Comparison of Hybrid Control Schemes for Vibration Suppression of Flexible Robot Manipulator

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Abstract—**This paper presents a comparative assessment of control schemes for vibration suppression and end-point trajectory tracking of a flexible robot manipulator. A constrained planar single-link flexible robot manipulator is considered and the dynamic model of the system is derived using the assumed mode method. To study the effectiveness of the controllers, initially a collocated PD controller is developed for control of rigid body motion. This is then extended to incorporate a non-collocated PID controller and a feedforward controller based on input shaping techniques for control of vibration (flexible motion) of the system. For input shaping controller, the positive input shapers with different derivatives are proposed and designed based on the properties of the system. Simulation results of the response of the manipulator with the controllers are presented in time and frequency domains. The performances of the control schemes are assessed in terms of level of vibration reduction, input tracking capability and speed of response. Finally, a comparative assessment of the control techniques is presented and discussed.**

I. INTRODUCTION

An important aspect of the flexible robot manipulator control that has received little attention is the interaction between the rigid and flexible dynamics of the links. An acceptable system performance with reduced vibration that accounts for system changes can be achieved by developing a hybrid control scheme that caters for rigid body motion and vibration of the system independently. This can be realised by utilising control strategies consisting of either non-collocated with collocated feedback controllers and feedforward with feedback controllers. A hybrid collocated and non-collocated controller has previously been proposed for control of a flexible robot manipulator [1]. The controller design utilises end-point acceleration feedback through a proportional-integral-derivative (PID) control scheme and a proportional-derivative (PD) configuration for control of rigid body motion. A PD feedback control with a feedforward control to regulate the position of a flexible manipulator has been proposed [2]. Simulation results have shown that although the pole-zero cancellation property of the feedforward control speeds up the system response, it increases overshoot and oscillation. Moreover, the investigation of hybrid control schemes of flexible manipulator with different polarities of input shaping also has been discussed [3]. Simulation results compare the performance of positive zero-vibration derivative-derivative and negative zero-vibration-derivative-derivative with collocated PD control respectively. A control law partitioning scheme which uses end-point sensing device has been reported [4]. The scheme uses end-point position signal in an outer loop controller to control the flexible modes, whereas the inner loop controls the rigid body motion independent of the flexible dynamics of the manipulator. Performance of the scheme has been demonstrated in both simulation and experimental trials incorporating the first two flexible modes. A combined feedforward and feedback method in which the end-point position is sensed by an accelerometer and fed back to the motor controller, operating as a velocity servo, has been proposed in the control a flexible manipulator system [5]. This method uses a single mass-spring-damper system to represent the manipulator and thus the technique is not suitable for high speed operation.

Manuscript submitted October 31, 2008. This work was supported by the Universiti Malaysia Pahang.
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II. THE FLEXIBLE MANIPULATOR SYSTEM

The single-link flexible manipulator system considered in this work is shown in Fig. 1, where X_oOY_o and XOY represent the stationary and moving coordinates frames respectively, τ represents the applied torque at the hub. E, I, ρ, A, I_h, $v(x,t)$ and $\theta(t)$ represent the Young modulus, area moment of inertia, mass density per unit volume, cross-sectional area, hub inertia, displacement and hub angle of the manipulator respectively. In this study, an aluminium type flexible manipulator of dimensions 900 × 19.008 × 3.2004 mm³, E = 71 × 10^9 N/m², I = 5.1924 × 10^{11} m^4 , ρ = 2710 kg/m^3 and I_H = 5.8598 × 10^{-4} kgm^2 is considered. These parameters constitute a single-link flexible robot manipulator experimental-rig developed for test and verification of control algorithms [6].

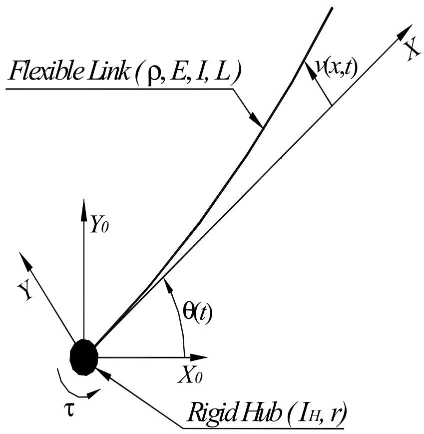

Fig. 1 Description of the flexible manipulator system.

III. Modelling of The Flexible Manipulator

This section provides a brief description on the modelling of the flexible manipulator system, as a basis of a simulation environment for development and assessment of the hybrid fuzzy logic control techniques. The assume mode method with two modal displacement is considered in characterizing the dynamic behaviour of the manipulator incorporating structural damping. The dynamic model has been validated with experimental exercises where a close agreement between both theoretical and experimental results has been achieved [7].

Considering revolute joints and motion of the manipulator on a two-dimensional plane, the kinetic energy of the system can thus be formulated as

$$T = \frac{1}{2}(I_H + I_b)\dot{\theta}^2 + \frac{1}{2}\rho\int_0^L (\dot{v}^2 + 2\dot{v}x\dot{\theta})dx \qquad (1)$$

where I_b is the beam rotation inertia about the origin O_0 as if it were rigid. The potential energy of the beam can be formulated as

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx \qquad (2)$$

This expression states the internal energy due to the elastic deformation of the link as it bends. The potential energy due to gravity is not accounted for since only motion in the plane perpendicular to the gravitational field is considered.

To obtain a closed-form dynamic model of the manipulator, the energy expressions in (1) and (2) are used to formulate the Lagrangian $L = T - U$. Assembling the mass and stiffness matrices and utilizing the Euler-Lagrange equation of motion, the dynamic equation of motion of the flexible manipulator system can be obtained as

$$M\ddot{Q}(t) + D\dot{Q}(t) + KQ(t) = F(t) \qquad (3)$$

where *M, D* and *K* are global mass, damping and stiffness matrices of the manipulator respectively. The damping matrix is obtained by assuming the manipulator exhibit the characteristic of Rayleigh damping. *F(t)* is a vector of external forces and *Q(t)* is a modal displacement vector given as

$$Q(t) = [\theta \quad q_1 \quad q_2 \quad \ldots \quad q_n]^T = [\theta \quad q^T]^T \qquad (4)$$

$$F(t) = [\tau \quad 0 \quad 0 \quad \ldots \quad 0]^T \qquad (5)$$

Here, q_n is the modal amplitude of the i th clamped-free mode considered in the assumed modes method procedure and n represents the total number of assumed modes. The model of the uncontrolled system can be represented in a state-space form as

$$\begin{aligned} \dot{x} &= \mathrm{A}x + \mathrm{B}u \\ y &= \mathrm{C}x \end{aligned} \qquad (6)$$

with the vector $x = [\theta \;\; \dot{\theta} \;\; q_1 \;\; q_2 \;\; \dot{q}_1 \;\; \dot{q}_2]^T$ and the matrices A and B are given by

$$\mathbf{A} = \left[\begin{array}{c|c} 0_{3\times3} & I_{3\times3} \\ \hline -M^{-1}K & -M^{-1}D \end{array}\right], \quad \mathbf{B} = \left[\begin{array}{c} 0_{3\times1} \\ \hline M^{-1} \end{array}\right]$$

$$\mathbf{C} = [I_{1\times3} \mid 0_{1\times3}], \qquad \mathbf{D} = [0] \qquad (7)$$

IV. Controller Design

In this section, control schemes for rigid body motion control and vibration suppression of a flexible robot manipulator are proposed. Initially, a collocated PD controller is designed. Then a non-collocated PID control and feedforward control based on input shaping are incorporated in the closed-loop system for control of vibration of the system.

A. Collocated PD Control

A common strategy in the control of manipulator systems involves the utilization of PD feedback of collocated sensor signals. In this work, such a strategy is adopted at this stage. A sub-block diagram of the PD controller is shown in Fig. 2, where K_p and K_v are proportional and derivative gains, respectively, θ and $\dot{\theta}$ represent hub angle and hub velocity, respectively, R_f is the reference hub angle and A_c is the gain of the motor amplifier. Here the motor/amplifier gain set is considered as a linear gain. To design the PD controller, a linear state-space model of the flexible manipulator was obtained by linearising the equations of motion of the system.

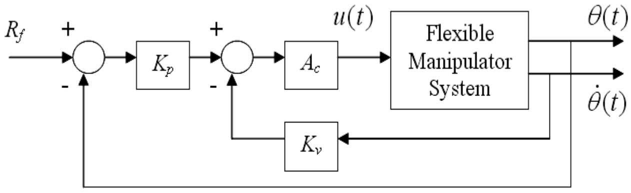

Fig. 2 Block diagram of collocated PD control structure.

The control signal $u(s)$ in Fig. 2 can be written as

$$u(s) = A_c[K_p\{R_f(s) - \theta(s)\} - K_v s\theta(s)] \quad (8)$$

where s is the Laplace variable. The closed-loop transfer function is, therefore, obtained as

$$\frac{\theta(s)}{R_f(s)} = \frac{K_p H(s) A_c}{1 + A_c K_v (s + K_p / K_v) H(s)} \quad (9)$$

where $H(s)$ is the open-loop transfer function from the input torque to hub angle, given by

$$H(s) = C(sI - A)^{-1} B \quad (10)$$

where A, B, and C are the characteristic matrix, input matrix and output matrix of the system, respectively, and I is the identity matrix. The closed-loop poles of the system are, thus, given by the closed-loop characteristics equation as

$$1 + A_c K_v (s + K_p / K_v) H(s) \quad (11)$$

where $Z = K_p / K_v$ represents the compensator zero which determines the control performance and characterises the shape of root locus of the closed-loop system. In this study, the root locus approach is utilized to design the PD controller.

B. Collocated PD with non-collocated PID controller

A combination of collocated and non-collocated control scheme for control of rigid body motion and vibration suppression of the system is presented in this section. The use of a non-collocated control system, where the end-point of the manipulator is controlled by measuring its position, can be applied to improve the overall performance, as more reliable output measurement is obtained. The control structure comprises two feedback loops: (1) The collocated PD control for rigid body motion control. (2) The end-point residual (elastic deformation) as input to a separate non-collocated control law for vibration control. These two loops are then summed together to give a torque input to the system. A block diagram of the control scheme is shown in Fig. 3 where α represents the end-point residual. r_α represents end-point residual reference input, which is set to zero as the control objective is to have zero vibration during movement of the manipulator.

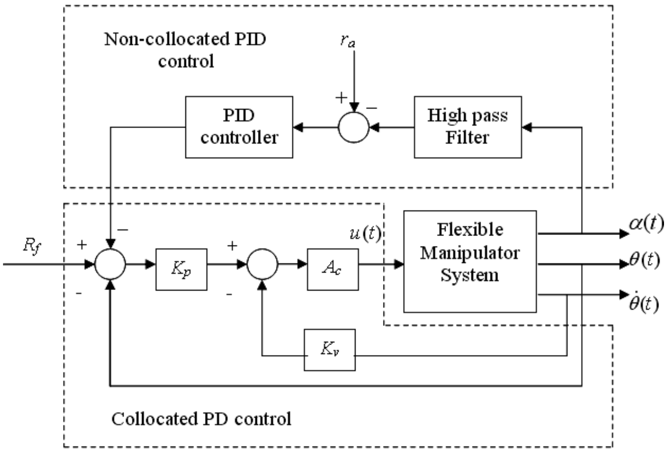

Fig. 3 Block diagram of collocated PD with non-collocated PID control structure.

For rigid body motion control, the collocated PD control strategy developed in the previous section is adopted whereas for the vibration control loop, the end-point residual feedback through a PID control scheme is utilized. The PID controller parameters were tuned using the Ziegler-Nichols method using a closed-loop technique, where the proportional gain K_p was initially tuned and the integral gain K_i and derivative gain K_d were then calculated [8]. Accordingly, the PID parameters K_p, K_i and K_d were deduced as 0.8, 5 and 0.03 respectively. To decouple the end-point measurement from the rigid body motion of the manipulator, a third-order infinite impulse response (IIR) Butterworth High-pass filter was utilised. In this investigation, a High-pass filter with cut-off frequency of 5 Hz was designed.

C. Collocated PD with input shaping controller

A control structure for control of rigid body motion and vibration suppression of the flexible robot manipulator based on PD and input shaping control is proposed in this section. A block diagram of the PD with input shaping control technique is shown in Fig. 4. The design objectives of input shaping are to determine the amplitude and time locations of the impulses in order to reduce the detrimental effects of system flexibility [9]. These parameters are obtained from the natural frequencies and damping ratios of the system. The corresponding design relations for achieving a zero residual single-mode vibration of a system and to ensure that the shaped command input produces the same rigid body motion as the unshaped command yields a two-impulse sequence (ZV shaper) with parameter as

$$t_1 = 0,\ t_2 = \frac{\pi}{\omega_d},\ A_1 = \frac{1}{1+K},\ A_2 = \frac{K}{1+K} \quad (12)$$

where

$$K = e^{-\zeta\pi/\sqrt{1-\zeta^2}},\ \ \omega_d = \omega_n\sqrt{1-\zeta^2}$$

(ω_n and ζ representing the natural frequency and damping ratio respectively) and t_j and A_j are the time location and amplitude of impulse j respectively. The robustness of the input shaper to errors in natural frequencies of the system can be increased by solving the derivatives of the system vibration equation. This yields a four-impulse sequence (ZVDD shaper) with parameter as

$$t_1 = 0,\ t_2 = \frac{\pi}{\omega_d},\ t_3 = \frac{2\pi}{\omega_d},\ t_4 = \frac{3\pi}{\omega_d} \quad (13)$$

$$A_1 = \frac{1}{1+3K+3K^2+K^3},\ A_2 = \frac{3K}{1+3K+3K^2+K^3}$$

$$A_3 = \frac{3K^2}{1+3K+3K^2+K^3},\ A_4 = \frac{K^3}{1+3K+3K^2+K^3}$$

where K as is equation (12).

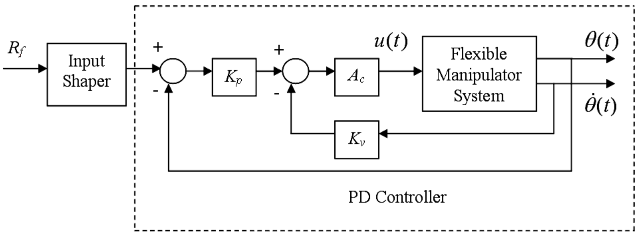

Fig. 4 Block diagram of collocated PD with input shaping control structure.

V. Implementation and Results

In this section, the proposed control schemes are implemented and tested within the simulation environment of the flexible manipulator and the corresponding results are presented. The manipulator is required to follow a trajectory within the range of ±0.8 radian. System responses namely the end-point trajectory and end-point acceleration are observed. To investigate the vibration of the system in the frequency domain, power spectral density (PSD) of the end-point acceleration response is obtained. The performances of the control schemes are assessed in terms of vibration suppression, input tracking and speed of response. Finally, a comparative assessment of the performance of the control schemes is presented and discussed.

Figs. 5-7 show the responses of the flexible robot manipulator to the reference input trajectory using collocated PD controller in time-domain and frequency domain (PSD). These results were considered as the system response under rigid body motion control and will be used to evaluate the performance of the non-collocated PID and input shaping control. The steady-state end-point trajectory of +0.8 radian for the flexible manipulator was achieved within the rise and settling times and overshoot of 0.506 s, 0.800 s and 0.5% respectively. It is noted that the manipulator reaches the required position from +0.8 rad to -0.8 rad within 1 s, with little overshoot. However, a noticeable amount of vibration occurs during movement of the manipulator. It is noted from end-point acceleration response, the vibration of the system settles within 2 s with a maximum acceleration of ±1000 m/s^2. Moreover, from the PSD of the end-point acceleration response the vibrations at the end-point are dominated by the first two vibration modes, which are obtained as 13.18 Hz and 55.91 Hz.

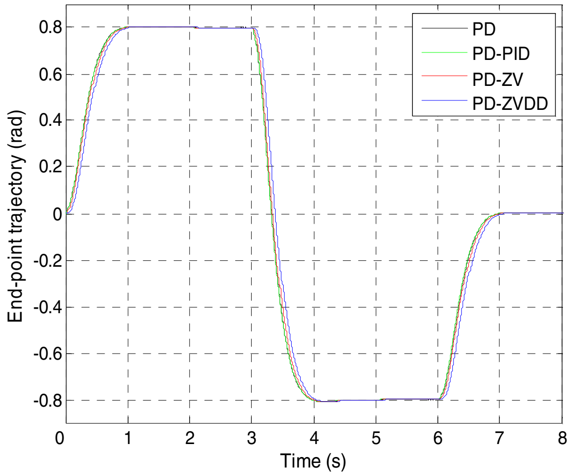

Fig. 5 End-point trajectory response.

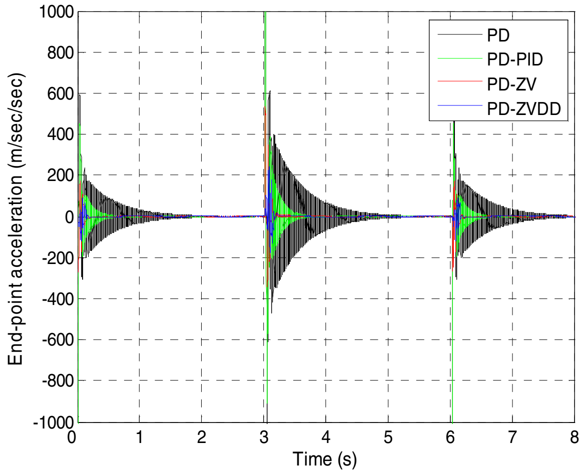

Fig. 6 End-point acceleration response.

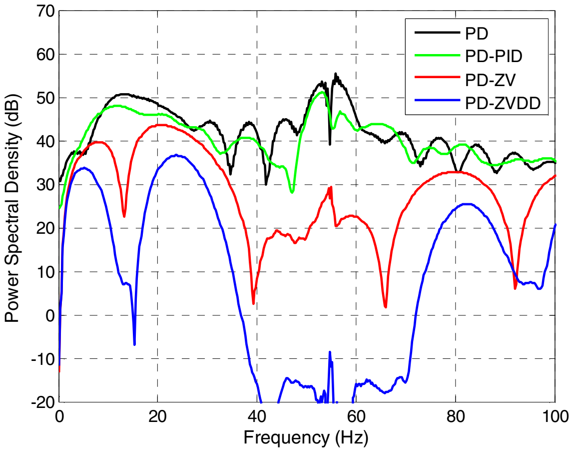

Fig. 7 PSD response.

The end-point trajectory, end-point acceleration and power spectral density responses of the flexible robot manipulator using PD with non-collocated PID (PD-PID), ZV shaper (PD-ZV) and ZVDD shaper (PD-ZVDD) control are shown in Figs. 5-7 respectively. Similar end-point trajectory and end-point acceleration responses were observed as compared to the PD controller. In overall, the highest levels of vibration reduction for the first two modes were obtained using PD-ZVDD followed by PD-ZV and PD-PID. However, the fastest system response was obtained using PD-PID followed by PD-ZV and PD-PZVDD. It is noted with the input shaping controller, the number of impulses sequence in input shaper increase the delay in the system response. Moreover, as demonstrated in the end-point trajectory response with PD-PID control, the minimum phase behaviour of the manipulator is unaffected. A significant amount of vibration reduction was demonstrated at the end-point of the manipulator with PD-PID and PD with input shaping. With the PD-PID control, the maximum end-point acceleration is ±500 m/s^2 while with the PD-ZVDD and PD-ZV control is ±400 m/s^2 and ±200 m/s^2 respectively. Hence, it is noted that the magnitude of oscillation was significantly reduced by using PD with input shaping control as compared to the case of PD with non-collocated PID control. In overall, the performance of the control schemes at input tracking capability is maintained as the PD control.

VI. Conclusions

The development of techniques for vibration suppression and end-point trajectory tracking of a flexible robot manipulator has been presented. Acceptable performance in end-point vibration suppression and input tracking control has been achieved with proposed control strategies. A comparative assessment of the control schemes has shown that the PD control with input shaping (feedforward) performs better than the PD with non-collocated PID control in respect of vibration reduction at the end-point of the manipulator. However, the speed of the response is slightly improved at the expenses of decrease in the level of vibration reduction by using the PD with non-collocated PID control. It is concluded that the proposed controllers are capable of reducing the system vibration while maintaining the input tracking performance of the manipulator.

Acknowledgment

This work was supported by Faculty of Electrical & Electronics Engineering, Universiti Malaysia Pahang, especially Control & Instrumentation (COINS) Research Group.

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