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Mixed convection boundary layer flow of viscoelastic nanofluid past a horizontal circular cylinder: Case of constant heat flux

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Abstract. The steady of two-dimensional convection boundary layer flow of a viscoelastic nanofluid over a circular cylinder is investigated in this paper. Carboxymethyl cellulose solution (CMC) is chosen as the base fluid and copper as a nanoparticle with the Prandtl number Pr = 6.2. The governing boundary layer partial differential equations are transformed into dimensionless forms. Then they are solved numerically by using the Keller-Box method. This paper focus on the effects of selected parameter on the flow and heat transfer characteristics and be presented in graphs. The results show that, the velocity profiles and the temperature profiles are increased by increasing the values of nanoparticles volume fraction. While velocity profile decreases when viscoelastic parameter is increase. The reverse trend is observed for the temperature profiles. Also, the values of reduced skin friction are increased by increasing mixed convection parameter, but the values of heat transfer coefficient produce an opposite behaviour with an increasing in mixed convection parameter.

1. Mathematical formulation

In this research, we focus on a horizontal circular cylinder with a radius a and constant heat flux. Viscoelastic nanofluid is filled in the circular cylinder and is placed in a constant free stream temperature T_∞. Following [1], the velocity outside the boundary layer is $\bar{u}_e(\bar{x}) = U_\infty \sin(\bar{x}/a)$ by assuming the constant free stream velocity is $(1/2U_\infty)$. To be assumed too, that the free stream velocity is direct vertically upward with $q_w > 0$ (assisting flow) and $q_w < 0$ (opposing flow). The end effects of the cylinder can be neglected by considering the cylinder long enough. By considering the nanofluid model proposed by [2], under the usual boundary layer and Boussinesq approximations, the basic governing equations can be written as,

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \quad (1)$$

$$\rho_{nf}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \rho_{nf}\bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}} + \mu_{nf}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$

$$+k_0\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2\bar{u}}{\partial \bar{y}^2}+\bar{v}\frac{\partial^3\bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2\bar{u}}{\partial \bar{x}\partial \bar{y}}\right)\right]+g\left(\rho\beta\right)_{nf}\left(T-T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right), \quad (2)$$

$$\left(\rho C_p\right)_{nf}\left[\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}\right]=k_{nf}\frac{\partial^2 T}{\partial \bar{y}^2}, \quad (3)$$

subjected to the boundary conditions:

$$\bar{u}=0,\quad \bar{v}=0,\quad \frac{\partial T}{\partial \bar{y}}=-\frac{q_w}{k}\quad \text{at}\quad \bar{y}=0,\quad \bar{x}\ge 0,$$
$$\bar{u}=\bar{u}_e(\bar{x}),\quad \frac{\partial \bar{u}}{\partial \bar{y}}=0,\quad T=T_\infty\quad \text{as}\quad \bar{y}\to\infty,\quad \bar{x}\ge 0, \quad (4)$$

where $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates along the surface of the cylinder. The value is starting from the lower stagnation point of the cylinder. While $\bar{y}$ is the coordinate measured normal to the surface of the cylinder, $\bar{u}$ and $\bar{v}$ are the velocity components, $\bar{u}_e(\bar{x})$ is the velocity outside the boundary layer, T is the temperature of selected fluid, $k_0(\ge 0)$ is the constant of the viscoelastic material with $k_0 = 0$, ρ_{nf} and μ_{nf} are the density and dynamic viscosity of nanofluid, $(\beta)_{nf}$ is the thermal expansion of nanofluid, k_{nf} is the effective thermal conductivity of the nanofluid and $\left(\rho C_p\right)_{nf}$ is the heat capacitance of nanofluid. These nanofluid constants are defined by,

$$\left(\rho C_p\right)_{nf}=(1-\phi)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s,\qquad \left(\rho\beta\right)_{nf}=(1-\phi)\left(\rho\beta\right)_f+\phi\left(\rho\beta\right)_s,$$
$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\quad \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\qquad k_{nf}=k_f\frac{\left(k_s+2k_f\right)-2\phi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\phi\left(k_f-k_s\right)}. \quad (5)$$

where ϕ is the nanoparticle volume fraction of the nanofluid. The thermophysical properties of nanoparticles and base fluid is given in Table 1.

Table 1. Thermophysical properties of nanoparticles and base fluid.

Physical Properties	$\rho\left(\text{kg } m^{-3}\right)$	$C_p\left(\text{J kg}^{-1}\text{K}^{-1}\right)$	$k\left(\text{Wm}^{-1}\text{K}^{-1}\right)$	$\beta\times 10^5\left(\text{K}^{-1}\right)$
CMC	997.1	4179	0.613	21
Cu	8933	385	401	1.67

By introducing the following non-dimensional variables

$$x=\bar{x}/a,\quad x=\text{Re}^{1/2}\left(\bar{y}/a\right),\quad u=\bar{u}/U_\infty,\quad v=\text{Re}^{1/2}\left(\bar{v}/U_\infty\right),$$
$$u_e(x)=\bar{u}_e(\bar{x})/U_\infty,\qquad \theta=\left(T-T_\infty\right)\text{Re}^{1/2}/\left(aq_w/k\right), \quad (6)$$

where $\text{Re}=U_\infty a/v$ is the Reynolds number. By substitution equation (6) into equations (1)-(3), the dimensionless equations becomes,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (7)$$

$$\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]u_e\frac{\partial u_e}{\partial x}+\frac{1}{(1+\phi)^{2.5}}\frac{\partial^2 u}{\partial y^2}$$
$$+K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right]+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\sin(x), \tag{8}$$

$$\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\frac{k_{nf}}{k_f}\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}, \tag{9}$$

wtih the new boundary conditions,

$$u=0,\quad v=0,\quad \frac{\partial\theta}{\partial y}=-1 \qquad \text{at } y=0,\ x\ge 0, \tag{10}$$
$$u=u_e(x),\qquad \frac{\partial u}{\partial y}=0,\ \ \theta=0 \qquad \text{as } y\to\infty,\ x\ge 0,$$

where Pr is the Prandtl number, K known as dimensionless viscoelastic parameter and λ is known as constant mixed convection parameter, that are defined by,

$$K=\frac{k_0U_\infty}{a\rho\upsilon},\qquad \lambda=\frac{Gr}{\mathrm{Re}^2}, \tag{11}$$

where $Gr=g\beta(T_w-T_\infty)a^3/v^2$ is also known as a Grashof number. $\lambda>0$ is correlate to an auxiliary flow while $\lambda<0$ is correlate to the opposing flow. But $\lambda=0$ is correlate to the forced convection flow, respectively. In this problem, if $K=0$ it is referring to the case of viscous (Newtonian) fluids.

2. Solution procedure

Equations (7)-(9) are solved according to the boundary condition (10), we follow [1] and assume that $u_e(x)=\sin x,$ and we assume the following variables:

$$\psi=xF(x,y),\qquad \theta=\theta(x,y), \tag{12}$$

where ψ is the stream function that define as

$$u=\frac{\partial\psi}{\partial y},\qquad v=\frac{\partial\psi}{\partial x}, \tag{13}$$

By using (12) and (13) into Eqs. (8) and (9), we obtain

$$\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[\left(\frac{\partial F}{\partial y}\right)^2+x\frac{\partial F}{\partial y}\left(\frac{\partial^2 F}{\partial x\partial y}\right)-x\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial y^2}-F\frac{\partial^2 F}{\partial y^2}\right]$$
$$=\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\frac{\sin x\cos x}{x}+\frac{1}{(1+\phi)^{2.5}}\frac{\partial^3 F}{\partial y^3}+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\frac{\sin x}{x} \tag{14}$$
$$+K\left[2\frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial y^3}-F\frac{\partial^4 F}{\partial y^4}-\left(\frac{\partial^2 F}{\partial y^2}\right)^2+x\left(\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^3 F}{\partial y^3}-\frac{\partial F}{\partial x}\frac{\partial^4 F}{\partial y^4}+\frac{\partial F}{\partial y}\frac{\partial^4 F}{\partial x\partial y^3}-\frac{\partial^2 F}{\partial y^2}\frac{\partial^3 F}{\partial x\partial y^2}\right)\right],$$

$$\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\left[x\frac{\partial F}{\partial y}\frac{\partial\theta}{\partial x}-x\frac{\partial F}{\partial x}\frac{\partial\theta}{\partial y}-F\frac{\partial\theta}{\partial y}\right]=\frac{k_{nf}}{k_f}\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}, \tag{15}$$

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are subject to the following boundary conditions

$$F=0,\quad \frac{\partial F}{\partial y}=0,\quad \frac{\partial F}{\partial y}=-1,\quad \text{at } y=0,\ x\geq 0,$$
$$\frac{\partial F}{\partial y}=\frac{\sin x}{x},\quad \frac{\partial^2 F}{\partial y^2}=0,\quad \theta=0,\quad \text{as } y\to\infty,\ \ x\geq 0. \tag{16}$$

The physical quantities interest of this problem are the wall temperature distribution $\theta_w(x)$ and the skin friction coefficient C_f, that are defined as,

$$C_f=\mathrm{Re}^{1/2}\frac{\tau_w}{\rho U_\infty^2},\qquad \theta_w=\mathrm{Re}^{-1/2}\frac{aq_w}{k\left(T_w-T_\infty\right)}, \tag{17}$$

where τ_w is the shear stress or skin friction while q_w is the heat flux from the surface of cylinder [3], that are given by

$$\tau_w=\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0}+k_0\left(\bar{u}\frac{\partial^2\bar{u}}{\partial\bar{x}\partial\bar{y}}+\bar{v}\frac{\partial^2\bar{u}}{\partial\bar{y}}+2\frac{\partial\bar{u}}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0},\quad q_w=-k\left(\frac{\partial T}{\partial\bar{y}}\right)_{\bar{y}=0}, \tag{18}$$

And it is known that μ is for dynamic viscosity while k is for thermal conductivity. By substitution of (6), (17) and (18), we obtain

$$C_f=x\left(\frac{\partial^2 F}{\partial y^2}\right)_{y=0},\qquad \theta(x,0)=\theta_w(x). \tag{19}$$

When $x\approx 0$, in the case of lower stagnation point, we introduce the similarity variable $xf(\eta)$, $\theta=\theta(\eta)$ and η as per below. Thus, it can be shown that equations (7)-(9) diminish to the ordinary differential equations as:

$$\frac{1}{(1+\phi)^{2.5}}f'''-\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[f'^2-ff''\right]$$
$$+K\left(2ff'''-ff^{iv}-f''^2\right)+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta=0, \tag{20}$$

$$\frac{k_{nf}}{k_f}\frac{1}{\mathrm{Pr}}\theta'+\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]f\theta'=0, \tag{21}$$

and the boundary condition (9) as

$$f(0)=0,\qquad f'(0)=0,\qquad \theta'(0)=-1,$$
$$f'(\infty)=1,\qquad f''(\infty)=0,\qquad \theta(\infty)=0, \tag{22}$$

In this case, we have

$$C_f=xf''(0),\qquad \theta_w(0)=\theta(0). \tag{23}$$

The present study in equations (20)-(21) reduces to that of [3] and [4] for a viscoelastic fluid when $\phi=0$ (regular fluid).

3. Results and discussion

Both equations (14-15) with the boundary conditions (16), and the ordinary differential equations (20-21) with the boundary conditions (22) that were solved numerically using the Keller-box method. The effect of different values of the previous parameters on the velocity profiles, temperature profiles, reduced skin friction and heat transfer coefficients are reviewed in particularly.

Table 2. Comparison values of $\theta(0)$ and $f''(0)$ for variety values of λ when $\Pr = 1$ and $K = 0$.

λ	**Nazar *et al.*** [4]	**Abdul Rahman *et al.*** [5]	**Present** $f''(0)$	**Nazar et al.** [4]	**Abdul Rahman *et al.*** [5]	**Present** $\theta(0)$
-0.2	1.0340	1.033028	1.033028	1.8157	1.816890	1.816890
0.4	1.5747	1.573759	1.573759	1.7018	1.702823	1.702823
3.0	2.4913	2.489892	2.489892	1.4015	1.402232	1.402232
10.0	5.7730	5.777805	5.777805	1.1770	1.178456	1.178456

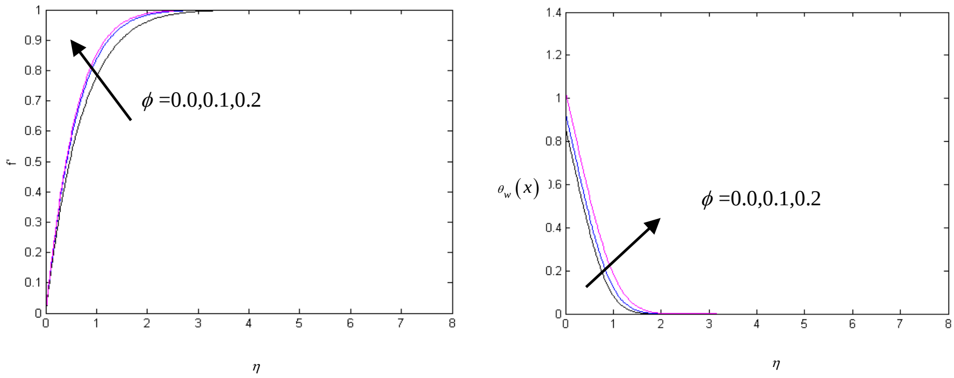

Figure 1. Velocity and temperature profiles for variety values of ϕ when $K = 0.1, \lambda = 1$.

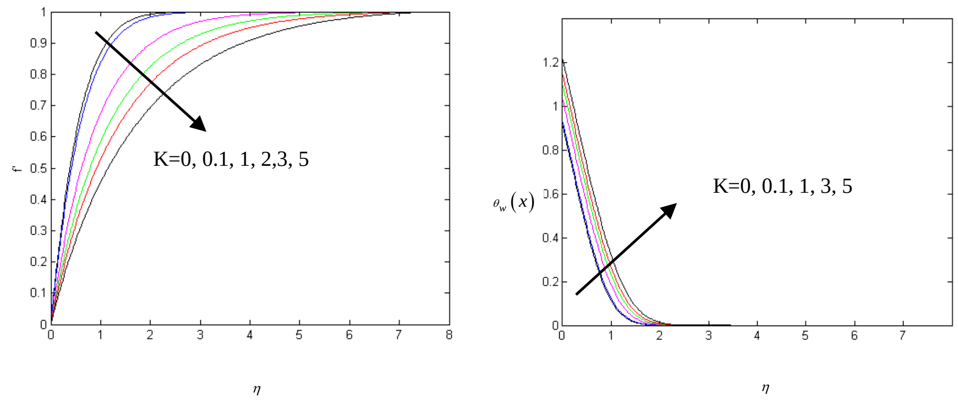

Figure 2. Velocity and temperature profiles for variety values of K when $\Pr = 6.2, \lambda = 1, \ \phi = 0.1$.

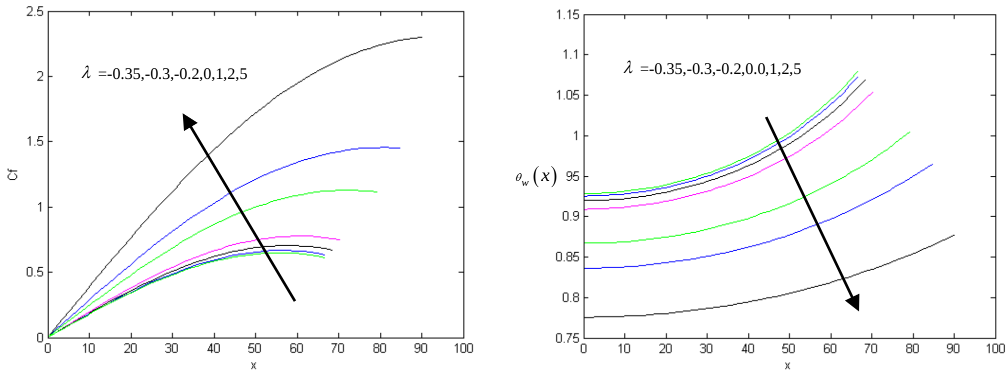

Figure 3. Variation of C_f and $\theta_w(x)$ with x for $K = 0.1, \phi = 0.1$.

Comparison of results on $f''(0)$ and $\theta(0)$ with the various values of λ obtained in this study, with those of [3] and [4] are presented in Table 2. These results showed a good agreement, which validates the use of present scheme. The distribution of velocity, temperature, skin friction and heat transfer coefficients are presented in Fig. 1 – 3. The figures show the effects of nanoparticles volume fraction ϕ , mixed convection parameter λ**,** and viscoelastic parameter *K*, on fluid flow and heat transfer characteristics.

4. Conclusions

Steady of mixed convection boundary layer flow of viscoelastic nanofluids past a horizontal circular cylinder with a case of constant heat flux problem has been solved numerically using Keller-box method. The numerical results for the skin friction, heat transfer coefficient, velocity and temperature profiles are presented for variety values of nanoparticles volume fraction, mixed convection parameter and viscoelastic parameter. Comparison of the result show that the present results agree very well with the previous published results reported by [3] and [4].

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