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Numerical study of unsteady Casson fluid flow and heat transfer over a stretching surface with modified magnetic field effects

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ABSTRACT – Numerical study of unsteady Casson fluid flow and heat transfer over a stretching surface with modified magnetic field effects is investigated. The governing partial differential equations are transformed into ordinary differential equations by using similarity transformation which then solved numerically by Keller box method. The validation process is conducted by comparing the present results to previous study. The effects of several parameters on the fluid flow and heat transfer are analysed and discussed. Results show that the increasing of Casson and aligned angle parameters suppressed the fluids velocity. Meanwhile, the unsteadiness parameter led to decrease the fluid temperature.

1. INTRODUCTION

Boundary layer flow over a stretching surface is an important type of a flow in several engineering process. For example, many extrusion processes involving the cooling of continuous strips or filaments by drawing them through a quiescent fluid. These phenomena can be related to the boundary layer along a moving flat plate. In order to obtain a thorough cognition of non-Newtonian fluids and their various applications, it is necessary to study their flow behaviours. It is well known that mechanics of non-Newtonian fluids present a special challenge to engineers, physicists and mathematicians.

In this study, we consider the non-Newtonian fluid which is Casson fluid because the Casson fluid model is stated to fit rheological data better than general viscoplastic models for many materials [1]. The non-linear Casson's constitutive equation has been found to describe accurately the flow curves of suspensions of pigments in lithographic varnishes used for preparation of printing inks and silicon suspensions. Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear [2].

Therefore, the present work will focus on the convection boundary layer flow and heat transfer over a stretching surface immersed in Casson fluid with modified magnetic field effects. The scope of this study is only limited to the mathematical computation.

2. METHODOLOGY

2.1 Mathematical formulation

Laminar boundary layer two-dimensional flow and heat transfer of an incompressible, conducting non-Newtonian Casson fluid over an unsteady stretching surface is considered. The unsteady fluid and heat flows start at $t = 0$. The surface emerges out of a slit at origin ($x = 0,\ y = 0$) and moves with non-uniform velocity $U(x,t) = cx(1-\alpha t)^{-1}$ where $c > 0$, $\alpha \geq 0$ are constants with dimensions $(\text{time})^{-1}$, and c is the initial stretching rate. The non-uniform aligned magnetic field is assumed to be of variable kind and is chosen in its special form as $B(x,t) = B_0(1-\alpha t)^{-1/2}$ with acute angle γ along y direction [3].

Under the boundary layer approximations, the continuity, momentum and energy equations of such type of flow can be written as [2].

$$\frac{\partial u}{\partial u} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B(x,t)^2}{\rho}\sin^2(\gamma)u \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the appropriate boundary conditions,

$$\begin{aligned} &u = U(x,t),\ v = 0,\ T = T_w(x,t) \ \text{ at } \ y = 0, \\ &u \to 0,\ T \to T_\infty \ \text{ as } \ y \to \infty \end{aligned} \tag{4}$$

It is noted that the continuity equation (1) is satisfied identically by introducing the stream function ψ such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The governing equations of the problem are simplified by using the following similarity transformation [4].

$$\eta = y\sqrt{\frac{c}{\nu(1-\alpha t)}}\,,\ \psi = \sqrt{\frac{c\nu}{(1-\alpha t)}}\,x f(\eta)\,, \qquad (5)$$
$$T = T_\infty + \frac{c}{2\nu x^2}(1-\alpha t)^{-3/2}\,\theta(\eta)$$

Substituting (5) into Equations (2)-(3) and boundary condition (4), we obtain:

$$\left(1+\frac{1}{\beta}\right)f''' + ff'' - f'^2 - A\left(f' + \frac{\eta}{2}f''\right) - M\sin^2(\gamma)f' = 0 \quad (6)$$

$$\frac{1}{\Pr}\theta'' + f\theta' + 2f'\theta - \frac{A}{2}(\eta\theta' + 3\theta) = 0 \quad (7)$$

where $A = \alpha/c$ is the unsteadiness parameter, $\Pr = \nu/\kappa$ is the Prandtl number, $M = \sigma B_0^{\,2}/\rho c$ is the magnetic parameter, β is the Casson fluid parameter and γ is the aligned angle. The boundary conditions (4) now becomes:

$$f' = 1,\ f = 0,\ \theta = 1 \ \text{at}\ \eta = 0, \qquad (8)$$
$$f' \to 0,\ \theta \to 0 \ \text{as}\ \eta \to \infty$$

The physical quantities of interest are the skin friction coefficient and the Nusselt number given by:

$$C_f\,\mathrm{Re}_x^{1/2} = f''(0)\,,\quad Nu_x/\mathrm{Re}_x^{1/2} = -\theta'(0)$$

2.2 Numerical computational

Equations (6) and (7) with boundary conditions (8) are solved numerically using the Keller box method and then programmed in Matlab software which comprises the following steps:

(a) Reduce the non-linear ordinary differential equations into first order system
(b) Finite Difference Discretization
(c) Linearization by using the Newton's Method
(d) Solve the linear system by the Block Tridiagonal Elimination Technique

3. RESULTS AND DISCUSSION

The results are validated by comparing the current results to previous results of Sharidan et al. [4] as shown in Table 1 and an excellent agreement between the results are exists. From Table 1, it is noted that Nusselt number, $-\theta'(0)$ increases with the increasing of Prandtl number, Pr. Meanwhile, skin-friction coefficient, $f''(0)$ is not affected by Pr.

Besides, the fluid velocity decreases with increasing values of β and γ as shown in Figure 1. Physically, rising values of γ strengthen the external magnetic field. Thus, it generates opposite force to the flow which is called Lorentz force. This force declines the velocity boundary layer thickness. Also, the increasing of A leads to decrease the fluid temperature as shown in Figure 2. In the steady case ($A = 0$), the fluid temperature overshoot its value at the surface. This behaviour is in similar trend with the result of Sharidan et al. [4].

Table 1 Comparison values of $f''(0)$ and $-\theta'(0)$ with published data when $A = 0.8, M = 0,\ \gamma = 0$ and $\beta \to \infty$

Pr	**Sharidan et al. [4]**		**Present Results**	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.1	-1.261042	0.229433	-1.2601097	0.2292686
1.0	-1.261042	0.471190	-1.2601097	0.4699101
10.0	-1.261042	0.510385	-1.2601097	0.4962170

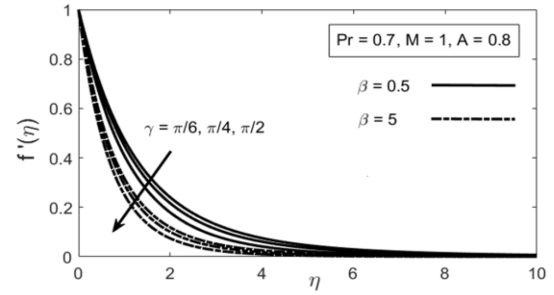

Figure 1 The velocity profiles for variation of β and γ

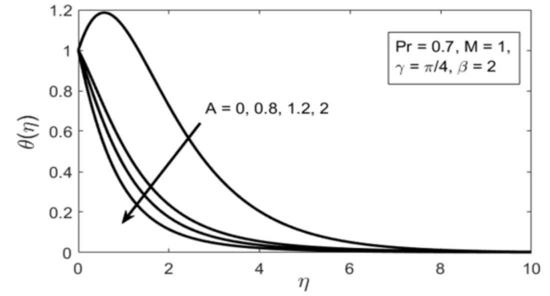

Figure 2 The temperature profiles for variation of A

4. CONCLUSION

Numerical study of the related problem is investigated. Overall, the finding from this study could benefit various sectors including industrial processes as well as to the engineers, physicists and mathematicians.

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