.B., Lin, T.Y., Yang, J., Dongjun, Y.L.A.: Combination of interval-valued fuzzy set and soft set. *Computers and Mathematics with Applications* 58, 521–527 (2009)
15. Jiang, Y., Tang, Y., Chen, Q., Liu, H., Tang, J.: Interval-valued intuitionistic fuzzy soft sets and their properties. *Computers and Mathematics with Applications* 60, 906–918 (2010)

16. Maji, P.K., Roy, A.R.: An application of soft sets in a decision making problem. *Computers and Mathematics with Applications* 44, 1077–1083 (2002)
17. Maji, P.K., Roy, A.R.: A fuzzy soft set theoretic approach to decision making problems. *Journal of Computational and Applied Mathematics* 203, 412–418 (2007)
18. Kong, Z., Gao, L.Q., Wang, L.F.: Comment on “A fuzzy soft set theoretic approach to decision making problems”. *Journal of Computational and Applied Mathematics* 223, 540–542 (2009)
19. Feng, F., Jun, Y.B., Liu, X., Li, L.: An adjustable approach to fuzzy soft set based decision making. *Journal of Computational and Applied Mathematics* 234, 10–20 (2010)
20. Feng, F., Li, Y., Leoreanu-Fotea, V.: Application of level soft sets in decision making based on interval-valued fuzzy soft sets. *Computers and Mathematics with Applications* 60, 1756–1767 (2010)
21. Jiang, Y., Tang, Y., Chen, Q.: An adjustable approach to intuitionistic fuzzy soft sets based decision making. *Applied Mathematical Modelling* 35, 824–836 (2011)
22. Gorzalczany, M.B.: A method of inference in approximate reasoning based on interval valued fuzzy sets. *Fuzzy Sets and Systems* 21, 1–17 (1987)
23. Atanassov, K., Gargov, G.: Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems* 31, 343–349 (1989)