Proceedings of 4th International Science Postgraduate Conference 2016 (ISPC2016)

STABILITY ANALYSIS OF EXPLICIT AND SEMI-IMPLICIT DERIVATIVE-FREE METHODS FOR STOCHASTIC DIFFERENTIAL EQUATIONS

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Abstract. This paper is devoted to investigate the mean-square stability of explicit and semi-implicit derivative-free methods to a class of stochastic differential equations (SDEs). The mean-square stability functions and regions of explicit and semi-implicit numerical approximation schemes are obtained for a linear stochastic differential equation with multiplicative noise. It is proved that the semi-implicit derivative-free method is mean-square stable compare than the explicit counterpart schemes. A numerical experiment is provided to illustrate the theory.

Keywords mean-square stability; stochastic differential equation; derivative-free method.

1.0 INTRODUCTION

Consider the Stratonovich form of SDE

$$dy(t) = f(y(t))dt + g(y(t)) \circ dW(t), \quad y(t_0) = y_0, \quad t \in [t_0, T], \quad y(t) \in \Re^m \tag{1}$$

where the deterministic term $f(y(t))$ is a drift coefficient, the stochastic function $g(y(t))$ is a diffusion coefficient and $W(t)$ is a Wiener process. The increment of Wiener process, $\Delta W(t) = W(t+\Delta t) - W(t)$ is a Gaussian random variable with zero mean and variance is given by the increment in time, Δt. Analytical solution of (1) is hardly to be found, hence solving SDE (1) numerically is required. Many authors have put their efforts in designing numerical methods for SDEs. Amongst of the references cited therein are Maruyama (1950), Milstein (1974), Rumelin (1982), Kloeden and Platen (1992) and Burrage (1999). The methods that have been proposed by Maruyama (1950), Milstein (1974) and Milstein (1995) are based on the truncation of stochastic Taylor series expansion. However, as the order increases, the difficulty of implementing those methods arises as one need to compute the partial derivatives of the drift and diffusion functions of high order. To overcome the aforementioned complexity, Rumelin (1982) proposed a derivative-free stochastic Runge-Kutta (SRK) method for strong approximations of SDEs. Kloeden and Platen (1992) develop a Platen's scheme based on the general formulation of Rumelin (1982). Then, Burrage and Burrage (1996) and Burrage (1999) presented a new formalisation of SRK method for solving SDE (1), hence the methods of order 1.0 and 1.5 were developed. All the existing aforementioned derivative free methods are in explicit form whose both drift and diffusion coefficients are explicit. New classes of semi-implicit derivative free methods for strong pathwise approximations were introduced by Tian and Burrage (2002). In semi-implicit approximation of SDEs, the drift coefficient is implicit while the diffusion coefficient is explicit.

Semi-implicit method is proposed in order to improve the stability properties of strong pathwise approximation to SDEs. Numerical stability analysis of SDEs is far more complex than ODEs. The stability analysis of SDEs has been investigated by many researchers. Amongst of the paper cited therein are Saito and Mitsui (1996), Norhayati (2010), Burrage (1999) and Platen and Shi (2008). It was Saito and Mitsui (1996) who investigated the stability analysis of various numerical schemes (EM, semi-implicit EM, Milstein scheme and semi-implicit Milstein scheme) in mean-square sense. Those of the schemes are based on the truncation of stochastic Taylor series whose require the computational of partial derivative in the computation. Platen and Shi (2008) provided a unified approach to the study of numerical stability of schemes for the discrete time approximations of SDEs. Numerical stability criterion was introduced and analysed. The corresponding numerical stability regions of the corresponding schemes were visualised. Then, Burrage (1999) investigated the stability of explicit derivative-free method to approximate the solution of SDEs. In this paper, we aim to investigate the stability property of two-stage stochastic Runge-Kutta (SRK2) method by imposing the stability criterion in mean-square sense that was proposed by Platen and Shi (2008).

This paper is organised as follows; Two-stage stochastic Runge-Kutta method of explicit and semi-implicit form are provided in Section 2. Section 3 concerns with the stability property of numerical method for SDEs. Then, in Section 4 the stability function and region for three different methods considered in this paper are presented. Numerical experiment to confirm the results is performed in Section 5.

2.0 TWO-STAGE STOCHASTIC RUNGE-KUTTA (SRK2)

A general form of two-stage stochastic Runge-Kutta is

$$Y_i = y_n + h\sum_{j=1}^{s} a_{ij} f(y_j) + J_1 \sum_{j=1}^{s} b_{ij} g(y_j), \qquad i = 1,\ldots,s$$

$$y_{n+1} = y_n + h\sum_{j=1}^{s} \alpha_j f(y_j) + J_1 \sum_{j=1}^{s} \gamma_j g(y_j) \tag{2}$$

where $A=(a_{ij})$ and $B=(b_{ij})$ are $s\times s$ matrices of real elements while $\alpha^T = (\alpha_1,\cdots,\alpha_s)$ and $\gamma^T = (\gamma_1,\cdots,\gamma_s)$ are row vectors $\in \mathbb{R}^s$ while J_1 integrals $\left(J_1 = \int_{t_n}^{t_{n+1}} \circ dW\right)$ represent stochastic components. In Butcher's tableu form the general formulation of (2) is written as

$$\begin{array}{c|cc} & a_1 & 0 \\ & a_2 & a_1 \\ \hline & \alpha_1 & \alpha_2 \end{array} \qquad \begin{array}{c|cc} & b_1 & 0 \\ & b_2 & b_1 \\ \hline & \beta_1 & \beta_2 \end{array}$$

They are explicit if $a_1 = b_1 = 0$ and semi-implicit if $a_1 \neq 0$ and $b_1 = 0$. We consider two types of explicit methods of Platen and Burrage scheme as well as a semi-implicit method that were proposed by Kloeden and Platen (1992), Burrage (1999) and Tian and Burrage (2002), respectively. Those SRK2 schemes are presented in tableu form as:

Explicit Platen scheme:
$$\begin{array}{c|cc} & 0 & 0 \\ & 1 & 0 \\ \hline & 1 & 0 \end{array} \qquad \begin{array}{c|cc} & 0 & 0 \\ & 1 & 0 \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

Explicit Burrage scheme:
$$\begin{array}{c|cc} & 0 & 0 \\ & \frac{2}{3} & 0 \\ \hline & \frac{1}{4} & \frac{3}{4} \end{array} \qquad \begin{array}{c|cc} & 0 & 0 \\ & \frac{2}{3} & 0 \\ \hline & \frac{1}{4} & \frac{3}{4} \end{array}$$

Semi-Implicit:
$$\begin{array}{c|cc} & 1 & 0 \\ & -\frac{2}{3} & 1 \\ \hline & \frac{1}{4} & \frac{3}{4} \end{array} \qquad \begin{array}{c|cc} & 0 & 0 \\ & \frac{2}{3} & 0 \\ \hline & \frac{1}{4} & \frac{3}{4} \end{array}$$

2.1 Mean-Square (MS) Stability Property

The linear equation of the Stratonovich type is used as a test equation
$$dy = \left(1 - \frac{3}{2}\delta\right)\lambda y dt + \sqrt{-\delta\lambda}\, y \circ dW(t) \qquad (2)$$
where $-\circ$" denotes the Stratonovich integral and for $\lambda h \in (-\infty, 0)$, $\delta \in [0,1)$, $t \geq 0$, $y_0 > 0$, $\lambda < 0$. We now turn our attention to the stability criterion of the approximation process $Y = \{y_t, t > 0\}$, which is given by Definition 1.

Definition 1: Numerical Stability Criterion [Platen and Shi, 2008]
For $p > 0$ a process $Y = \{y_t, t > 0\}$ is called p-stable if $\lim_{t\to\infty}\left(E|y_t|^p\right) = 0.$
A process is $p-$stable if in the long run its p^{th} moment vanishes. The discrete time approximation, $Y = \{y_t, t > 0\}$ is stable if this process and the analytical solution of a stochastic process counterpart, X have similar stability properties according to Definition 1. Definition 2 provides the concept of the stability region for a discrete time approximation. The stability region will permit the visualization of its numerical stability properties.

Definition 2: Stability Region [Platan and Shi, 2008]
The stability region Γ is determined by the triplets $(\lambda h, \delta, p) \in (-\infty, 0) \times (0, \infty)$ for which the discrete line approximation Y with line step size h, when applied to the test equation (2), is p-stable.
For many discrete time approximation Y with a step size $\Delta > 0$ when applied to the test equation (2) with a given $\delta \in [0,1)$, the ratio $\left|\frac{Y_{n+1}}{Y_n}\right| = G_{n+1}(\lambda h, \delta)$, for $n \in \{0,1,......\}$, $Y_n > 0$, $\lambda < 0$ is of crucial interest.
$G_{n+1}(\lambda h, \delta)$ is called the transfer function of the approximation Y at time t_n. It transfers the previous approximate value Y_n into the approximate value Y_{n+1} of the next time step.

For a given scheme and $\lambda < 0$, $h \in (0,1)$ and $\delta \in [0,1)$, the random variables $G_{n+1}(\lambda h, \delta)$ are for $n \in \{0,1,...\}$ non-negative, independent and identically distributed with $E((\ln(G_{n+1}(\lambda h, \delta)))^2) < \infty$. The corresponding numerical stability criterion is given in Lemma 1.

Lemma 1: Numerical Stability Property [Platen and Shi, 2008]
A discrete time approximation is for given $\lambda h < 0$, $\delta \in [0,1)$, and $p > 0$, p-stable if and only if $E((G_{n+1}(\lambda h, \delta))^p) < 1$.

2.2 MS-Stability of SRK2 Approximations Scheme

The MS-stability functions and MS-stability region for the discrete time approximation derivative-free method of SRK2 of explicit and semi-implicit schemes are presented in this section.

2.2.1 MS-Stability of Explicit SRK2 Method

By applying the Platen scheme to the linear test equation of (2), the following approximation solution for the process, y at time, t_{n+1} is

$$y_{n+1} = \left(1 + h\left(1 - \frac{3}{2}\delta\right)\lambda + J1\left(\frac{1}{2}\sqrt{-\delta\lambda} + \frac{1}{2}\sqrt{-\delta\lambda}\left(1 + h\left(1 - \frac{3}{2}\delta\right)\lambda + J1\sqrt{-\delta\lambda}\right)\right)\right)y_n \tag{3}$$

with the intermediate stages $Y_1 = y_n$ and $Y_2 = y_n + hf(Y_1) + J1g(Y_1)$, where

$$f(Y_1) = \left(1 - \frac{3}{2}\delta\right)\lambda y_n, \quad g(Y_1) = \sqrt{-\delta\lambda}\, y_n,$$

$$f(Y_2) = \left(1 - \frac{3}{2}\delta\right)\lambda\left(y_n + h\left(1 - \frac{3}{2}\delta\right)\lambda y_n + J1\sqrt{-\delta\lambda}\, y_n\right),$$

$$g(Y_2) = \sqrt{-\delta\lambda}\left(y_n + h\left(1 - \frac{3}{2}\delta\right)\lambda y_n + J1\sqrt{-\delta\lambda}\, y_n\right).$$

The transfer function, $G_{n+1}(\lambda h, \delta)$ at time, t_{n+1} can be computed by taking the ratio of $\left|\frac{y_{n+1}}{y_n}\right|$ of (3). This yield

$$G_{n+1_{Platen\ scheme}} = 1 + h\left(1 - \frac{3}{2}\delta\right)\lambda + J1\left(\frac{1}{2}\sqrt{-\delta\lambda} + \frac{1}{2}\sqrt{-\delta\lambda}\left(1 + h\left(1 - \frac{3}{2}\delta\right)\lambda + J1\sqrt{-\delta\lambda}\right)\right) \tag{4}$$

Square both sides of (4) and then take the expectation of the Startonovich stochastic integrals J_1^4, J_1^3, J_1^2, J_1, where $E(J_1^4) = \frac{h^2}{2}$, $E(J_1^3) = E(J_1) = 0$, $E(J_1^2) = h$, we have

$$G_{1_{Platen\ Scheme}} = 1 + \left(\frac{3}{4}\delta^2 - \frac{1}{4}\delta - \frac{9}{16}\delta^3\right)\lambda^3 h^3 + (2 - 5\delta)h\lambda + \left(-5\delta + 1 + \frac{43}{8}\delta^2\right)h^2\lambda^2$$

Let $\overline{\lambda} = \lambda h$ yield

$$G_{1,Platen\ Scheme} = 1 + \left(\frac{3}{4}\delta^2 - \frac{1}{4}\delta - \frac{9}{16}\delta^3\right)\overline{\lambda^3} + (2 - 5\delta)\overline{\lambda} + \left(-5\delta + 1 + \frac{43}{8}\delta^2\right)\overline{\lambda^2} \tag{5}$$

The stability region of a stability function (5) is plotted in Maple 16 and the region is illustrated in Figure 1.

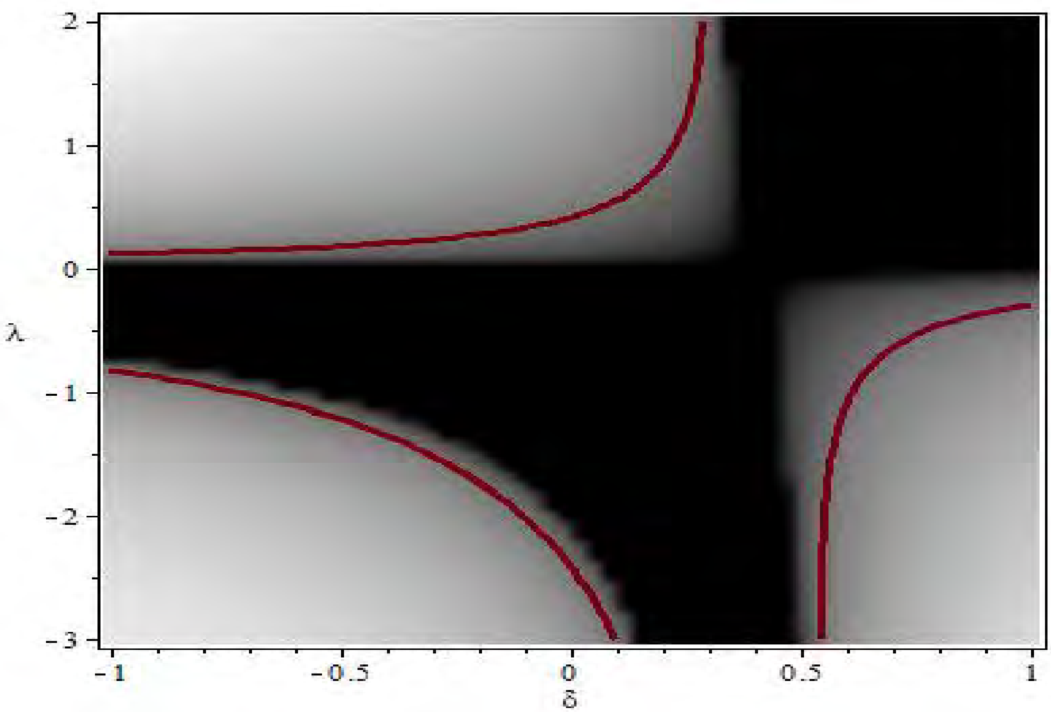

Figure 1 The stability region of a stability function (5)

Equation (5) is MS-stability function of explicit SRK2 of Platen scheme. Next, we compute the MS-stability function of an explicit SRK2 Burrage scheme. By applying the Burrage scheme to the linear test equation of (2), the following approximation solution for the process, y at time, t_{n+1} is

$$\begin{aligned} y_{n+1} = &\left(1 + h\left(\frac{1}{4}\left(1-\frac{3}{2}\delta\right)\lambda + \frac{3}{4}\left(1-\frac{3}{2}\delta\right)\lambda\left(1+\frac{2}{3}h\left(1-\frac{3}{2}\delta\right)\lambda + \frac{2}{3}J1\sqrt{-\delta\lambda}\right)\right)\right. \\ &\left. + J1\left(\frac{1}{4}\sqrt{-\delta\lambda} + \frac{3}{4}\sqrt{-\delta\lambda}\left(1+\frac{2}{3}h\left(1-\frac{3}{2}\delta\right)\lambda + \frac{2}{3}J1\sqrt{-\delta\lambda}\right)\right)\right)y_n \end{aligned} \tag{5}$$

where the intermediate stages, the drift and diffusion functions are

$$Y_1 = y_n, \quad Y_2 = y_n + \frac{2}{3}hf(Y_1) + \frac{2}{3}J1g(Y_1), \; f(Y_1) = \left(1-\frac{3}{2}\delta\right)\lambda y_n, \quad g(Y_1) = \sqrt{-\delta\lambda}\,y_n,$$

$$f(Y_2) = \left(1-\frac{3}{2}\delta\right)\lambda\left(y_n + \frac{2}{3}h\left(1-\frac{3}{2}\delta\right)\lambda y_n + \frac{2}{3}J1\sqrt{-\delta\lambda}\,y_n\right),$$

$$g(Y_2) = \sqrt{-\delta\lambda}\left(y_n + \frac{2}{3}h\left(1-\frac{3}{2}\delta\right)\lambda y_n + \frac{2}{3}J1\sqrt{-\delta\lambda}\,y_n\right).$$

The transfer function, $G_{n+1}(\lambda h, \delta)$ at time, t_{n+1} is given by

$$G_{n+1_{Burrage\ Scheme}} = 1 + h\left(\frac{1}{4}\left(1-\frac{3}{2}\delta\right)\lambda + \frac{3}{4}\left(1-\frac{3}{2}\delta\right)\lambda\left(1+\frac{2}{3}h\left(1-\frac{3}{2}\delta\right)\lambda + \frac{2}{3}J1\sqrt{-\delta\lambda}\right)\right)$$
$$+ J1\left(\frac{1}{4}\sqrt{-\delta\lambda} + \frac{3}{4}\sqrt{-\delta\lambda}\left(1+\frac{2}{3}h\left(1-\frac{3}{2}\delta\right)\lambda + \frac{2}{3}J1\sqrt{-\delta\lambda}\right)\right) \quad (6)$$

By using the same procedure as Platen scheme, the MS-stability function, G_1 of Burrage scheme is obtained as

$$G_{1_{Burrage\ Scheme}} = 1 + \left(\frac{73}{8}\delta^2 + 2 - 9\delta\right)\overline{\lambda^2} + \left(1 - 6\delta - \frac{27}{4}\delta^3 + \frac{45}{4}\delta^2\right)\overline{\lambda^3}$$
$$+\left(\frac{27}{8}\delta^2 + \frac{1}{4} + \frac{81}{64}\delta^4 - \frac{27}{8}\delta^3 - \frac{3}{2}\delta\right)\overline{\lambda^4} + (2-5\delta)\overline{\lambda} \quad (7)$$

The stability region of the stability function of (7) is illustrated in Figure 2.

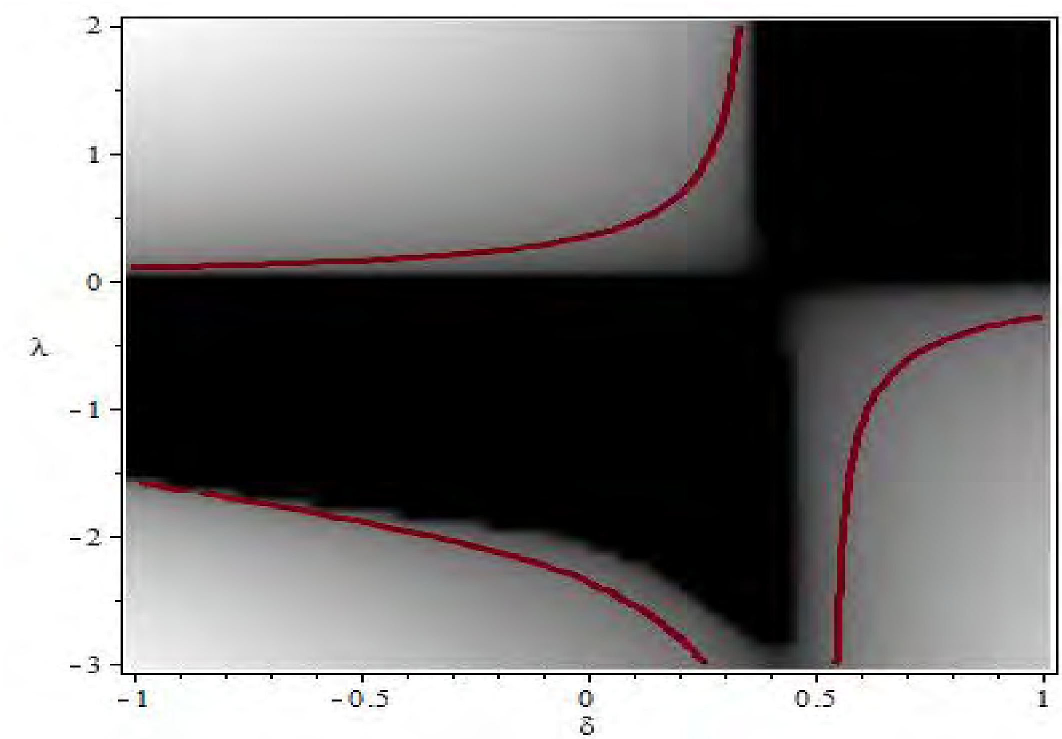

Figure 2 The stability region of a stability function (7)

2.2.2 MS-Stability of Semi-Implicit SRK2 Method

The MS-stability function of semi-implicit SRK2 that was proposed by Tian and Burrage (2002) is calculated in this section. We apply SRK2 of semi-implicit to the linear test equation (2). The approximate solution for the process, y at time, t_{n+1} is

$$y_{n+1}=\Bigg(1+h\Bigg(\frac{1}{4}\Bigg(\frac{1}{4}\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)+\frac{3}{4}\Big(1-\frac{3}{2}\delta\Big)\lambda\Bigg(1+h\Bigg(-\frac{2}{3}\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)+\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h+\sqrt{-\delta\lambda}J1\Big)\Bigg)+\frac{2}{3}\sqrt{-\delta\lambda}J1\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)\Bigg)\Bigg)\Bigg)$$
$$+J1\Bigg(\frac{1}{4}\sqrt{-\delta\lambda}\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)+\frac{3}{4}\sqrt{-\delta\lambda}\Bigg(-\frac{2}{3}\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)+\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h+\sqrt{-\delta\lambda}J1\Big)\Bigg)+\frac{2}{3}\sqrt{-\delta\lambda}J1\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)\Bigg)\Bigg)y_n \tag{8}$$

with the intermediate stages, drift and diffusion functions are

$$\overline{Y_1}=y_n,\quad Y_1=y_n+hf(\overline{Y_1}),\quad \overline{Y_2}=y_n+hf(\overline{Y_1})+J1g(\overline{Y_1}),$$
$$Y_2=y_n+h\left(-\frac{2}{3}f(Y_1)+f(\overline{Y_2})\right)+\frac{2}{3}J1g(Y_1),\ f(\overline{Y_1})=\left(1-\frac{3}{2}\delta\right)\lambda y_n,\ g(\overline{Y_1})=\sqrt{-\delta\lambda}\,y_n,$$
$$f(Y_1)=\left(1-\frac{3}{2}\delta\right)\lambda\left(y_n+\left(1-\frac{3}{2}\delta\right)\lambda h y_n\right),\ g(Y_1)=\sqrt{-\delta\lambda}\left(y_n+\left(1-\frac{3}{2}\delta\right)\lambda h y_n\right).$$

The transfer function, $G_{n+1}(\lambda h,\delta)$ at time, t_{n+1} of (8) is

$$G_{n+1_{\text{SemiImplici}}}=1+h\Bigg(\frac{1}{4}\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)+\frac{3}{4}\Big(1-\frac{3}{2}\delta\Big)\lambda\Bigg(1+h\Bigg(-\frac{2}{3}\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)+\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h+\sqrt{-\delta\lambda}J1\Big)\Bigg)+\frac{2}{3}\sqrt{-\delta\lambda}J1\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)\Bigg)\Bigg)$$
$$+J1\Bigg(\frac{1}{4}\sqrt{-\delta\lambda}\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)+\frac{3}{4}\sqrt{-\delta\lambda}\Bigg(1+h\Bigg(-\frac{2}{3}\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)+\Big(1-\frac{3}{2}\delta\Big)\lambda\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h+\sqrt{-\delta\lambda}J1\Big)\Bigg)+\frac{2}{3}\sqrt{-\delta\lambda}J1\Big(1+\Big(1-\frac{3}{2}\delta\Big)\lambda h\Big)\Bigg)\Bigg)\Bigg) \tag{9}$$

and the MS-stability function of SRK2 (9) is

$$G_{1_{\text{SemiImplicit}}}=1+\left(\frac{3}{2}-\frac{55}{4}\delta-\frac{87}{4}\delta^3+\frac{127}{4}\delta^2\right)\overline{\lambda^3}+\left(\frac{3}{4}-9\delta-\frac{1371}{32}\delta^3+\frac{2655}{128}\delta^4+\frac{997}{32}\delta^2\right)\overline{\lambda^4}$$
$$+\left(\frac{1}{4}+\frac{2889}{64}\delta^4-\frac{1053}{64}\delta^5+\frac{183}{8}\delta^2-\frac{189}{4}\delta^3-\frac{19}{4}\delta\right)\overline{\lambda^5}$$
$$+\left(-\frac{9}{16}\delta+\frac{135}{64}\delta^2+\frac{1215}{256}\delta^4+\frac{729}{1024}\delta^6-\frac{729}{256}\delta^5+\frac{1}{16}-\frac{135}{32}\delta^3\right)\overline{\lambda^6}$$
$$+\left(-\frac{23}{2}\delta+\frac{103}{8}\delta^2+2\right)\overline{\lambda^2}+(2-5\delta)\overline{\lambda} \tag{10}$$

The stability region of the stability function (9) is illustrated in Figure 3.

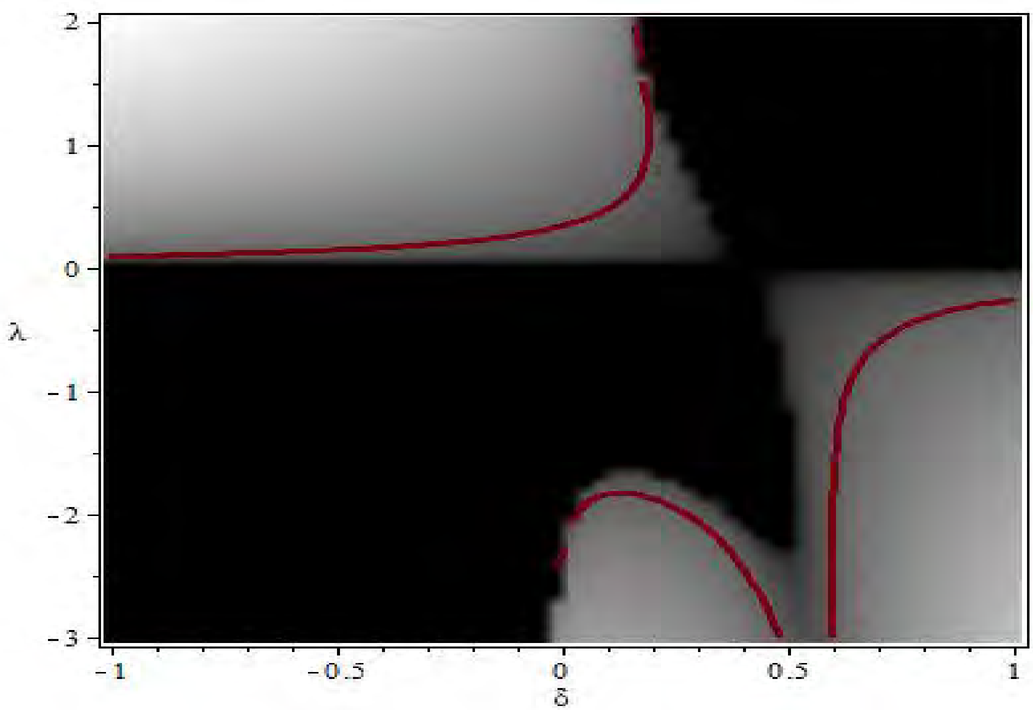

Figure 3 The stability region of a stability function (9)

Based on Figures 1, 2 and 3, it is clear that the semi-implicit SRK2 method shows better stability result compare than the explicit Burrage and Platen's scheme. It can be confirmed by performing numerical experiments that is presented in the next section.

3.0 NUMERICAL EXPERIMENT

We carried out the numerical experiment to examine the stability properties of the explicit and semi-implicit of SRK2 methods. The following numerical experiments show that the step size, h influences the mean-square stability of the SRK2 methods. Linear SDE (2) is used as a test equation by choosing a set of parameters $\lambda = -2$ and at the critical point of $\delta = 0.5$ with a step size of 1.0, 0.5, 0.25 and 0.125. Therefore, we have

$$dy = -0.4y dt + 0.6325 y \circ dW(t) \tag{11}$$

We estimate the second moment of y_T for $T \in [0,10]$. We compute the expectation of $|y_n|^2$ for $N = 10$ sample paths with 5 batches, that is

$$E|y_n|^2 = \frac{1}{5\times 10}\sum_{i=1}^{50} |y_n(\varpi_i)|^2 .$$

The results are illustrated in Figures 4a, 4b and 4c.

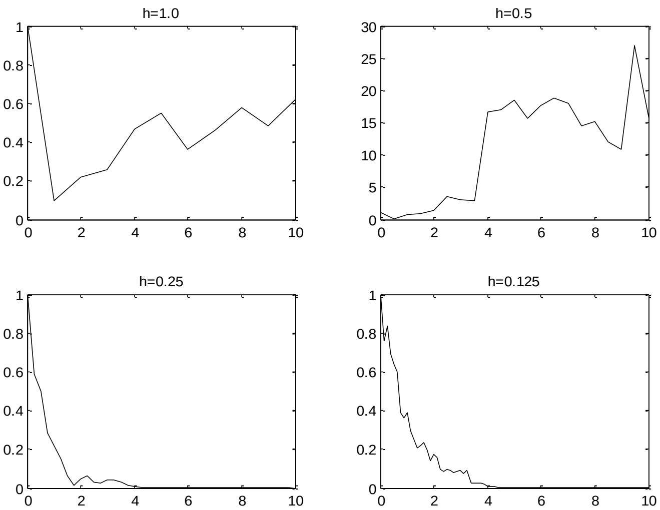

Figure 4a: Numerical solution of SDE (11) via SRK2 of Platen's scheme.

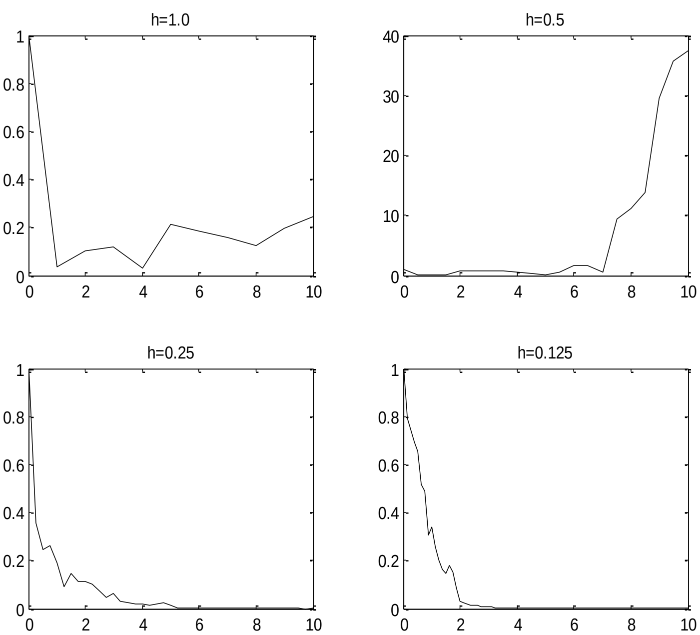

Figure 4b: Numerical solution of SDE (11) via SRK2 of Burrage's scheme.

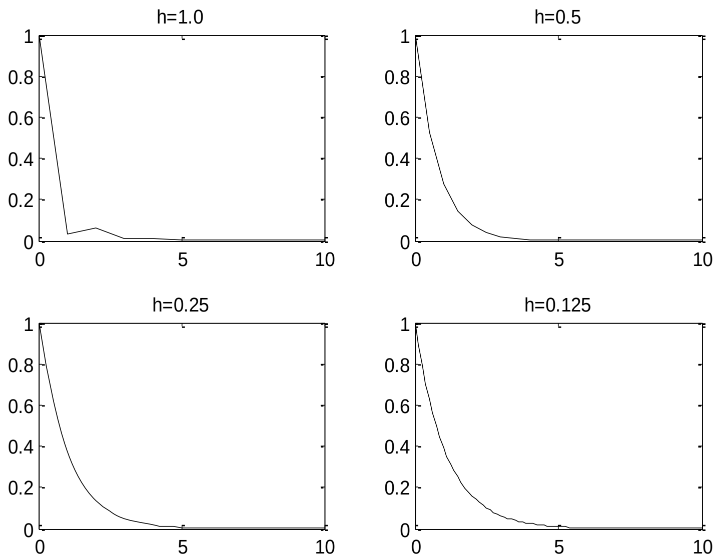

Figure 4c: Numerical solution of SDE (11) via semi-implicit of SRK2 method.

In Figure 4a and 4b, we apply SRK2 of explicit Platen and Burrage schemes, respectively to simulate the solution of (11). As the values of step size increases ($h = 1.0, 0.5$), the results are numerically unstable. However, for $h = 0.25, 0.125$ the numerical solution of SDE (11) show the stability of the solution. When the semi-implicit method of SRK2 is used to solve SDE (11), the solutions tend to zero for all values of $h = 1.0, 0.5, 0.25$ and 0.125 as shown in Figure 4c. This indicates that the semi-implicit method is numerically stable compare than the explicit methods of Burrage and Platen schemes.

4.0 CONCLUSION

We have presented the stability function and stability region for explicit and semi-implicit derivative-free SRK2 methods for a linear test equation (1). It can be seen that, the semi-implicit SRK2 method shows better stability region compare than explicit schemes. The theoretical finding is confirmed by the numerical experiment. For various values of step size, semi-implicit method that was performed to a linear test equation indicates numerical stability. Whereas, the explicit SRK2 methods of Platen and Burrage schemes show numerical instability for certain values of step size.

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