

# Forward Problem Solving for Non-Invasive Electrical Resistance Tomography System

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## Abstract

This paper aims to provide a forward problem solving for non-invasive electrical resistance tomography. A finite element model (FEM) using COMSOL Multiphysics is implemented for generating the sensitivity map for ERT system. Later, a masking data for a better sensitivity map was done to optimize the map. As a result, the sensitivity map can be used later for reconstructing the image of the medium of interest.

**Keywords:** ERT, forward problem, sensitivity map, COMSOL Multiphysics, MATLAB

## 1. Introduction

The image reconstruction of process tomography, can be divided into two parts: forward problem and inverse problem [1]. The forward problem is solved first in order to know the theoretical value of each of the sensors output based on the signal projection, whereas the algorithm is solved later in the inverse problem of getting the tomogram. The ERT is a non-linear model and it is difficult to be solved analytically. Therefore, a finite element model (FEM) is preferable to solve the forward problem in ERT.

In this paper, a forward problem solving will be the main focus. The finite element model will be utilized to get the sensitivity map of the ERT system followed by the masking data for a better sensitivity map.

## 2. Forward Problem

The forward problem also known as the sensitivity map of the system and can be divided into three solutions: linearization solution, numerical solution and analytical solution. The linearization solution is applied when the linear approximation to the relationship of the signal

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projection of sensors output is made for the system [2]. In addition, the numerical solution – for example, the finite element method (FEM) – is normally used for complex geometry, and the analytical method is used for simple geometry [1]. Most researchers applied numerical solution to solve the forward problem in industrial applications owing to the complex geometry. Also, the availability of FEM software in the market such as COMSOL Multiphysics software, ANSYS software, and EIDORS software help researchers to apply a numerical solution for solving the sensitivity map.

The sensitivity coefficient of a two-dimensional electrode pair for non-invasive ERT is based on Equation (1) [3].

$$M_{i,j}(x,y) = \frac{\Delta G}{\Delta \sigma} = \int_{A(x,y)} \frac{\mathbf{E}_i}{V_i} \cdot \frac{\mathbf{E}_j}{V_j} dx dy \quad (1)$$

According to Equation (1), the sensitivity coefficient for pair electrode  $i$ - $j$ ,  $M_{i,j}(x,y)$  to the conductivity change of the pixel at position  $(x,y)$  within the area  $A(x,y)$  is solved by using dot-multiplying between the  $i$ -electric field,  $\mathbf{E}_i$ , and  $j$ -electric field,  $\mathbf{E}_j$ , when  $V_i$  and  $V_j$  are applied, respectively. In addition, the soft-field nature produces a poor condition for sensitivity distribution in electrical tomography, which becomes gradually poorer when it moves towards the center of the vessel region [4]. In other words, the sensitivity distribution for electrical tomography is more sensitive near to the electrodes. The electrical field distribution in soft-field tomography also depends on the material characteristic inside the pipe region and it is inhomogeneous. Later on, based on the sensitivity map equation, the next sub-topic will discuss in detail how to generate the sensitivity map using finite element model software.

### 3. Generation of sensitivity map

A numerical approach using the finite element model (FEM) was utilized to obtain the sensitivity map distribution. For simplicity, the sensitivity map through the FEM was applied by using commercial software COMSOL Multiphysics. The electric field projection inside the pipe was exported according to how many pixels are involved. For this research,  $128 \times 128$  pixels are the pixels concerned. The parameters of acrylic pipe and water were set as in Table 1.

**Table1.** Parameters for system

Item	Dimension
Thickness of pipe	2 mm
Inner diameter of pipe	96 mm
Electrical Permittivity	$\epsilon_r = 3.45$ (acrylic), $\epsilon_r = 80$ (water)
Electrical Conductivity	$\sigma = 3 \times 10^{-14}$ S/m (acrylic), $\sigma = 7 \times 10^{-3}$ S/m (water)

A specific step for generating the map can be done manually under the results branch in COMSOL, by exporting how many regular grid data of x axis and y axis points are involved. The x and y points represent the pixels of the model. Similarly, the user also can use coding in MATLAB that is linked to COMSOL. In this research, this forward problem was solved in COMSOL via a live link with MATLAB software.

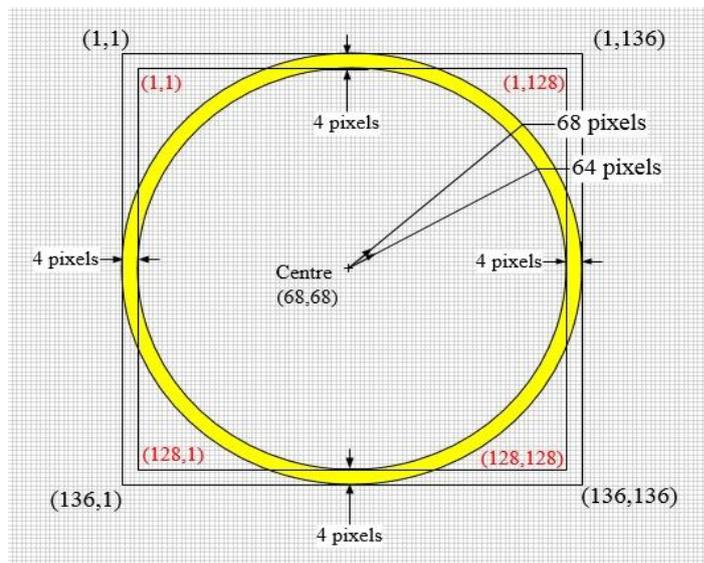
Firstly, the user must open COMSOL with MATLAB. This will open MATLAB which then links to the COMSOL software. Then, the coding for generating map was developed in MATLAB. The idea for writing the coding is as follows:

1. The single projection of each electrode out of sixteen was obtained. The  $136 \times 136$  pixels data consisting of an electric field in the export function in COMSOL was generated. In this case, there are sixteen data each representing sixteen single projections.
2. Next, the pair projection electrode as in Equation (1) was determined by multiplying each single projection in step 1. Moreover, it would have 256 data representing each pair projection of sixteen sensors such as  $E_{1,2}$ ,  $E_{1,3}$ ,  $E_{1,4}, \dots$  till  $E_{16,16}$ . The multiplication of pair projection was done until all pair projections had been completed. Hence, the map for each pair projections that contains the concerned pixels was achieved.

#### 4. Masking data for better sensitivity map

As illustrated in Figure 1, the pipe drawing consists of the thickness of the pipe. The thickness of the pipe is needed in COMSOL software because the electrodes were applied to the periphery of the pipe wall. However, the sensitivity map is only the map inside the column excluding the thickness of the pipe. Hence, the  $136 \times 136$  data representing the pixel values of the overall drawing were reduced to  $128 \times 128$  pixels to enhance the quality of the sensitivity projection of each of the electrodes.

The idea is illustrated in Figure 1. The circle frame coloured by yellow was eliminated to get the affected  $128 \times 128$  pixels. Accordingly, MATLAB software is used to remove the thickness of the pipe by using the masking approach. If the resolution used was  $136 \times 136$  pixels, the radius of the circle was 68 pixels. However, it was interested in a radius of 64 pixels to get  $128 \times 128$  pixels. Roughly, 4 pixels were assumed to be eliminated for each side (left, right, top and bottom) of the circle. However, at the curve side, it was difficult to assume how many pixels should be eliminated. Therefore, to ensure that the radius was consistent with 64 pixels; the approach was done based on Pythagorean Theorem and coded in MATLAB, as shown in Figure 2.



**Figure 1.** Basic drawing of  $136 \times 136$  pixels to  $128 \times 128$  pixels

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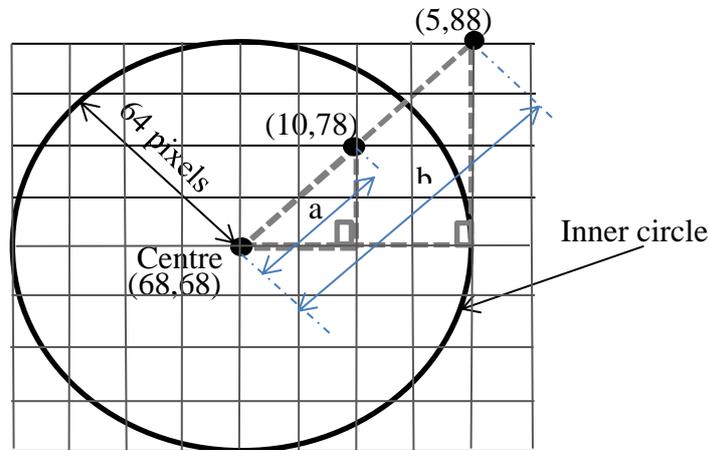
%% eliminate the thickness of pipe
pixel= 136; %diameter of pipe in pixel
r=pixel/2; %radius of pipe in pixel

for x=1:pixel
    for y=1:pixel
        %if pixel distance (between (x,y) and centre) bigger than 64
        pixels, set value to 0
        if
    
```

**Figure 2.** Coding for eliminating thickness of pipe written in MATLAB

For each point of pixel  $(x,y)$ ; the distance between every point of the pixels and the centre  $(68,68)$  was compared. If the distance was bigger than 64 pixels, then, the value at the current

point becomes zero. If not, the value will remain and therefore only the thickness of the pipe will be eliminated. The example of how it works is illustrated in Figure 3 and Equations (2) and (3).



**Figure 3.** Example of illustration to eliminate pipe thickness

Based on Figure 3, let's say at point  $(x,y) = (10,78)$ , distance  $a$  is

$$\sqrt{\left(10 - \frac{136}{2}\right)^2 + \left(78 - \frac{136}{2}\right)^2} = 59 \quad (2)$$

$$\therefore a = 59 < 64$$

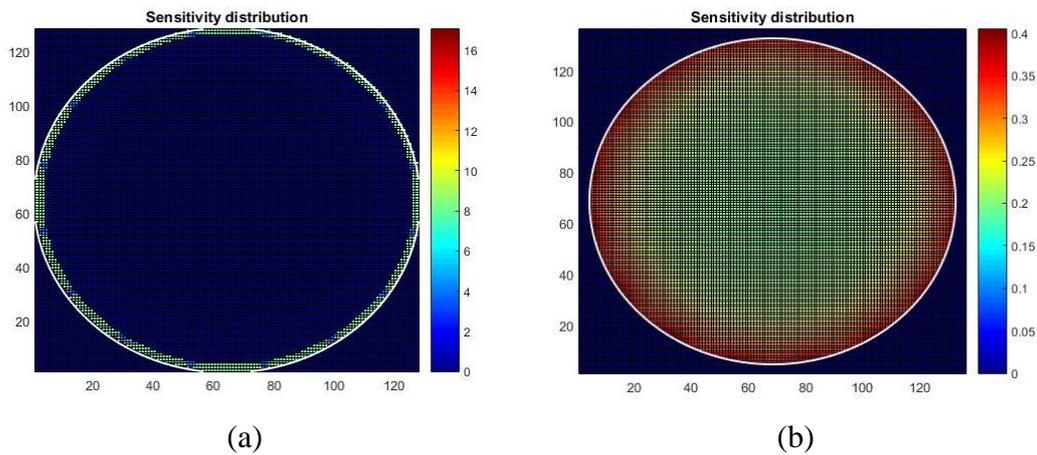
Thus, the value at point  $(10, 78)$  has not changed. But let's say at point  $(x,y) = (5,88)$ , distance  $b$  is

$$\sqrt{\left(5 - \frac{136}{2}\right)^2 + \left(88 - \frac{136}{2}\right)^2} = 66 \quad (3)$$

$$\therefore b = 66 > 64$$

Hence, the value at point  $(5, 88)$  was set to zero. Consequently, the array data of  $128 \times 128$  pixels representing the inner area of the circle for each single projection can be obtained and applied for getting the sensitivity map.

If the direct data are applied without being manipulated, the map projection almost cannot be seen compared with after manipulation. Thus, by manipulating the data exported from COMSOL, the sensitivity screening for all pairing of electrodes rose. The total map of the sensitivity distribution between before manipulated and after manipulated is shown in Figure 4.



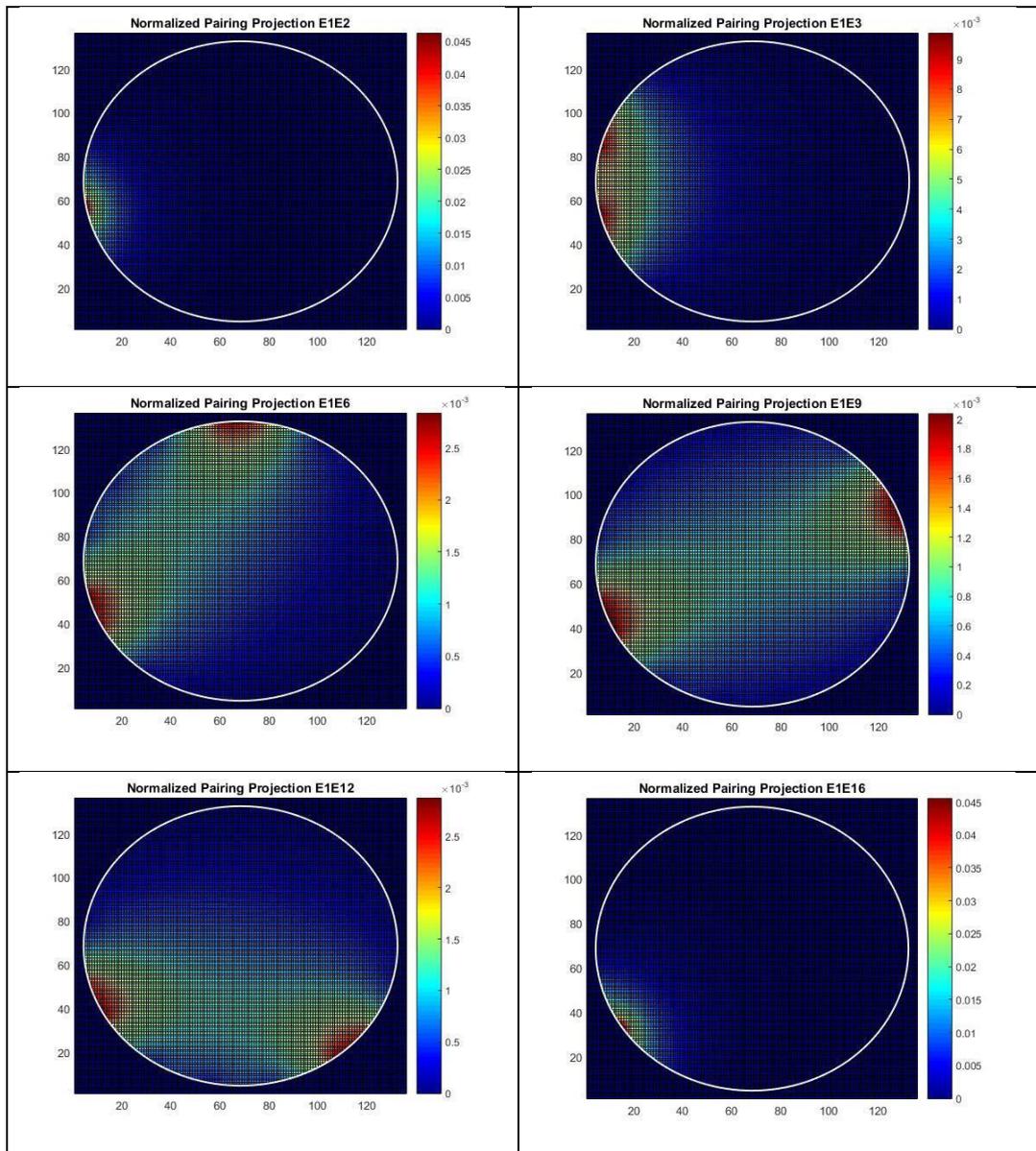
**Figure 4.** Total map of the system before and after eliminating the thickness of the pipe: (a) before; (b) after

It can be seen that the intensity of the sensitivity distribution of the overall electrodes was rose significantly particularly in the periphery and the centre of the pipe. Consequently, the manipulation data to eliminate pipe thickness approach was chosen in this research. Moreover, the sensitivity distribution for each of the pair projections is then normalized to get the standardized map. The normalization sensitivity distribution for each pair projection,  $\overline{M}_{i,j}(x, y)$  was done by dividing each pair's sensitivity map by the sum of all pair projections, as in Equation (4) [5].

$$\overline{M}_{i,j}(x, y) = \frac{M_{i,j}(x, y)}{\sum_{i=1}^{16} \sum_{j=1}^{16} M_{i,j}(x, y)} \quad (4)$$

By implementing the manipulated data of the map distribution, this also raises the normalized pair projection. As sixteen electrodes act as transmitters and receivers, there are 256 sensitivity matrices overall in the system. Examples of normalized sensitivity distribution when channel one was set as the excitation electrode is shown in Figures 5. It can be seen that the projection for each pair of electrodes produces a curved line. The curved line occurred owing to the soft-field behaviour of the electrical tomography. Also, the projection value of adjacent electrodes pairs (E1E2 and E1E16) are larger compared to the opposite electrode pair, which indicates that it failed to give a uniform and even sensitivity distribution. This is because of distinctive

soft-field sensing characteristics [6]. The normalized sensitivity map obtained for each of the pair projections is later can be implemented in the inverse problem part.



**Figure 5.** Sensitivity distribution for transmitter 1 (E1) with selective receiver (E2, E3, E6, E9, E12 and E16)

### **Conclusion**

In short, the forward problem for non-invasive ERT system can solved using finite element model (FEM) through COMSOL Multiphysics software. A masking data also can be applied to obtain a better sensitivity map for the system. It is believed that, this approach can be applied in other electrical tomography types system in providing a sensitivity map.

## **Acknowledgment**

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## **References**

- [1] G. Dong, J. Zou, R. H. Bayford, X. Ma, S. Gao, W. Yan, M. Ge, and A. G. Equation, "The Comparison Between FVM and FEM for EIT Forward Problem," *IEEE Transactions on Magnetics*, vol. 41, no. 5, pp. 1468–1471, 2005.
- [2] G. R. Shaw, Y. Goussard, and R. Guardo, "Linearization of The Forward Problem in Electrical Impedance Tomography," in *Proceedings of the IEEE International Conference on Engineering in Medicine and Biology Society*, October 28-31, 1993, San Diego, USA, 1993, no. 2, pp. 82–83.
- [3] Yasmin Abdul Wahab, Ruzairi Abdul Rahim, Mohd Hafiz Fazalul Rahiman, Leow Pei Ling<sup>2</sup>, Suzanna Ridzuan Aw<sup>5</sup>, Muhammad Jayasuman Puspanathan, Mohd Fadzli Abd Shaib, Herlina Abdul Rahim, and Elmy Johana Mohamad, Image Reconstruction for Solid Profile Measurement in ERT using Non-invasive Approach, *Telecommunication Computing Electronics and Control, Telkomnika*, vol. 15, Issue 4, pp. 1554-1564. 2017.
- [4] S. Mohammad, "Segmented Excitation for Electrical Capacitance Tomography", Master Thesis, Universiti Teknologi Malaysia, 2012.
- [5] Y. S. Kim, S. H. Lee, U. Z. Ijaz, K. Y. Kim, and B. Y. Choi, "Sensitivity Map Generation in Electrical Capacitance Tomography using Mixed Normalization Models," *Measurement Science and Technology*, vol. 18, no. 7, pp. 2092–2102, 2007.
- [6] E. J. Mohamad, "A Segmented Capacitance Tomography for Visualising Material Distributions in Pipeline Conveying Crude Plam Oil", Ph.D. Thesis, Universiti Teknologi Malaysia, 2012.