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Frequency Based Substructuring for Structure with Double Bolted Joints: A Case Study

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ABSTRACT

Adopting dynamic substructuring schemes is a common practice in the field of structural dynamics, where the dynamic behaviour of a structure is predicted by combining the multiple subsystems that are analysed individually. This paper investigated the dynamic behaviour of a structure with doubled bolted joints using the frequency based substructuring (FBS) method. In the attempt, the substructures were combined with the frequency domain that can be numerically derived or experimentally measured. However, the applicability of this method suffered from several issues where most of them were related to the frequency response function (FRF) of the rotational degree of freedom (DOF) at the subsystem's interface. In some cases, the system's interface vibrated in the rotary motion of certain modes, for instance, a car side rear view mirror. Therefore, excluding the rotational FRF during the coupling process could lead to a completely different result. This paper presents the use of the FBS method for a structure with double bolted joints by using the equivalent multipoint constraint (EMPC) method through which rotational DOFs can be completely neglected. The actual tested structure for this study was an assembled structure consisting of two substructures: a simple beam and an irregular plate steel structure. The FRFs of both substructures were derived from using the FRF synthesis from finite element models and combined together via the FBS method. This study reveals that the use of the FBS method with the EMPC method for a structure double bolted joints has led to very promising results.

Keywords: Frequency based substructuring; equivalent multipoint connection; double bolted joints.

INTRODUCTION

In structural dynamics, the dynamic behaviour of a structure which consists of several components or substructures can be efficiently and economically calculated by using dynamic substructuring method [1, 2]. This method allows a built-up structure to be dissembled into several substructures and analysing analysis of the dynamic behaviour of the substructures can be done individually. Thus, combining the analysed dynamic data can predict the dynamic behaviour of the built-up structure [3, 4]. Dynamic substructuring also permits a combination between the dynamic behaviour obtained from numerical and experimental model in the frequency domain [5, 6]. This dynamic substructuring method is called frequency based substructuring (FBS) or admittance coupling. The FBS method combines the substructures frequency response function (FRF) by balancing the forces and enforcing continuity at the interfaces. The summarized review, history and general framework of FBS are available in De Klerk [7].

The FBS method has the capability to efficiently and economically solve several complex vibrational issues, especially where some parts of the system are very difficult to be analytically modelled or are merely required to be analysed. For example, Law, Rentzsh and Ihlenfeldt [8-10] predicted the dynamic behaviour of a machining tool by combining the measured FRF of a complex operating machine with other several substructures via the FBS method. Another striking example is that the FBS method was used in a cost-effective way for predicting the dynamic behaviour of the isolation system for an operating machinery [11, 12]. Both examples have demonstrated a strong link between efficiency and economics of the FBS method in the use of predicting the dynamic behaviour of a complex structure.

The dynamic substructuring method is a common practice and well accepted method in the domain of the finite element analysis in particular. However, its experimental equivalence is more likely susceptible to the unacceptable results [13]. One of the major issues which have been the major contribution to the FBS errors that have been critically discussed, is the rotational degrees of freedom (DOFs) from experimental work [14]. In certain cases, the system's interface vibrates in rotary motion for certain particular modes, for instance, a car side rear view mirror. The car side mirror needs to be rigid as the car moves. This is to ensure that there are no excessive vibrations when the car operates. Therefore, directly neglecting the rotational DOFs in the coupling process will lead to completely different results.

Figure 1. A car side rear view mirror that is subjected to rotary motion.

Several research studies have been performed in order to estimate the rotational FRF of a substructure. For example, an X-block attachment was used to allow the measurement of rotational FRF at a component in order to perform a successful structural modification of a helicopter tail [15]. Another approach that had been used was to attach a known model of T-block mass at the interface of the experimental substructure in order to bring more FRF peak towards lower modes [16, 17]. Later, the numerical model of the T-block was decoupled in order to expand and obtain the rotational FRFs at the interfaces. This method was also called as transmission simulator in [18].

The development of applying pure torque excitation and rotational FRF measurement equipment have therefore caught many dynamicists' attention [19, 20].

Recently, the rotational accelerometer was introduced by Kristler Instrument which allows the direct measurement of the rotational FRF [21]. However, the force-moment excitation of the experimental substructure still needed to be expanded based on the modal model in order to obtain full DOF at the coupling interface [22]. The successful direct measurement of the rotational DOF's FRF however remains to be unexplored.

In most cases, the dynamicists tend to ignore the rotational DOF by combining the substructure's interfaces to a multipoint connection. This approach is named "Equivalent Multipoint Connection (EMPC)" [23]. The approach was adopted based on the discrete DOFs at the interfaces from the 3D finite element model, where the interface consists of multiple nodes and directions [13]. By this manner, the rotational DOF can be completely neglected. However, there were very limited studies related to the capabilities of EMPC approach. Therefore, this paper aims to investigate the capabilities of the EMPC method in order to predict the dynamic behaviour of a double bolted joint structure, which is subject to rotary motion at the interface in certain modes. The test structure for this study consists of a single beam and an irregular shape structure.

Theoretical Explanations for FBS method

In this section, the FBS method is explained based on the schematic diagram in Figure 2. The coupling method is physically and mathematically described based on the Generalized Receptance Coupling (GRC) [24]. To be physically general, consider a system (S) which consists of independent subsystems (A) and (B), which the DOFs have been classified either as internal (subscript *a* and *b*) or coupling (subscript *c*) DOFs.

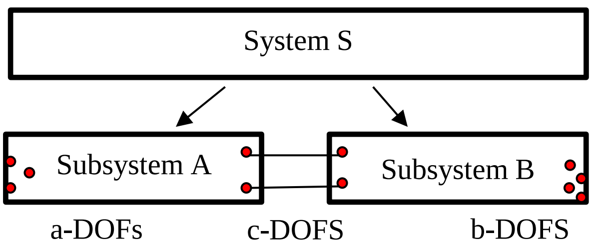


Figure 2. Schematic diagram for subsystem S.

The equation of motion of subsystem A and B in the frequency domain for the complex displacement $\mathbf{X}_n$ is described as:

$$\mathbf{X}_n = \mathbf{H}_{nn}\mathbf{F}_n \tag{1}$$

where $\mathbf{H}_{nn}$ is the complex admittance matrix in term of displacement/force and force $\mathbf{F}_n$ is the applied force vector. The total number of DOFs for each subsystem are defined as subscript *n*, while subscript *s* represents the coupled system. For the rigid connections and force equilibrium between subsystems A and B, compatibility implies that;

$$\mathbf{X}^{\mathrm{A}}{}_{\mathrm{c}} = \mathbf{X}^{\mathrm{B}}{}_{\mathrm{c}} = \mathbf{X}^{\mathrm{S}}{}_{\mathrm{c}} \tag{2}$$

$$\mathbf{F^{A}}{}_{\mathbf{c}} = \mathbf{F^{B}}{}_{\mathbf{c}} = \mathbf{F^{S}}{}_{\mathbf{c}} \tag{3}$$

Based on Eq. (1), (2) and (3), the partitions matrix of subsystems A and B is derived to form the whole coupled system FRF matrix. The FRF matrix for a coupled system can be represented as:

$$\begin{Bmatrix} \mathbf{X}^{s}{}_{a} \\ \mathbf{X}^{s}{}_{c} \\ \mathbf{X}^{s}{}_{b} \end{Bmatrix}_{n} = \begin{bmatrix} \mathbf{H}^{s}{}_{aa} & \mathbf{H}^{s}{}_{ac} & \mathbf{H}^{s}{}_{ab} \\ \mathbf{H}^{s}{}_{ca} & \mathbf{H}^{s}{}_{cc} & \mathbf{H}^{s}{}_{cb} \\ \mathbf{H}^{s}{}_{ba} & \mathbf{H}^{s}{}_{bc} & \mathbf{H}^{s}{}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{F}^{s}{}_{a} \\ \mathbf{F}^{s}{}_{c} \\ \mathbf{F}^{s}{}_{b} \end{Bmatrix}_{n} \tag{4}$$

The above further matrix can be derived to obtain the FRF at the coupling or internal DOFs of both subsystems. The detailed explanation of the GRC based FBS can be found in [25].

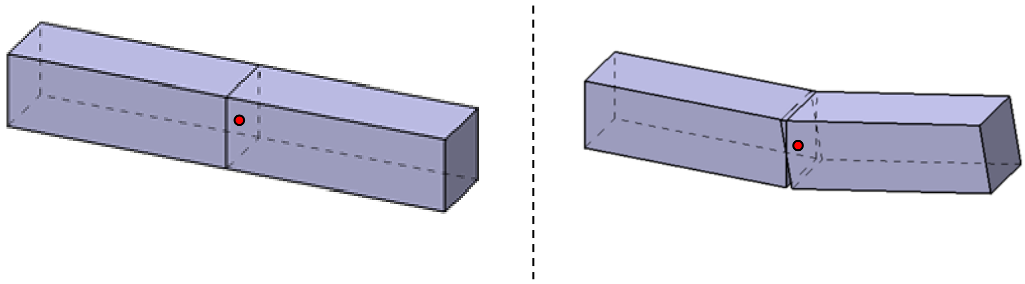
The Importance of Rotational DOF

Theoretically, in order to accurately calculate the assembled system's FRF via GRC based FBS, all the degrees of freedom at the interface need to be coupled, including the rotational DOFs. The previous study reveals that including the rotational DOFs in the process of coupling has led to successful and accurate coupled FRF between FE-experimental dynamic substructuring [26]. Consider a single node system described in a matrix:

$$\begin{bmatrix} x \\ y \\ z \\ \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} = \begin{bmatrix} Y_{xx} & Y_{xy} & Y_{xz} & Y_{x\phi_x} & Y_{x\phi_y} & Y_{x\phi_z} \\ Y_{yx} & Y_{yy} & Y_{yz} & Y_{x\phi_x} & Y_{x\phi_y} & Y_{x\phi_z} \\ Y_{zx} & Y_{zy} & Y_{zz} & Y_{x\phi_x} & Y_{x\phi_y} & Y_{x\phi_z} \\ Y_{\phi_x x} & Y_{\phi_x y} & Y_{\phi_x z} & Y_{\phi_x x} & Y_{\phi_x \phi_y} & Y_{\phi_x \phi_z} \\ Y_{\phi_y x} & Y_{\phi_y y} & Y_{\phi_y z} & Y_{\phi_y x} & Y_{\phi_y \phi_y} & Y_{x\phi_z} \\ Y_{\phi_z x} & Y_{\phi_z y} & Y_{\phi_z z} & Y_{\phi_z x} & Y_{\phi_z \phi_y} & Y_{\phi_z \phi_z} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} \tag{5}$$

where *Y*, *F*, and *M* describe the FRF matrix, force-excite and moment-excite respectively. The subscripts [*x*,*y*,*z*] represent the translational DOFs while $[\phi_x \phi_y \phi_z]$ represent the rotational DOFs. The matrix in Eq. (5) clearly shows that 75% of the single node matrix contains rotational information. Therefore, neglecting the rotational DOFs at the connection during the coupling can lead to an incomplete DOF matrix at the interface. In that manner, 75% of information that is transferred through the joint has been removed. The illustration of neglecting the rotational DOFs at the connection of two simple beams is presented in Figure 3.

In structural dynamics, the rotational FRFs can easily be calculated analytically or via the finite element method. However, the major problem is dealing with the experimental subsystem. Currently, there are two ways to obtain the rotational information for experimental dynamic substructuring. They are by measurement techniques [15, 22] and modal expansion method.



(a) (b)

Figure 3. (a) Coupling all connection DOFs (b) Neglecting the connection of rotational DOF.

EQUIVALENT MULTIPOINT CONNECTION (EMPC) APPROACH

The EMPC method consists of obtaining the FRFs of the substructure's interface at multiple nodes and multiple directions. This method implies that the connection interfaces are always surface connections. Therefore, real-life structures should formally be coupled by infinite DOFs at the interfaces. In finite element method, the translational DOFs information can only provide a good estimate of the interface flexibility, if it is determined on multiple points. Figure 4 shows the illustration of the EMPC method by combining multiple nodes at the interface. A minimum of three nodes and six coupling DOFs are required to sufficiently describe all motions of the rigid interface as recommended [23].

The substructure interface can roughly be assumed as a rigid interface, as it is stiff and small surface. The advantage of regarding the interface as rigid is the possibility to build the flexibility matrix of the interface by the geometric relations between the nodes. For example, for a 9 translational DOFs forming 3 nodes at the interface, the flexibility matrix can be described as:

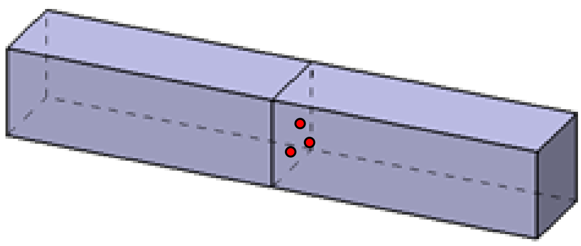


Figure 4. Illustration of EMPC approach of a beam structure.

$$\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{1z} \\ u_{2x} \\ u_{2y} \\ u_{2z} \\ u_{3x} \\ u_{3x} \\ u_{3x} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & a_{1z} & -a_{1y} \\ 0 & 1 & 0 & a_{1z} & 0 & a_{1x} \\ 0 & 0 & 1 & a_{1y} & a_{1x} & 0 \\ 1 & 0 & 0 & 0 & a_{2z} & -a_{2y} \\ 0 & 1 & 0 & a_{2z} & 0 & a_{2x} \\ 0 & 0 & 1 & a_{2y} & a_{2x} & 0 \\ 1 & 0 & 0 & 0 & a_{3z} & -a_{3y} \\ 0 & 1 & 0 & a_{3z} & 0 & a_{3x} \\ 0 & 0 & 1 & a_{3y} & a_{3x} & 0 \end{bmatrix} \begin{bmatrix} q_{1x} \\ q_{1y} \\ q_{1z} \\ q_{1\phi_x} \\ q_{1\phi_y} \\ q_{1\phi_z} \end{bmatrix} + m \tag{6}$$

where *u* is the interface DOF in a main content *q*, *a* is a relative distance between nodes and *m* is the residual vector. The six resulting unique DOFs can be interpreted as three translational DOFs and three rotational DOFs [13].

DESCRIPTION OF THE TEST STRUCTURE

The applicability of the EMPC approach for FBS can be demonstrated as an assembled structure consisting of a 380mm beam and an irregular shape substructure as shown in Figure 5. Both substructures should be modelled based on steel material properties and combined by using two bolted joints. These simple configurations allow an initial study of a structure that is subjected to rotational motion at the interface (bending and torsion mode) as shown in Figure 6, where the rotational DOF plays an important role for a successful FRF coupling. The frequency of interest for this case study is between 0 to 2000Hz which contains several bending and torsional modes.

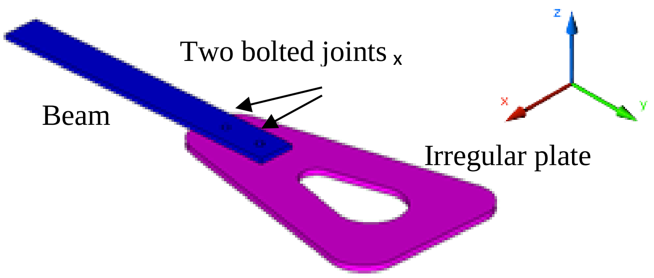


Figure 5. The test structure for the study.

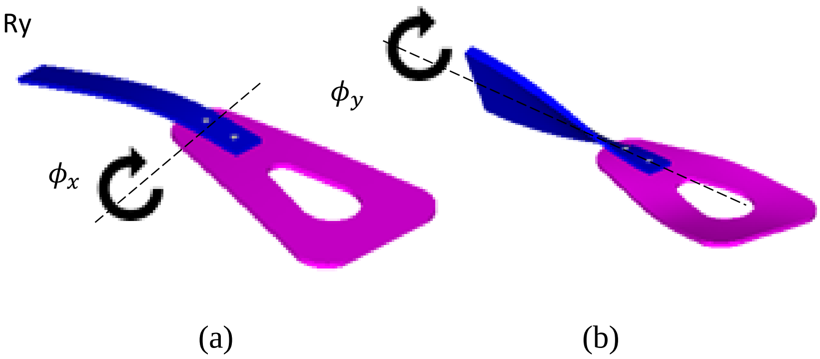


Figure 6. The test structure subjected to rotational motion about (a) rotational x-axis (1st mode) and; (b) rotational y-axis (4th mode) at the interface.

FREQUENCY BASED SUBSTRUCTURING METHOD

The FBS method was performed by coupling the FE FRF of both substructures in order to evaluate the capabilities of EMPC method for combining a structure with two bolted joints. Prior to combining the FE FRF of a simpe beam and irregular plate substructure, both FE model of the substrcutrues had been constructred as shown in Figure 7 and Figure 8. The frequency bandwidth for both substructures were set between 0-10,000Hz to include the rigid body modes and minimised the truncation error for dynamic substructuring [27]. The natural frequencies and mode shapes of the finite element model were solved by using normal modes solution in the CAE software [28, 29]. The FRFs at the interface of the FE models were derived by using FRF synthesized method based on the calculated natural frequencies and mode shapes [30].

In this study, two types of coupling configuration had been performed. The first one was the traditional single node FRF coupling [31, 32] as shown in Figure 7. Initially, the two substructures were coupled by using both translational and rotational DOFs in order to obtain the actual and reference FRF for benchmarking purposes. Subsequently, the substructures were coupled by using only the translational DOFs. This coupling configuration allows an intial study of the accuracy of the coupled FRF by neglecting the rotational DOFs during the process of coupling.

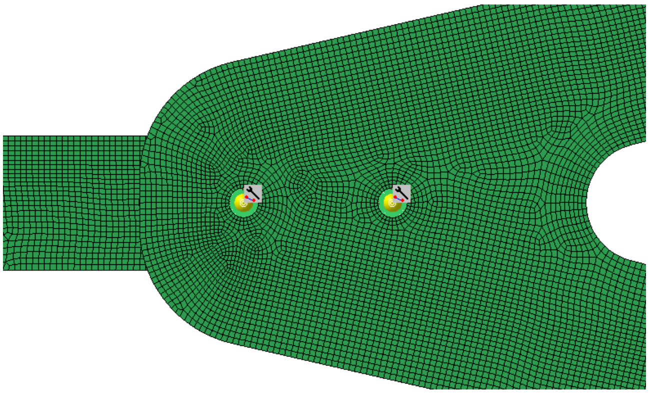


Figure 7. FRF coupling using single node interface.

The second configuration was by implimenting EMPC approach for FRF coupling by using three nodes at each of the substructure connection interfaces as shown in Figure 8 and 9. The interfaces were assumed as rigid and only translatioanl DOFs were involved in the process of coupling. The calculated coupled FRF for both coupling configuration were compared, validated and discussed in the next section of this article.

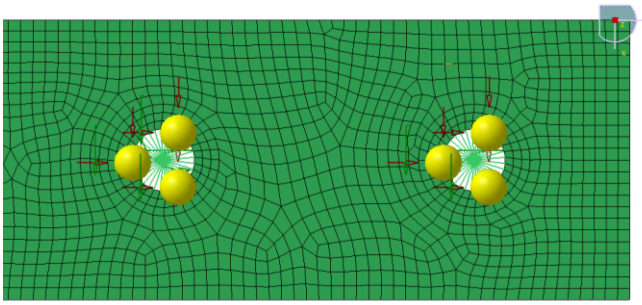


Figure 8. EMPC node interface of beam substructure.

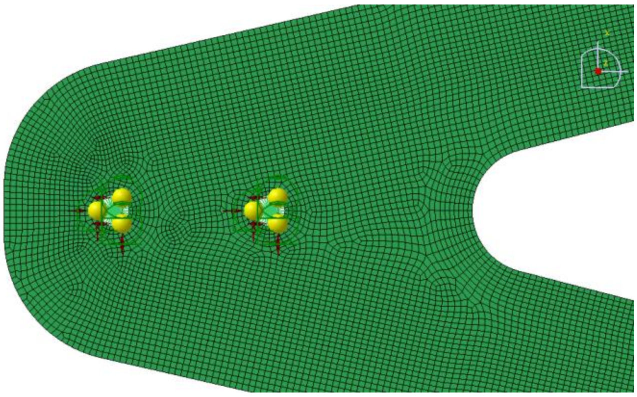


Figure 9. EMPC node interface of irregular shape structure.

RESULTS AND DISCUSSION

Several previous studies revealed that excluding one of the important DOFs during the process of coupling has usually led to completely different results [33, 34]. There have been numerous attempts made by researchers to enhance the predicted coupled FRF by considering the rotational FRF. The common roadblocks for introducing the rotational FRF to the flexibility matrix found in the attempts were (1) poor quality of measured FRF and (2) the incomplete moment excite FRF needed to be expanded. The expanded FRF seems to be acceptable for a simple beam structure, but very challenging for a complex structure as the process of FRF coupling is case-dependent. Therefore, the first part of this section discusses the effect of the coupled FRF of the actual tested structure by excluding the rotational FRF from the interface matrix.

In this study, the actual tested structure was subjected to torsional or rotational motion at certain modes, where neglecting the rotational DOFs at the substructure's connection interface seemed to be unacceptable. In that manner, 75% of the information that was transferred to the connections was not taken into account for representing the flexibility of the connection interface. The comparison between the predicted coupled FRF based on translational DOFs (blue) and the reference FRF (red) is shown in Figure 10, revealing the attempt to couple the substructure using only translational DOFs at the single node at the interfaces of the substructure had led to an inaccurate prediction of the coupled FRF, especially for the higher modes.

From Figure 10, it can clearly be seen that the pattern and amplitudes of the resonance as well as anti-resonance of coupled FRF are almost identical with the benchmark FRF for the first three modes. The good correlation of FRFs obtained for the first three modes is mainly because of the translational DOFs can well represent the rigidity of the interface in ϕ_x–axis. However, the pattern of the coupled FRF began to shift for the higher mode, especially starting from 1000 Hz above. The significant reason for the shift in the coupled FRF is mainly because the rotational motion in the ϕ_y – axis starts to develop at the fourth mode as shown in Figure 6. Therefore, it is worth noting that the coupled FRF will only be reliable for the lower modes if the rotational DOFs are not included in the coupling process [34].

In order to completely exclude the rotational DOFs in the FRF coupling process, the EMPC method has been introduced. This method assumes that the interfaces are always surface connections and have infinite DOFs, where having at least three-point connections and nine DOFs can describe the flexibility of the interface. Therefore, each substructure interface is described by multiple point-connections which have translational

DOFs information only. The second coupling configuration for the actual tested structure by using the EMPC approach has been successfully calculated and presented in Figure 11.

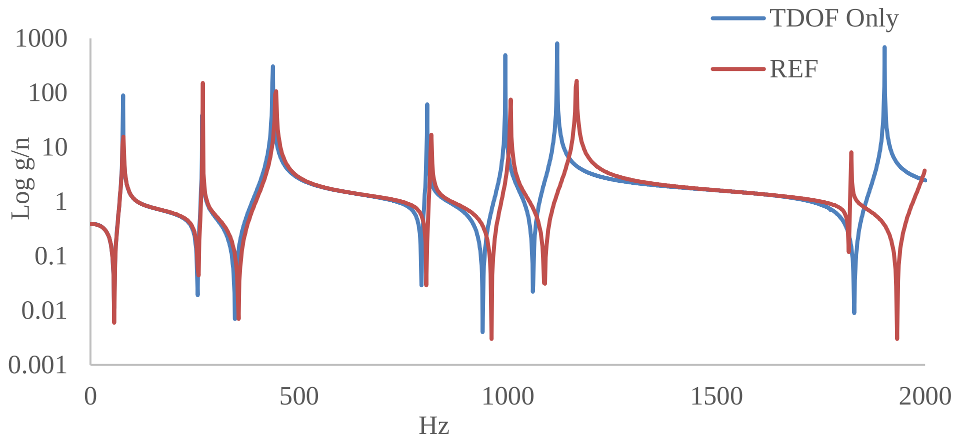


Figure 10. Coupled FRF using single node translational DOFs and reference FRF.

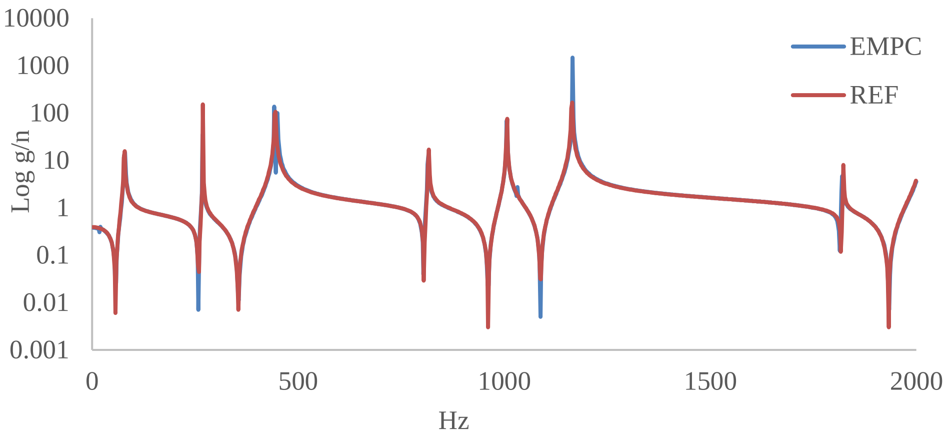


Figure 11. Coupled FRF using EMPC approach and reference FRF.

Figure 11 shows that there is a perfect match for all resonance as well as anti-resonance peaks between the coupled FRF from the EMPC method and the benchmark FRF. This remarkable achievement has proved that a good estimation of the interface flexibility can also be attained from solely using the translational DOFs in the coupling process. Since measuring the rotational DOFs FRF is reported to be very difficult and not a common practice among modal analysts, the EMPC method proposed in this study can be very practical and useful to be used by the modal analysts.

CONCLUSION

This study investigated the dynamic behaviour of a structure with double bolted joints by using the frequency based substructuring (FBS) method. The structure consisted of two substructures and they were combined via two different types of coupling configurations, namely the single node and the equivalent multipoint connection (EMPC) method. The evaluation of the accuracy and capability of both coupling methods has been presented and discussed. Based on the traditional single node FRF coupling, completely excluding the rotational FRF in the coupling process, has rendered the coupled FRF incapable of matching the benchmark FRF, mainly because 75 percent of the joint information has been neglected. On the other hand, the EMPC approach has led to a successful and accurate calculation of the FRF as the flexibility matrix of the interface was built based on the multiple translational degrees of freedom for the surface connection of the interface. Therefore, the EMPC method, which implicitly accounts for rotational degrees of freedom information, can considerably improve the predicted results. This significant achievement has led to the conclusion that the EMPC method has many advantages over the conventional methods for preparing the experimental model due to the moment excitation and rotational FRF measurements being very problematic to be performed.

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REFERENCES

[1] Mirza WIIWI, Rani MNA, Yunus MA, Roslan L, Basri ABA. The investigation of the dynamic behaviour of a complex assembled structure using the frequency response function based substructuring method. MATEC Web of Conferences2016.

[2] Peeters P, Manzato S, Tamarozzi T, Desmet W. Reducing the impact of measurement errors in FRF-based substructure decoupling using a modal model. Mechanical Systems and Signal Processing. 2018;99:384-402.

[3] Mirza WIIWI, Rani MNA, Othman MH, Kasolang S, Yunus MA. Reduced order model for model updating of a jointed structure. Journal of Engineering and Applied Sciences. 2016;11:2383-6.

[4] Tol Ş, Özgüven HN. Dynamic characterization of bolted joints using FRF decoupling and optimization. Mechanical Systems and Signal Processing. 2015;54:124-38.

[5] Van Der Seijs MV, De Klerk D, Rixen DJ. General framework for transfer path analysis: History, theory and classification of techniques. Mechanical Systems and Signal Processing. 2016;68-69:217-44.

[6] De Klerk D, Rixen DJ, Voormeeren SN. General framework for dynamic substructuring: History, review, and classification of techniques. AIAA Journal. 2008;46:1169-81.

[7] Law M, Ihlenfeldt S. A frequency-based substructuring approach to efficiently model position-dependent dynamics in machine tools. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics. 2015;229:304-17.

[8] Law M, Rentzsch H, Ihlenfeldt S. Predicting mobile machine tool dynamics by experimental dynamic substructuring. International Journal of Machine Tools and Manufacture. 2016;108:127-34.

[9] Law M, Rentzsch H, Ihlenfeldt S, Putz M. Application of Substructure Decoupling Techniques to Predict Mobile Machine Tool Dynamics: Numerical Investigations. Procedia CIRP2016. p. 537-40.

[10] Peng W. FRF-based substructure technique for modeling machinery isolation system. 21st International Congress on Sound and Vibration 2014, ICSV 20142014. p. 182-9.

[11] Peng W, Huang HZ, Li Y, Zuo MJ, Xie M. Life cycle reliability assessment of new products - A Bayesian model updating approach. Reliability Engineering and System Safety. 2013;112:109-19.

[12] De Klerk D, Rixen DJ, Voormeeren SN, Pasteuning F. Solving the RDoF problem in experimental dynamic substructuring. Conference Proceedings of the Society for Experimental Mechanics Series2008.

[13] Nicgorski D, Avitabile P. Experimental issues related to frequency response function measurements for frequency-based substructuring. Mechanical Systems and Signal Processing. 2010;24:1324-37.

[14] Mottershead JE, Ghandchi Tehrani M, Stancioiu D, James S, Shahverdi H. Structural modification of a helicopter tailcone. Journal of Sound and Vibration. 2006;298:366-84.

[15] Goldenberg S, Shapiro M. A study of modal coupling procedures for the space shuttle. Bethpage, New York: Grunman Aerospace Coorperation; 1972.

[16] Allen MS, Mayes RL, Bergman EJ. Experimental modal substructuring to couple and uncouple substructures with flexible fixtures and multi-point connections. Journal of Sound and Vibration. 2010;329:4891-906.

[17] Pasma E.A., Seijs M.V., Klaassen S.W.B., M.W. K. Frequency Based Substructuring with the Virtual Point Transformation, Flexible Interface Modes and a Transmission Simulator. In: Linderholt A., Allen M., Mayes R., D. R, editors. Dynamics of Coupled Structures. Cham: Springer; 2018.

[18] Montalvão D, Ribeiro AMR, Maia NMM, Silva JMM. Estimation of the rotational terms of the dynamic response matrix. Shock and Vibration. 2004;11:333-50.

[19] De Klerk D, Voormeeren SN, Petzsche T. Enhanced FRF determination using a 3D impedance head. Conference Proceedings of the Society for Experimental Mechanics Series2008.

[20] Dumont M, Kinsley N. Rotational accelerometers and their usage in investigating shaker head rotations. Conference Proceedings of the Society for Experimental Mechanics Series. 2015 ed2015. p. 85-92.

[21] Drozg A, Čepon G, Boltežar M. Full-degrees-of-freedom frequency based substructuring. Mechanical Systems and Signal Processing. 2018;98:570-9.

[22] Voormeeren S. Coupling Procedure Improvement & Uncertainty Quantification in Experimental Dynamic Substructuring–Validation and Application in Automotive Research [Doctoral Thesis]. Delft, Netherlands: Delft University of Technology; 2007.

[23] Ren Y, Beards CF. Generalized receptance coupling technique. Proceedings of SPIE - The International Society for Optical Engineering. pt 2 ed1993. p. 868-71.

[24] Urgueira PVA. Dynamic Analysis of Coupled Structures Using Experimental Data [Doctoral Thesis]: University Of London; 1989.

[25] Williams A, Chipman C, Avitabile P. Modal and frequency based substructuring using rotational DOF consideratations. International Modal Analysis Conference XXVI. Orlando, Florida2008.

[26] Van Der Valk PLC, Van Wuijckhuijse JB, De Klerk D. A Benchmark Test Structure for Experimental Dynamic Substruturing. Conference Proceedings of the Society for Experimental Mechanics Series2011. p. 113–23.

[27] Rani MNA, Kasolang S, Mohd MHO, Yunus A, Wan Iskandar Mirza WII, Ouyang H. Finite element modelling and modal based updating for the dynamic behaviour of a laser spot welded structure. ICSV 2016 - 23rd International Congress on Sound and Vibration: From Ancient to Modern Acoustics2016.

[28] Abdullah NAZ, Sani MSM, Husain NA, Rahman MM, Zaman I. Dynamics properties of a Go-kart chassis structure and its prediction improvement using model updating approach. International Journal of Automotive and Mechanical Engineering. 2017;14:3887-97.

[29] Zin MSM, Rani MNA, Yunus MA, Sani MSM, Wan Iskandar Mirza WII, Mat Isa AA. Frequency response function (FRF) based updating of a laser spot welded structure. AIP Conference Proceedings2018.

[30] Sani MSM, Nazri NA, Rani MNA, Yunus MA. Dynamic analysis of i cross beam section dissimilar plate joined by TIG welding. AIP Conference Proceedings2018.

[31] Yunus MA, Rani MNA, Sani MSM, Shah MASA. Finite element model updating of riveted joints of simplified model aircraft structure. AIP Conference Proceedings2018.

[32] Manzato S, Risaliti E, Napoli C, Tamarozzi T, Peeters B. A Review of Frequency-Based Substructuring Methods and Their Applicability to Engineering Structures. Proceeding of International Conference of Structural Engineering Dynamics (ICEDyn). Lagos, Portucal2015.

[33] Liu W, Ewins D. Importance assessment of RDOF in FRF coupling analysis. Proceedings of the International Modal Analysis Conference - IMAC1999. p. 1481-7.

[34] Abdul Rani MN, Stancioiu D, Yunus MA, Ouyang HJ, Deng H, James S. Model Updating for a Welded Structure Made from Thin Steel Sheets. Applied Mechanics and Materials. 2011;70:117-22.