

The convergence problems of eigenfunction expansions of elliptic differential operators

Anvarjon Ahmedov

Faculty of Industrial Sciences & Technology, Universiti Malaysia Pahang

ABSTRACT

In the present research we investigate the problems concerning the almost everywhere convergence of multiple Fourier series summed over the elliptic levels in the classes of Liouville. The sufficient conditions for the almost everywhere convergence problems, which are most difficult problems in Harmonic analysis, are obtained. The methods of approximation by multiple Fourier series summed over elliptic curves are applied to obtain suitable estimations for the maximal operator of the spectral decompositions. Obtaining of such estimations involves very complicated calculations which depends on the functional structure of the classes of functions. The main idea on the proving the almost everywhere convergence of the eigenfunction expansions in the interpolation spaces is estimation of the maximal operator of the partial sums in the boundary classes and application of the interpolation Theorem of the family of linear operators. In the present work the maximal operator of the elliptic partial sums are estimated in the interpolation classes of Liouville and the almost everywhere convergence of the multiple Fourier series by elliptic summation methods are established. The considering multiple Fourier series as an eigenfunction expansions of the differential operators helps to translate the functional properties (for example smoothness) of the Liouville classes into Fourier coefficients of the functions which being expanded into such expansions. The sufficient conditions for convergence of the multiple Fourier series of functions from Liouville classes are obtained in terms of the smoothness and dimensions. Such results are highly effective in solving the boundary problems with periodic boundary conditions occurring in the spectral theory of differential operators. The investigations of multiple Fourier series in modern methods of harmonic analysis incorporates the wide use of methods from functional analysis, mathematical physics, modern operator theory and spectral decomposition. New method for the best approximation of the square-integrable function by multiple Fourier series summed over the elliptic levels are established. Using the best approximation, the Lebesgue constant corresponding to the elliptic partial sums is estimated. The latter is applied to obtain an estimation for the maximal operator in the classes of Liouville.

KEYWORDS:

Approximation algorithms; Boundary value problems; Computation theory