

# Diagonalization of Covariance Matrix in Simultaneous Localization and Mapping of Mobile Robot

Maziatun Mohamad Mazlan<sup>1</sup>, Nur Aqilah Othman<sup>2</sup> and Hamzah Ahmad<sup>2</sup>

<sup>1</sup>Electrical Engineering Department, Politeknik Sultan Haji Ahmad Shah, 25350 Kuantan, Pahang, Malaysia

<sup>2</sup>Faculty of Electrical and Electronics Engineering, Universiti Malaysia Pahang, 26600 Pekan, Pahang, Malaysia  
nuraqilah@ump.edu.my

**Abstract.** The purpose of this study is to analyze the effects of covariance state update by means of modified algorithm of diagonal matrix using eigenvalue, and diagonalization function in MATLAB on the computational cost of extended Kalman filter based Simultaneous Localization and Mapping (SLAM). The multiplications of the covariance matrix with other parameters increase its dimension, which is twice the number of landmarks and might result in erroneous estimation. This motivates this study in searching for ways to reduce the computational cost of the covariance matrix without minimizing the accuracy of the state estimation using eigenvalue method. The matrix diagonalization strategy which is applied to the covariance matrix in EKF-based SLAM must be examined to simplify the multiplication procedure. Therefore, improvement is needed to find better diagonalization method. Simulation results demonstrate that MATLAB's built-in diagonalization function can reduce the computational cost.

**Keywords:** covariance, diagonalization, eigenvalue, extended Kalman filter, localization.

## 1 Introduction

These days, there has been expanding business enthusiasm for new utilizations of a robot other than modern creation. Not restricted to just mechanical robots, portable robots have broadening their applications in various distinctive areas from indoor robots to outside, space, in military or unsafe region, submerged, and airborne framework. Nevertheless, it has shown a great deal of difficulties to defeat in seeking after a genuinely self-governing robot. One of the investigation districts that is acknowledged to give a response for this issue is known as Simultaneous Localization and Mapping (SLAM) issue.

The Simultaneous Localization and Mapping (SLAM) problem asks if it is possible for a mobile robot to move autonomously and observing its surrounding in an obscure situation with no earlier data on its area, and have it simultaneously decided its location.

In realizing a solution for SLAM problem, researcher has to deal with several issues such as uncertainties, data association, and feature extraction. This examination reveals the vulnerabilities impact to the covariance state as it adds to computational cost of Extended Kalman Filter based Simultaneous Localization and Mapping issue.

In recent years, there are few methodologies have been presented, for example, extended Kalman filter (EKF), unscented Kalman filter (UKF), and Particle filter for estimation purposes. A large portion of those strategies share the indistinguishable specialized properties where they depend on Bayesian strategy. Between previously mentioned approaches, EKF are the most commended technique for SLAM arrangement. The reason could be because of the channel offers simpler calculation to take after and has brought down computational cost contrasted with others.

Extended Kalman filter (EKF) is a good way to learn about SLAM because of simplicity whereas probabilistic method is complex, but they handle uncertainty better contrasted with different methodologies, for example particle filter. Nevertheless, the entire covariance matrix in EKF-based SLAM should be refreshed each time another point of interest is identified. This methodology includes bunches of numerical tasks, subsequently will raise the computational cost. Besides, the measurement of covariance matrix will increase to double the quantity of historic point, as more landmarks are detected. It is known that the cost of  $O(m^2)$ , in which  $m$  is a total landmark of the map in EKF-based SLAM. For large environment the used of EKF is limited (only hundred landmarks). As the landmarks increase it will affect the full covariance structure which is responsive to the effects of linearization errors which build up through time.

Therefore, this study is focused on MATLAB simulation coding modification where the MATLAB command for finding eigenvalue and diagonal matrix directly used without changing the structure of diagonal

matrix. One of the process of finding the eigenvalue is done by some MATLAB simulation coding modification that designs so that the structure of covariance matrix will be diagonalized.

Guivant and Nebot developed a decorrelation calculation with improve the covariance matrix [1]. A subset of the states will decorrelate the calculation that is feebly associated and drop the pitifully cross-relationship terms in the covariance matrix. To diminish both computational and capacity costs in SLAM a positive semi definite matrix is added to the covariance matrix.

Besides decorrelation algorithm, Julier and Uhlmann presented a covariance convergence strategy for SLAM, a combination system that consolidates two covariances when the relationships between them are obscure [2], and this technique has been implemented not only in SLAM, but also in other applications [3] and this procedure has been executed in SLAM, as well as in different applications. In this strategy, the refresh procedure is completed in two autonomous advances; refreshing the robot, at that point refreshing the historic point. In addition, there exists a parameter  $\omega$  in the calculation that should be defined beforehand.

Besides in Kalman filter, a method of decorrelating some of the updated state covariance of the H infinity filter has been suggested by Ahmad and Othman [4], in order to reduce the cost computation. This is due to Finite Escape Time problem in H infinity filter based Simultaneous Localization and Mapping problem as the Finite escape time has been one of the obstacles that holding the realization of H infinity Filter in many applications. As their discoveries, they proposed an outcome if the full rank of delta P is added to the covariance and without association with the other state components. However, there is also a possibility that may result unbounded uncertainties in the estimation of SLAM problem.

This study is conducted to analyze the behavior effects of covariance state update of different MATLAB simulation coding between modification algorithm of diagonal matrix using eigenvalue [5] and using algorithm build-in function of diagonalization directly. The results of the cause on the estimation and covariance behavior are presented, which have been obtained through simulations.

## 2 Issue of Formulation

### 2.1 Model of Simultaneous Localization and Mapping (SLAM)

The equation of discrete time dynamic system can be represented for SLAM concerning process and observation model. The movements of the mobile robot illustrated from process model while the measurement of the map features described by the observation model with reference to the mobile robot position. Both models (i.e. process and observation model) of SLAM depicts on Fig. 1. An equation that represent SLAM process model from time  $k$  to time  $k + 1$  for linear system is stated as

$$X_{k+1} = F_k X_k + G_k U_k + w_k \quad (1)$$

in which the state of landmarks and mobile robot represented by  $X_k$ , the state transition matrix represented by  $F_k$ , the control matrix represented by  $G_k$ , the control inputs represented by  $U_k$ , and the zero-mean Gaussian process noise with covariance  $Q$  represented  $w_k$ .

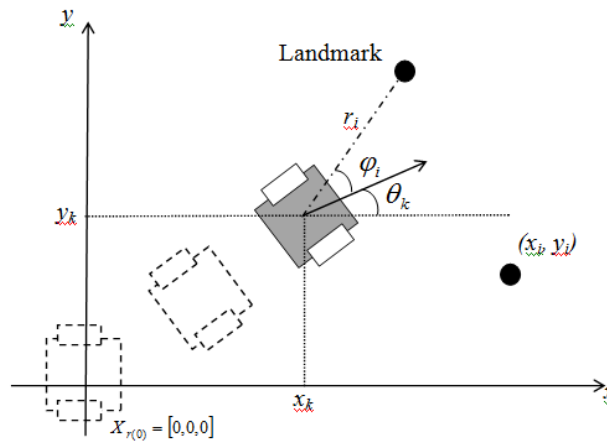


Fig. 1. SLAM model.

A combined state vector of landmarks  $X_m$  and robot  $X_r$  represent the state vector of 2D SLAM at time  $k$  as follows

$$X_k = \begin{bmatrix} X_r \\ X_m \end{bmatrix} = \begin{bmatrix} \theta_k \\ x_k \\ y_k \\ x_i \\ y_i \end{bmatrix} \quad (2)$$

where the middle location of the mobile robot with reference to global coordinate frame represented by  $x_k$  and  $y_k$  and the direction angle of the mobile robot represented by  $\theta_k$ . The Cartesian coordinate  $(x_i, y_i)$ ,  $i=1, 2, \dots, m$  is correspond to the landmark model where  $m$  is number of landmarks. A model of two-wheel mobile robot is applied through this study.  $X_r = [\theta_k \ x_k \ y_k]^T$  is applied to signify the robot position or in this study sometimes we denote it as robot pose. The kinematic movement of mobile robot that illustrate the process form defined as  $X_{r(k+1)} = f(X_{r(k)}, u_k, \delta\omega, \delta v)$  and  $u_k = [\omega_k \ v_k]^T$  in which

$$\begin{aligned} \theta_{k+1} &= \theta_k + (\omega_k + \delta\omega)T \\ x_{k+1} &= x_k + (v_k + \delta v)T \cos(\theta_k) \\ y_{k+1} &= y_k + (v_k + \delta v)T \sin(\theta_k) \end{aligned} \quad (3)$$

with mobile robot angular acceleration control inputs defined as  $\omega_k$  and mobile robot velocity with related process noises,  $\delta\omega$  and  $\delta v$  represent by  $v_k$ . The time interval of one movement step defined as  $T$ . As landmarks are assumed to be static, the process model for the landmarks  $[x_i, y_i]^T$  for  $i=1, 2, \dots, m$  is unaffected with zero noise.

$$X_{m(k+1)} = X_{m(k)} \quad (4)$$

State observation or measurement processes are represented using an observation model

$$z_{(k+1)} = \begin{bmatrix} r_i \\ \phi_i \end{bmatrix} = H_{k+1} X_{k+1} + V_{r_i \phi_i} \quad (5)$$

where the measurement matrix represented by  $H_k$  and the zero-mean Gaussian noise with covariance matrix  $R$  represented by  $V_{r_i \phi_i}$ . At time  $k+1$ , the observation of  $i^{\text{th}}$  landmark is a range  $r_i$  and bearing  $\phi_i$  which shows the relative distance and angle of the mobile robot to the observed landmarks. It is assumed that the sensor in the robot operates with a range sensor and a bearing that keeps the observation of the landmark in the environment as well as the encoder on the wheel for vehicle speed measurement. Range and bearing are defined as

$$r_i = \sqrt{(y_i - y_{k+1})^2 + (x_i - x_{k+1})^2} + v_{r_i} \quad (6)$$

$$\phi_i = \arctan\left(\frac{y_i - y_{k+1}}{x_i - x_{k+1}}\right) - \theta_{k+1} + v_{\theta_i} \quad (7)$$

where  $(x_{k+1}, y_{k+1}, \theta_{k+1})$  is current robot position,  $(x_i, y_i)$  is position of observed landmark,  $v_{r_i}$  and  $v_{\theta_i}$  are the noises on the measurements

## 2.2 SLAM Based on Extended Kalman Filter

In this study the extended Kalman filter (EKF) is applied to estimate the location of mobile robot and landmarks. First, based on the earlier system information, the state vector is predicted. After that using the measurement data received from the sensors, the state vector will be estimated. Parameters of interest are the updated state vector  $\hat{X}_k$  and the covariance matrix of the estimation  $P_k$ . The elaborations of prediction and estimation of EKF are stated as follows.

### A. Prediction (update of time)

The estimation of the state vector at the instant  $k$  stated as

$$\hat{X}_k = [\hat{X}_r \quad \hat{X}_1 \quad \hat{X}_2 \quad \dots \quad \hat{X}_m]^T \quad (8)$$

and  $P_k$  is the covariance matrix of the estimation error. The process model (Equations 1 to 4) is linearized as an extension of the Taylor series about  $\hat{X}_k$  and thus leads to the next predicted state vector  $\hat{X}_{k+1}^-$  and error covariance matrix  $P_{k+1}^-$

$$\begin{aligned} \hat{X}_{k+1}^- &= f(X_k, u_k, 0, k) \\ P_{k+1}^- &= \nabla F_X P_k \nabla F_X^T + \nabla F_w Q_k \nabla F_w^T \end{aligned} \quad (9)$$

where the Jacobian of  $f$  with respect to  $X_k$  is represented by  $\nabla F_X$  and the Jacobian with respect to  $\omega_k$  is represented by  $\nabla F_\omega$  the Jacobian with respect to  $\omega_k$ . These Jacobians are valued from the Eq. 3 at  $\hat{X}_k$  and have the subsequent expressions:

$$\nabla F_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\omega T \sin \hat{\theta}_k & 1 & 0 & 0 \\ \omega T \cos \hat{\theta}_k & 0 & 1 & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix}, \nabla F_\omega = \begin{bmatrix} \nabla F_{\gamma\omega} \\ 0_m \end{bmatrix} \quad (10)$$

$I_m$  and  $0_m$  is the character and null matrix individually with fitting measurements relying upon the quantity of points of landmark watched while the inspecting rate is  $T$ . There is no procedure noise for the landmarks as they are assumed to be stationary constantly.

### B. Updated (update estimation)

The equation of the state vector and the error covariance matrix in the update procedure is framed by linearizing the observation model (Eq. 5) through the Taylor series development about  $\hat{X}_{k+1}^-$ . With the accessibility of estimation data information  $z_{k+1}$ , the mobile robot updates its current position relative to the position of observed landmarks

$$P_{k+1}^+ = P_{k+1}^- - K_{k+1} (\nabla H_i P_{k+1}^- \nabla H_i^T + R_k) K_{k+1}^T$$

$K_{k+1}$  is a Kalman gain and has the following definition:

$$K_{k+1} = P_{k+1}^- \nabla H_i^T (\nabla H_i P_{k+1}^- \nabla H_i^T + R_k)^{-1} \quad (11)$$

The Jacobian is represented by  $\nabla H_i$  in Eq. 5 with refer to  $X_k$  assessed at  $\hat{X}_{k+1}^-$  and expressed as follows:

$$\nabla H_i = \begin{bmatrix} 0 & -\frac{dx}{r} & -\frac{dy}{r} & \frac{dx}{r} & \frac{dy}{r} \\ -1 & \frac{dy}{r^2} & -\frac{dx}{r^2} & -\frac{dy}{r^2} & \frac{dx}{r^2} \end{bmatrix} \quad (12)$$

with

$$dx = \hat{x}_i^- - \hat{x}_{k+1}^-, \quad dy = \hat{y}_i^- - \hat{y}_{k+1}^-, \quad r = \sqrt{dx^2 + dy^2} \quad (13)$$

## 2.3 State Error Covariance Matrix

Commonly, the covariance of two variants is the measure of the correlation between two variables. The amount of linear dependence between variables can be measured by the correlation theory. Matrix arrangement of robot position and landmarks covariance matrices and correlation among the robot and landmarks is the covariance matrix of a state estimate in SLAM. In SLAM the covariance matrix,  $P$  is defined as

$$P = \begin{bmatrix} P_{RR} & P_{RM} \\ P_{RM}^T & P_{MM} \end{bmatrix} \quad (14)$$

Covariance matrix of robot position is represented  $P_{RR}$ , covariance matrix of the landmark position is represented by  $P_{MM}$  and cross-covariance matrix of the robot and landmarks or cross-correlation between them is represented by  $P_{RM}$ .

In SLAM, the covariance matrix indicates the error associated with the estimates of the state of the robot and of the reference point. As of the covariance matrix information, the uncertainties and errors of estimation can be monitor either raise or decrease, in which they stand for the accuracy and consistency of the estimate. Consequently, it is very crucial to research the behavior of the covariance matrix as it contributes significant issue in SLAM.

**Proposition 1:** *The determinant of the error covariance matrix is a measure of the volume of the uncertainty ellipsoid associated with the state estimate, which indicate the total uncertainty of that particular state estimation [6].*

The state error covariance in SLAM having dimension of  $(3 + 2m)^2$ , where  $m$  is a landmark. As robot detected the latest landmarks in its area the size of state error covariance will be increased. Eq. 15 represents the state error covariance matrix for SLAM. The covariance of the state error indicates the error associated with the estimates of the status of the reference points and of the robot, as defined in Proposition 1. The precision and consistency of the estimation could be represented by the state error covariance where the increment and decrement of uncertainties could be observed. The smaller covariance value shows the better estimation. Nevertheless, if the actual value is bigger than the covariance value, the estimation is look like to have an inaccuracy, but the covariance indicates lesser value, then the estimation in this condition is said to be an optimistic estimation. The optimistic estimation is one of the problems in SLAM which have to take into account in EKF-based SLAM.

$$\begin{bmatrix} P_{\theta\theta} & P_{\theta x} & P_{\theta y} & P_{\theta m_1, x} & P_{\theta m_1, y} & \dots & \dots & P_{\theta m_n, x} & P_{\theta m_n, y} \\ P_{x\theta} & P_{xx} & P_{xy} & P_{xm_1, x} & P_{xm_1, y} & \dots & \dots & P_{xm_n, x} & P_{xm_n, y} \\ P_{y\theta} & P_{yx} & P_{yy} & P_{ym_1, x} & P_{ym_1, y} & \dots & \dots & P_{ym_n, x} & P_{ym_n, y} \\ P_{m_1, x, \theta} & P_{m_1, x, x} & P_{m_1, x, y} & P_{m_1, x, m_1, x} & P_{m_1, x, m_1, y} & \dots & \dots & P_{m_1, x, m_n, x} & P_{m_1, x, m_n, y} \\ P_{m_1, y, \theta} & P_{m_1, y, x} & P_{m_1, y, y} & P_{m_1, y, m_1, x} & P_{m_1, y, m_1, y} & \dots & \dots & P_{m_1, y, m_n, x} & P_{m_1, y, m_n, y} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \dots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \dots & \vdots & \vdots \\ P_{m_n, x, \theta} & P_{m_n, x, x} & P_{m_n, x, y} & P_{m_n, x, m_1, x} & P_{m_n, x, m_1, y} & \dots & \dots & P_{m_n, x, m_n, x} & P_{m_n, x, m_n, y} \\ P_{m_n, y, \theta} & P_{m_n, y, x} & P_{m_n, y, y} & P_{m_n, y, m_1, x} & P_{m_n, y, m_1, y} & \dots & \dots & P_{m_n, y, m_n, x} & P_{m_n, y, m_n, y} \end{bmatrix} \quad (15)$$

### 3 Matrix Diagonalization

A matrix in which the top and bottom elements are all null is called the diagonal matrix. The contents of diagonal elements may fill up either with value or also null. For a  $n \times n$  square matrix is said to be diagonal matrix if it stated as

$$\begin{aligned} \text{Let the elements of } D &= (d_{i,j}) \\ (d_{i,j}) &= 0 \text{ if } i \neq j \forall i, j \in \{1, 2, \dots, n\} \end{aligned} \quad (16)$$

The multiplication step of the matrix is easier for a diagonal matrix where just diagonal elements are concerned and this will make the operation faster and will require a lower computational cost if applied in SLAM. Let  $A$  be a  $n \times n$  square matrix. It is believed that there be present a number and a column matrix  $B$  with dimension of such that

$$AB = \lambda B$$

With the matching eigenvector  $B$ ,  $\lambda$  is define as an eigenvalue of  $A$ . Then  $A$  is diagonalizable to a matrix  $D$ . There will usually be  $n$  number of eigenvalues for each  $n \times n$  matrix, in which the eigenvalues could be actual, complex or the join of mutually numbers.

**Definition 1:** Let  $A$  be a  $n \times n$  square matrix and  $D$  is a diagonal matrix in which its diagonal elements are the eigenvalues of  $A$ , such as follows:

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \lambda_n \end{bmatrix} \quad (17)$$

Therefore, there exists the following relationship between matrix  $A$  and matrix  $D$ :

$$\begin{aligned} \det(A) &= \det(D) \\ \text{norm}(A) &= \text{norm}(D) \end{aligned} \quad (18)$$

Therefore, referring to Proposition 1 and to the behavior of the diagonal matrix presented by Eq.18, the possibility exists that the diagonalization through eigenvalues may be one of the alternative techniques to minimize the computational cost of SLAM based on EKF. The method was also motivated by the earlier works of [1] and [7] that mainly investigated and discussed about the diagonalization of the updated state covariance matrix.

### 3.1 Diagonalization of Covariance Matrix for EKF-based SLAM

The behavior of covariance and estimation performance is examined through simulation by the effect of diagonalizing the covariance matrix. This is because the multiplication steps in covariance calculation become simple, as an effort to decrease the computational difficulty as well as computational cost. As the only diagonal elements involve in multiplication of a matrix with another diagonal matrix, it is much simpler and quicker. The study has been conducted based on two case studies:

- (1) Estimated covariance for both (robot and landmark) is diagonalized using eigenvalue MATLAB function and reconstruction of diagonal matrix as in [5].
- (2) Estimated covariance for both (robot and landmark) is diagonalized using MATLAB function of diagonalization directly.

For the case study 1 the eigenvalue of estimated covariance is first calculated using the function stated as follows:

$$\lambda_n = \text{eig}(P_{k+1}^+)$$

where  $\lambda_n$  represent the eigenvalue and  $(P_{k+1}^+)$  represent the estimated covariance. Then the diagonal matrix will be build using the next function stated as follows:

$$P_{(D),k+1}^+ = \text{diag}(\lambda_n)$$

where  $P_{(D),k+1}^+ = \text{diag}(\lambda_n)$  is a diagonal matrix that built from eigenvalue.

For the case study 2 the eigenvalue of estimated covariance is directly calculated using the MATLAB function stated as follows:

$$[V, D] = \text{eig}(P_{k+1}^+)$$

where  $[V, D]$  is the MATLAB function of finding diagonal matrix directly. Next the diagonal matrix will be build using the function stated as follows:

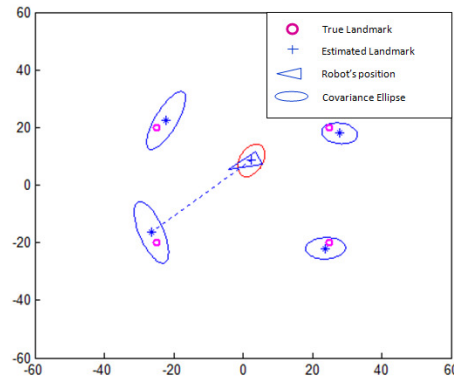
$$P_{(D),k+1}^+ = D$$

The above two cases are analyzed separately to find out the consistency and reliability of the proposed method.

## 4 Simulation Results and Discussions

The analyses through simulation for two case studies of different diagonalization algorithm coding are presented to examine the behavior of the estimation and covariance matrix of EKF-based SLAM.

Fig. 2 depicts the estimation of the the landmarks and mobile robot position under normal condition, (i.e. using normal covariance matrix). The simulation time for the mobile robot is 1000s and continuously detects the landmarks for every loop of motion, with constant speed. The uncertainties of the estimation are represented by the covariance-ellipses. Better estimation will show smaller ellipse.



**Fig. 2.** Position estimation and covariance under normal condition.

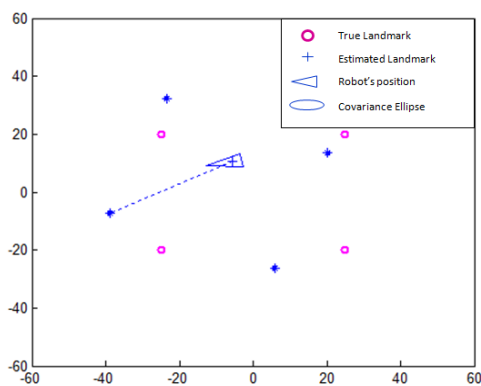
The simulations for two case studies as defined in Section 3 are conducted using the same parameters. The behavior of covariance through estimation in the first case as depicted in Fig. 3, while Fig. 4 shows the result of the second case study. It is apparent in the previously mentioned figures, that the estimation of the position of mobile robot and landmarks is possible even when the whole covariance is diagonalized through the technique defined in Section 3. However, the estimations demonstrated some acceptable errors. Estimation of landmark position in case 2 is better than that of in case 1. Moreover, this shows that the diagonalization method through finding of eigenvalues and rebuilding a diagonal matrix as defined in [5], is able to produce similar prediction behavior as compared to built-in MATLAB function of matrix diagonalization.

On the other hand, the covariance behaves unusual in both cases, where the covariance decreases suddenly, and it is too tiny compared to the normal covariance as illustrated in Fig. 2. This situation illustrates the optimistic estimation as described in Section 2.3 of this paper.

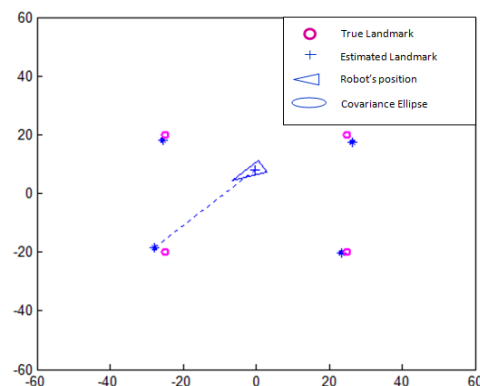
The comparison of processing time of all methods is depicted in Table 1. Diagonalization method using built-in function in MATLAB (Case 2) was found to be the fastest among all cases, about 15% faster than the normal condition. Case 1 also completed about 13% faster than the normal condition, despite additional steps taken in diagonalizing the covariance matrix. Furthermore, Case 2 produces more accurate estimation compared to Case 1 as shown in Fig. 3 and Fig. 4. This is demonstrated by the true landmark being closer to the estimated landmark in Case 2.

**Table 1.** Processing time for all cases.

Covariance type	Simulation time (s)	Total processing time (s)	Percentage of processing time reduction (%)
Normal	1000	96.3977	
Case 1	1000	83.9984	12.86
Case 2	1000	81.639	15.31



**Fig. 3.** Estimation of the state and covariance behavior of case one.



**Fig. 4.** Estimation of the state and covariance behavior of case two.

## 5 Conclusion

This paper presented an analysis of EKF-based SLAM execution under the states of diagonalized covariance of two cases as discussed in Section 3. Case 1 diagonalizes the covariance matrix through eigenvalues, while Case 2 makes use of built-in diagonalization function in MATLAB. It was found that Case 2 completed the fastest compared to Case 1 and the normal condition. In addition, it was found that Case 2 produces more accurate estimation than Case 1. Future work will be using the build-in MATLAB function to further investigate on how to reduce computational cost of mobile robot SLAM. In addition, correction of optimistic value of covariance matrix is the subject of future investigation.

## 6 Acknowledgement

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