# TECHNIQUES OF VIBRATION AND END-POINT TRAJECTORY CONTROL OF FLEXIBLE MANIPULATOR

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## ABSTRACT

This paper presents investigations into the development of control schemes for end-point vibration suppression and input trajectory of a flexible manipulator. A constrained planar single-link flexible manipulator is considered and the dynamic model of the system is derived using the assumed mode method. To study the effectiveness of the controllers, initially a Linear Quadratic Regulator (LQR) is developed for control of rigid body motion. This is then extended to incorporate a non-collocated PID controller and a feedforward controller based on input shaping techniques for control of vibration (flexible motion) of the system. For feedforward controller, the positive input shapers are proposed and designed based on the properties of the system. Simulation results of the response of the manipulator with the controllers are presented in time and frequency domains. The performances of the control schemes are assessed in terms of level of vibration reduction, input tracking capability and time response specifications. Finally, a comparative assessment of the control techniques is presented and discussed.

## 1. INTRODUCTION

An important aspect of the flexible manipulator control that has received little attention is the interaction between the rigid and flexible dynamics of the links. An acceptable system performance with reduced vibration that accounts for system changes can be achieved by developing a hybrid control scheme that caters for rigid body motion and vibration of the system independently. This can be realized by utilizing control strategies consisting of either non-collocated with collocated feedback controllers and feedforward with feedback controllers. In both cases, the former can be used for vibration suppression and the latter for input tracking of a flexible manipulator. Practically, a combination of the control techniques would position the end-point of the flexible manipulator from one point to another with reduced vibration. Both feedforward and feedback control structures have been utilized in the control of flexible manipulator systems. A hybrid collocated and non-collocated controller has previously been proposed for control of a flexible manipulator [1]. The controller design utilizes end-point acceleration feedback through a proportional-integral-derivative (PID) control scheme and a proportional-derivative (PD) configuration for control of rigid body motion. Experimental investigations have shown that the control structure gives a satisfactory system response with significant vibration reduction as compared to a response with a collocated controller. A PD feedback control with a feedforward control to regulate the position of a flexible manipulator has been proposed [2]. Simulation results have shown that although the pole-zero cancellation property of the feedforward control speeds up the system response, it increases overshoot and oscillation. A control law partitioning scheme which uses end-point sensing device has been reported [3]. The scheme uses end-point position signal in an outer loop controller to control the flexible modes, whereas the inner loop controls the rigid body motion independent of the flexible dynamics of the manipulator. Performance of the scheme has been demonstrated in both simulation and experimental trials incorporating the first two flexible modes. A combined feedforward and feedback method in which the end-point position is sensed by an accelerometer and fed back to the motor controller, operating as a velocity servo, has been proposed in the control a flexible manipulator system [4]. This method uses a single mass-spring-damper system to represent the manipulator and thus the technique is not suitable for high speed operation.

This paper presents investigations into the development of techniques for end-point vibration suppression and input tracking of a flexible manipulator. A constrained planar single-link flexible manipulator is considered. Control strategies based on feedforward with LQR controllers and with combined non-collocated and LQR controllers are investigated. A simulation environment is developed within Simulink and Matlab for evaluation of performance of the control schemes. In this work, the dynamic model of the flexible manipulator is derived using the assumed mode method (AMM). Previous simulation and experimental studies have shown that the AMM method gives an acceptable dynamic characterization of the actual system [5]. Moreover, two mode of vibration is sufficient to describe the dynamic behavior of the manipulator reasonably well. To demonstrate the effectiveness of the proposed control schemes, initially an LQR controller utilizing full-state feedback is developed for control of rigid body motion. This is then extended to incorporate non-collocated and feedforward controllers for vibration suppression of the manipulator. For non-collocated control, end-point displacement feedback through a PID control configuration is developed whereas in the feedforward scheme, the positive input shapers are utilized as these have been shown to be effective in reducing system vibration. Simulation results of the response of the manipulator with the controllers are presented in time and frequency domains. The performances of the control schemes are assessed in terms of level of vibration reduction, input tracking capability and time response specifications. Finally, a comparative assessment of the control techniques is presented and discussed.

## 2. THE FLEXIBLE MANIPULATOR SYSTEM

Figure 1 shows the single-link flexible manipulator system considered in this work, where $X_oOY_o$ and $XOY$ represent the stationary and moving coordinates frames respectively, $\tau$ represents the applied torque at the hub. $E$, $I$, $\rho$, $L$, $A$ and $I_h$ represent the Young modulus, area moment of inertia, mass density per unit volume, length, cross-sectional area and hub inertia of the manipulator respectively. In this work, the motion of the manipulator is confined to $X_oOY_o$ plane. Transverse shear and rotary inertia effects are neglected, since the manipulator is long and slender. Thus, the Bernoulli-Euler beam theory is allowed to be used to model the elastic behavior of the manipulator. The manipulator is assumed to be stiff in vertical bending and torsion, allowing it to vibrate dominantly in the horizontal direction and thus, the gravity effects are neglected. Moreover, the manipulator is considered to have a constant cross-section and uniform material properties throughout. In this study, an aluminium type flexible manipulator of dimensions 900 × 19.008 × 3.2004 mm³, $E = 71 \times 10^9$ N/m², $I = 5.1924 \times 10^{11}$ m$^4$ , $\rho = 2710$ kg/m$^3$ and $I_h = 5.8598 \times 10^{-4}$ kgm$^2$ is considered. These parameters constitute a single-link flexible manipulator experimental-rig developed for test and verification of control algorithms [6].

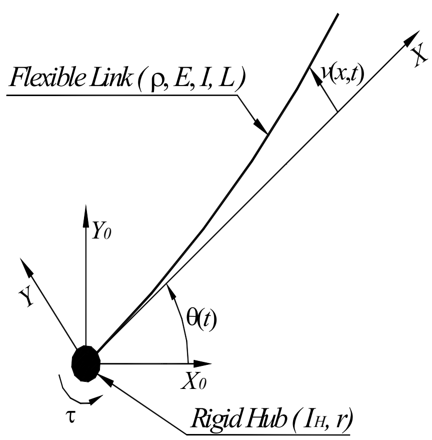


Figure 1. *Description of the flexible manipulator system*.

## 3. MODELLING OF THE FLEXIBLE MANIPULATOR

This section provides a brief description on the modelling of the flexible manipulator system, as a basis of a simulation environment for development and assessment of the hybrid control techniques. The assume mode method with two modal displacement is considered in characterizing the dynamic behaviour of the manipulator incorporating structural damping. The dynamic model has been validated with experimental exercises where a close agreement between both theoretical and experimental results has been achieved [5].

Considering revolute joints and motion of the manipulator on a two-dimensional plane, the kinetic energy of the system can thus be formulated as

$$T = \frac{1}{2}(I_H + I_b)\dot{\theta}^2 + \frac{1}{2}\rho\int_0^L (\dot{v}^2 + 2\dot{v}x\dot{\theta})dx \qquad (1)$$

where $I_b$ is the beam rotation inertia about the origin $O_0$ as if it were rigid. The potential energy of the beam can be formulated as

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx \qquad (2)$$

This expression states the internal energy due to the elastic deformation of the link as it bends. The potential energy due to gravity is not accounted for since only motion in the plane perpendicular to the gravitational field is considered.

To obtain a closed-form dynamic model of the manipulator, the energy expressions in (1) and (2) are used to formulate the Lagrangian $L = T - U$ . Assembling the mass and stiffness matrices and utilizing the Euler-Lagrange equation of motion, the dynamic equation of motion of the flexible manipulator system can be obtained as

$$M\ddot{Q}(t) + D\dot{Q}(t) + KQ(t) = F(t) \qquad (3)$$

where $M$, $D$ and $K$ are global mass, damping and stiffness matrices of the manipulator respectively. The damping matrix is obtained by assuming the manipulator exhibit the characteristic of Rayleigh damping. $F(t)$ is a vector of external forces and $Q(t)$ is a modal displacement vector given as

$$Q(t) = \begin{bmatrix}\theta & q_1 & q_2 & \dots & q_n\end{bmatrix}^T = \begin{bmatrix}\theta & q^T\end{bmatrix}^T \qquad (4)$$

$$F(t) = \begin{bmatrix}\tau & 0 & 0 & \dots & 0\end{bmatrix}^T \qquad (5)$$

Here, $q_n$ is the modal amplitude of the $i$ th clamped-free mode considered in the assumed modes method procedure and $n$ represents the total number of assumed modes. The model of the uncontrolled system can be represented in a state-space form as

$$\begin{aligned}\dot{x} &= \mathrm{A}x + \mathrm{B}u \\ y &= \mathrm{C}x\end{aligned} \qquad (6)$$

with the vector $x = \begin{bmatrix}\theta & \dot{\theta} & q_1 & q_2 & \dot{q}_1 & \dot{q}_2\end{bmatrix}^T$ and the matrices $A$ and $B$ are given by

$$\mathbf{A} = \left[\begin{array}{c:c} 0_{3\times3} & I_{3\times3} \\ \hdashline -M^{-1}K & -M^{-1}D \end{array}\right], \quad \mathbf{B} = \left[\begin{array}{c} 0_{3\times1} \\ \hdashline M^{-1} \end{array}\right]$$
$$\mathbf{C} = \left[ I_{1\times3} \;\vdots\; 0_{1\times3} \right], \qquad \mathbf{D} = [0] \tag{7}$$

## 4. CONTROL SCHEMES

In this section, control schemes for rigid body motion control and vibration suppression of a flexible manipulator are proposed. Initially, an LQR controller is designed. Then a non-collocated PID control and feedforward control based on input shaping are incorporated in the closed-loop system for control of vibration of the system.

### 4.1. LQR controller

A more common approach in the control of manipulator systems involves the utilization linear quadratic regulator (LQR) design [7]. Such an approach is adopted at this stage of the investigation here. In order to design the LQR controller a linear state-space model of the flexible manipulator was obtained by linearising the equations of motion of the system. For a LTI system

$$\dot{x} = Ax + Bu \;, \tag{8}$$

the technique involves choosing a control law $u = \psi(x)$ which stabilizes the origin (i.e., regulates $x$ to zero) while minimizing the quadratic cost function

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t)dt \tag{9}$$

where $Q = Q^T \geq 0$ and $R = R^T > 0$ . The term "linear-quadratic" refers to the linear system dynamics and the quadratic cost function.

The matrices $Q$ and $R$ are called the state and control penalty matrices, respectively. If the components of $Q$ are chosen large relative to those of $R$, then deviations of $x$ from zero will be penalized heavily relative to deviations of $u$ from zero. On the other hand, if the components of $R$ are large relative to those of $Q$, then control effort will be more costly and the state will not converge to zero as quickly.

A famous and somewhat surprising result due to Kalman is that the control law which minimizes $J$ always takes the form $u = \psi(x) = -Kx$ . The optimal regulator for a LTI system with respect to the quadratic cost function above is always a linear control law. With this observation in mind, the closed-loop system takes the form

$$\dot{x} = (A - BK)x \tag{10}$$

and the cost function J takes the form

$$J = \int_0^\infty x(t)^T Qx(t) + (-Kx(t))^T R(-Kx(t))dt \tag{11}$$

$$J = \int_0^\infty x(t)^T (Q + K^T RK)x(t)dt \tag{12}$$

Assuming that the closed-loop system is internally stable, which is a fundamental requirement for any feedback controller, the following theorem allows the computation value of the cost function for a given control gain matrix $K$.

In this investigation, the tracking performance of the LQR applied to the flexible manipulator was investigated by setting the value of vector $K$ and $\overline{N}$ which determines the feedback control law and for elimination of steady state error capability respectively. For the single-link flexible manipulator described by the state-space model given by Equation (6) and with $M$, $K$, and $D$ matrices calculated earlier, the LQR gain matrix for

$$Q = \left[\begin{array}{c:c} I_{3\times3} & 0_{3\times3} \\ \hdashline 0_{3\times3} & 0_{3\times3} \end{array}\right] \quad \text{and} \quad R = 1$$

was calculated using Matlab and was found to be

$$K = [1.000 \;\; 1.0848 \;\; 9.8810 \;\; 0.2705 \;\; 0.3416 \;\; 3.4663]$$

$$\overline{N} = [1.000]$$

### 4.2. LQR with non-collocated PID controller

A combination of full-state feedback and non-collocated control scheme for control of rigid body motion and vibration suppression of the system is presented in this section. The use of a non-collocated control system, where the end-point of the manipulator is controlled by measuring its position, can be applied to improve the overall performance, as more reliable output measurement is obtained. The control structure comprises two feedback loops: (1) The full-state feedback as input to optimize the control gain matrix for rigid body motion control. (2) The end-point residual (elastic deformation) as input to a separate non-collocated control law for vibration control. These two loops are then summed together to give a torque input to the system. A block diagram of the control scheme is shown in Figure 2 where $\alpha$ represents the end-point residual. $r_\alpha$ represents end-point residual reference input, which is set to zero as the control objective is to have zero vibration during movement of the manipulator. Equations should be placed on separate lines and numbered.

For rigid body motion control, the LQR control strategy developed in the previous section is adopted whereas for the vibration control loop, the end-point residual feedback through a PID control scheme is utilized. The PID controller parameters were tuned using the Ziegler-Nichols method using a closed-loop technique, where the proportional gain $K_p$ was initially tuned and the integral gain $K_i$ and derivative gain $K_d$ were then calculated [8]. Accordingly, the PID parameters $K_p$, $K_i$ and $K_d$ were deduced as 0.7, 5 and 0.03 respectively. To decouple the end-point measurement from the rigid body motion of the manipulator, a third-order infinite impulse response (IIR) Butterworth High-pass filter was utilised. In this investigation, a High-pass filter with cut-off frequency of 5 Hz was designed.

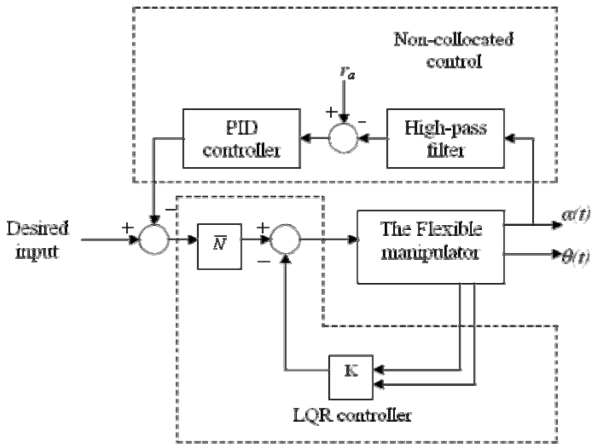


Figure 2. *The LQR and non-collocated PID control structure.*

### 4.3. LQR with input shaping control

A control structure for control of rigid body motion and vibration suppression of the flexible manipulator based on LQR and input shaping control is proposed in this section. The positive input shapers are proposed and designed based on the properties of the system. In this study, the input shaping control scheme is developed using a Zero-Vibration-Derivative-Derivative (ZVDD) input shaping technique [9]. Previous experimental study with a flexible manipulator has shown that significant vibration reduction and robustness is achieved using a ZVDD technique [10]. A block diagram of the LQR with input shaping control technique is shown in Figure 3.

The input shaping method involves convolving a desired command with a sequence of impulses known as input shaper. The design objectives are to determine the amplitude and time location of the impulses based on the natural frequencies and damping ratios of the system. The positive input shapers have been used in most input shaping schemes. The requirement of positive amplitude for the impulses is to avoid the problem of large amplitude impulses. In this case, each individual impulse must be less than one to satisfy the unity magnitude constraint. In addition, the robustness of the input shaper to errors in natural frequencies of the system can be increased by solving the derivatives of the system vibration equation. This yields a positive ZVDD shaper with parameter as

$$t_1 = 0,\ t_2 = \frac{\pi}{\omega_d},\ t_3 = \frac{2\pi}{\omega_d},\ t_4 = \frac{3\pi}{\omega_d}$$

$$A_1 = \frac{1}{1+3K+3K^2+K^3},\ A_2 = \frac{3K}{1+3K+3K^2+K^3}$$
$$A_3 = \frac{3K^2}{1+3K+3K^2+K^3},\ A_4 = \frac{K^3}{1+3K+3K^2+K^3} \qquad (13)$$

where

$$K = e^{-\zeta\pi/\sqrt{1-\zeta^2}},\quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$\omega_n$ and $\zeta$ representing the natural frequency and damping ratio respectively. For the impulses, $t_j$ and $A_j$ are the time location and amplitude of impulse $j$ respectively.

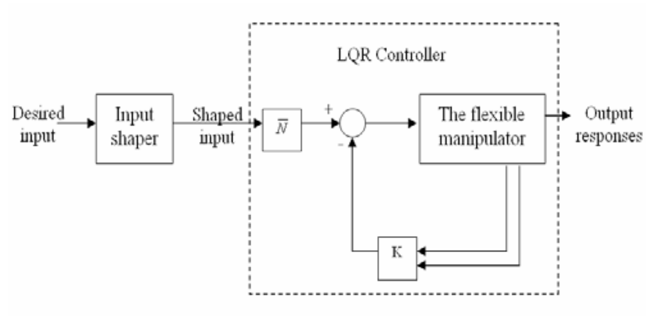


Figure 3. *The LQR and input shaping control structure.*

## 5. IMPLEMENTATION AND RESULTS

In this section, the proposed control schemes are implemented and tested within the simulation environment of the flexible manipulator and the corresponding results are presented. The manipulator is required to follow a trajectory within the range of $\pm 0.8$ radian. System responses namely the end-point trajectory, displacement and end-point acceleration are observed. To investigate the vibration of the system in the frequency domain, power spectral density (PSD) of the end-point acceleration response is obtained. The performances of the control schemes are assessed in terms of vibration suppression, input tracking and time response specifications. Finally, a comparative assessment of the performance of the control schemes is presented and discussed.

Figures 4-6 show the responses of the flexible manipulator to the reference input trajectory using LQR controller in time-domain and frequency domain (PSD). These results were considered as the system response under rigid body motion control and will be used to evaluate the performance of the non-collocated PID and input shaping control. The steady-state end-point trajectory of +0.8 radian for the flexible manipulator was achieved within the rise and settling times and overshoot of 0.421 s, 1.233 s and 6.06% respectively. It is noted that the manipulator reaches the required position from +0.8 rad to -0.8 rad within 2 s, with little overshoot. However, a noticeable amount of vibration occurs during movement of the manipulator. It is noted from end-point acceleration response, the vibration of the system settles within 0.5 s with a maximum acceleration of $\pm 600$ m/s$^2$. Moreover, from the PSD of the end-point acceleration response the vibrations at the end-point are dominated by the first two vibration modes, which are obtained as 16 and 56 Hz with magnitude of $1.367\times 10^5$ m/s$^2$/Hz and 138.4 m/s$^2$/Hz respectively.

The end-point trajectory, end-point acceleration and power spectral density responses of the flexible manipulator using LQR with non-collocated PID (LQR-PID) and input shaping (LQR-IS) control are shown in Figures 4-6 respectively. It is noted that the proposed control schemes are capable of reducing the system vibration while maintaining the input tracking performance of the manipulator. Similar end-point trajectory, end-point acceleration and power spectral density of end-point acceleration responses were observed as compared to the LQR controller.

Table 1 summarizes the levels of vibration reduction of the system responses at the first two modes in comparison to the LQR control. In overall, higher levels of vibration reduction for the first two modes were obtained using LQR-IS as compared to LQR-PID. However, the system response using LQR-PID is faster than the case of LQR-IS. It is noted with the input shaping controller, the impulses sequence in input shaper increase the delay in the system response. The corresponding rise time, setting time and overshoot of the end-point trajectory response using LQR-IS and LQR-PID is depicted in Table 1. Moreover, as demonstrated in the end-point trajectory response with LQR-PID control, the minimum phase behavior of the manipulator is unaffected. A significant amount of vibration reduction was demonstrated at the end-point of the manipulator with both control schemes. With the LQR-PID control, the maximum acceleration at the end-point is ±500 $m/s^2$ while with the LQR-IS control is ±100 $m/s^2$. Hence, it is noted that the magnitude of oscillation was significantly reduced by using LQR with input shaping control as compared to the case of LQR with non-collocated PID control. In overall, the performance of the control schemes at input tracking capability is maintained as the LQR control.

The simulation results show that performance of LQR-IS control scheme is better than LQR-PID schemes in vibration suppression of the flexible manipulator. This is further evidenced in Figure 7 that demonstrates the level of vibration reduction at the resonance modes of the LQR with non-collocated and input shaping control respectively as compared to the LQR controller. It is noted that higher vibration reduction is achieved with LQR-IS at the first two modes of vibration. Almost twofold and more than fourfold improvement in the vibration reduction at the first and second resonance mode respectively were observed with LQR-IS as compared to LQR-PID. Moreover, implementation of LQR with input shaping control is easier than LQR with non-collocated PID control as a large amount of design effort is required to determine the best PID parameters. Note that a properly tuned PID could produce better results. However, as demonstrated in the end-point trajectory response, slightly slower response is obtained using LQR with input shaping control as compared to the LQR with non-collocated control. Further comparisons of the specifications of the end-point trajectory responses are summarized in Figure 8 for the rise and settling times. The work thus developed and reported in this paper forms the basis of design and development of hybrid control schemes for input tracking and vibration suppression of multi-link flexible manipulator systems and can be extended to and adopted in practical applications.

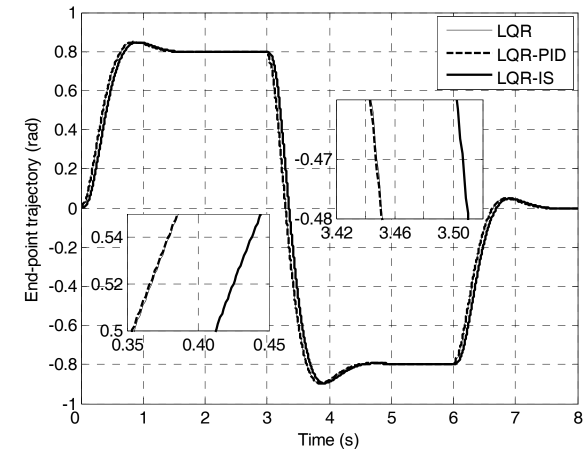


Figure 4. *End-point trajectory response with LQR and LQR-PID and LQR-IS.*

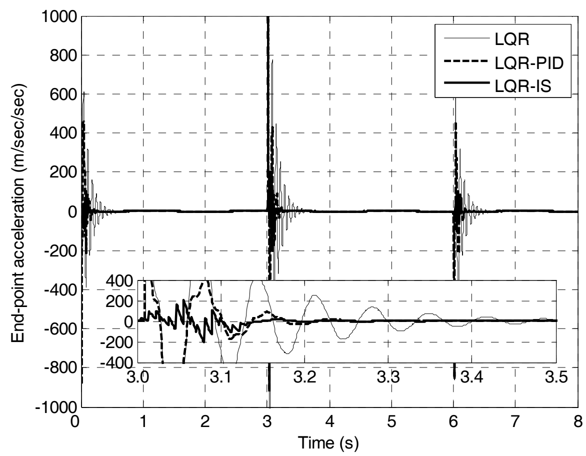


Figure 5. *End-point acceleration response with LQR and LQR-PID and LQR-IS.*

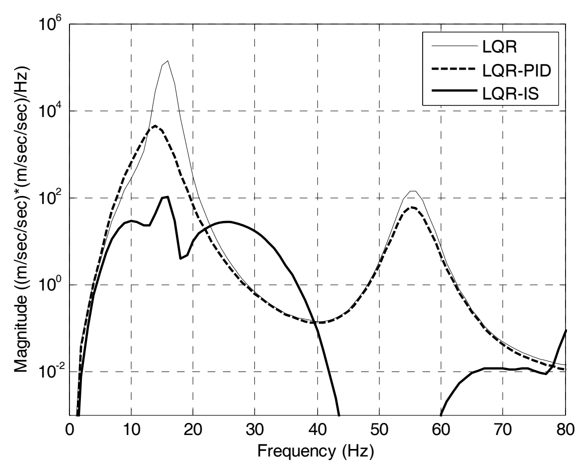


Figure 6. Power spectral density response with LQR and LQR-PID and LQR-IS.

Table 1. Level of vibration reduction of the end-point acceleration and specifications of end-point trajectory response for hybrid control schemes.

| Controller | Attenuation (dB) of vibration end-point acceleration | | Specifications of end-point trajectory response | | |
|---|---|---|---|---|---|
| | Mode 1 | Mode 2 | Rise time (s) | Settling time (s) | Overshoot (%) |
| LQR - PID | 37.14 | 8.04 | 0.418 | 1.232 | 6.06 |
| LQR - IS | 62.59 | 146.73 | 0.423 | 1.291 | 6.00 |

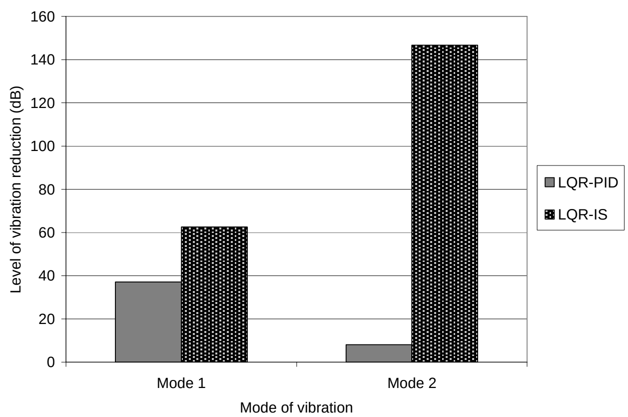


Figure 7. *Level of vibration reduction with LQR-IS and LQR-PID. at the end-point of the manipulator.*

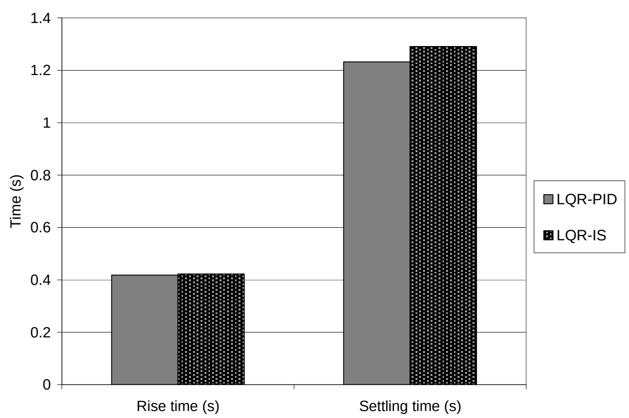


Figure 8. *Rise and settling times of the end-point trajectory with LQR-IS and LQR-PID.*

## 6. CONCLUSIONS

The development of techniques for end-point vibration suppression and input tracking of a flexible manipulator has been presented. The control schemes have been developed based on LQR with non-collocated PID control and LQR with input shaper technique. The proposed control schemes have been implemented and tested within simulation environment of a single-link flexible manipulator. The performances of the control schemes have been evaluated in terms of end-point vibration suppression and input tracking capability at the resonance modes of the manipulator. Acceptable performance in end-point vibration suppression and input tracking control has been achieved with proposed control strategies. A comparative assessment of the control schemes has shown that the LQR control with input shaping performs better than the LQR with non-collocated PID control in respect of vibration reduction at the end-point of the manipulator. However, the speed of the response is slightly improved at the expenses of decrease in the level of vibration reduction by using the LQR with non-collocated PID control. It is concluded that the proposed controllers are capable of reducing the system vibration while maintaining the input tracking performance of the manipulator.

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