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Hybrid Harmony Search Algorithm with Grey Wolf Optimizer and Modified Opposition-based Learning

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ABSTRACT Most metaheuristic algorithms, including harmony search (HS), suffer from parameter selection. Many variants have been developed to cope with this problem and improve algorithm performance. In this paper, a hybrid algorithm of HS with grey wolf optimizer (GWO) has been developed to solve the problem of HS parameter selection. Then, a modified version of opposition-based learning technique has been applied on the hybrid algorithm to improve the HS exploration because HS easily gets trapped into local optima. Two HS parameters were automatically updated using GWO, namely, pitch adjustment rate and bandwidth. The proposed hybrid algorithm for global optimization problems is called GWO-HS. GWO-HS was evaluated using 24 classical benchmark functions with 30 state-of-the-art benchmark functions from CEC2014. Then, GWO-HS has been compared with recent HS variants and other well-known metaheuristic algorithms. Results show that the GWO-HS is superior over the old HS variants and other well-known metaheuristics in terms of accuracy and speed process.

INDEX TERMS Computational Intelligence, Grey wolf optimizer, Harmony search, Hybrid algorithm, Metaheuristic, Optimization algorithm, CEC2014.

I. INTRODUCTION

Solving the NP-hard problem using an exhaustive search is an impractical technique because of long-time consumption and complex application. A well-known solution to solve the NP-hard problem with minimal time consumption is using a heuristic technique that can find a near-optimal solution. Heuristic algorithm sacrifices optimality or completeness to obtain quickly the best result.

Meta-heuristic algorithms are higher-level heuristic algorithms that can cover a wider range of problems, with a lack of information or high computation time [1]. The main functionality of meta-heuristic algorithms is obtained by merging rules and randomness to simulate natural phenomena, such as physical annealing in a simulated annealing (SA) algorithm [2], the human intelligence in the harmony search (HS) algorithm [3], the biological evolutionary process in an evolutionary algorithm (EA) [4], and animal behavior in Tabu search [5].

The efficiency of metaheuristic algorithms depends on the utilization of explorative and exploitative ranges through the search process [6]. The exploitative process is accomplished by utilizing the information obtained to guide the search toward its goal. The explorative process is the capability of an algorithm to examine uncovered areas quickly within considerable search sizes. Overall performance develops if the balance between these two characteristics is established [7].

Harmony search (HS) algorithm is a well-known metaheuristic algorithm, introduced by Geem et al. [3] by mimicking the musician's process in creating a new musical harmony[8, 9]. The HS algorithm is used in different fields of optimization problems, such as engineering [10, 11], water distribution [12], structural optimization [6], music ensemble [13], and university timetable [14], Software testing [15-18]. Many other applications and variants of the HS algorithm were made according to previous survey articles [19, 20].

The success of using HS in different research fields is attributed to its characteristics. The main advantage of HS is its capability to utilize exploration and exploitation simultaneously through the search process [14].

Most metaheuristic algorithms, including HS, suffer from parameter selection, and premature convergence. Many variants have been developed to cope with this problem and improve algorithm performance [21-26].

Generally, researchers have two ways of setting metaheuristic parameter values, namely, by using parameter tuning or by using parameter control.

A. PARAMETER TUNING

The use of parameter tuning is achieved by finding the best values for algorithm parameters before running the algorithm to fix the problem. Parameter tuning involves a number of difficulties, such as longtime consumption because of the need to cover all possibilities, which is practically impossible; another difficulty is high complexity because parameters are not independent; moreover, choosing a fixed parameter as optimal value through the search process is against the idea of EA of a dynamic and adaptive process[27].

B. PARAMETER CONTROL

The other way to modify algorithm parameter values is through the search process, which can be accomplished in three ways.

1: **First method:** The algorithm parameter values can be modified using a deterministic function to replace the static value of the parameters in the search process; an example of this process is the improved HS by Mahdavi et al. [21], who replaced the static values of pitch adjustment rate (PAR) and bandwidth (BW) with new functions to modify their values throughout the search process. The following equations present the dynamic BW:

$$C = \left(ln \left(\frac{BW_{\ min}}{BW_{\ max}} \right) \div \mathrm{NI} \right) \tag{1}$$

$$BW\ (t\ ) = BW_{\ max} \times e^{(\mathrm{c} \times t)}. \tag{2}$$

(BWmin; BWmax) are the minimum and maximum values of BW, t is the current number of iterations. The following equation present the dynamic PAR:

$$\mathrm{PAR(t)} = \mathrm{PAR_{min}} + \frac{(\mathrm{PAR_{max}} - \mathrm{PAR_{min}})}{NI} \times \mathrm{t}. \tag{3}$$

(*PARmin; PARmax*) are minimum and maximum values of PAR, *t* is the current number of iterations, *NI* is the total number of iterations.

2: **Second method:** The algorithm can use feedback from the search process to improve the search parameter values, such as updating step size (by decreasing or increasing it) on the basis of the success rate of the search process.

3: **Third method**: The third method uses the self-adaptive values of the algorithm parameters. The adapted parameters can change in chromosomes and mutation processes on the basis of the previous results; an example of this approach is the self-adaptive global best HS algorithm by Pan et al. who constructed the mutated values of harmony memory consideration rate (HMCR) and PAR through the search process.

In the current article, we present a hybrid algorithm of HS and grey wolf optimizer (GWO). GWO is a newly developed algorithm inspired by the hunting and leadership of grey wolf packs [28]. Inspired by the idea of finding the best values using optimization algorithms, GWO was used in the current paper to modify the HS parameters as a self-adaptive process. Hence, instead of tuning the PAR and BW parameters before the search start, the GWO algorithm modifies the parameter values throughout the search process.

To improve HS exploration and avoid premature convergence, a modified version of the original opposition-based learning (OBL) [29] is implemented in the hybrid algorithm. This paper mainly aims to design, implement, and evaluate a new hybrid algorithm of HS and GWO with self-adaptive parameter selection. This paper also aims to improve HS algorithm exploration using a modified version of the OBL technique.

To evaluate the effectiveness of the suggested hybrid algorithm, the hybrid algorithm has been tested using 24 classical benchmark functions with 30 state-of-the-art benchmark functions from CEC and compared them with previous HS variants as well as with well-known metaheuristic algorithms. Parametric tests, namely, Wilcoxon's rank test and Friedman test, were used. The tests were used to provide an insight into the new hybrid algorithm in contrast to the previous variants and hybrid algorithm at α = 5% significance level. The new hybrid algorithm shows highly competitive results in all experiments. To find the best values of harmony memory size (HMS) and HMCR for the hybrid algorithm, some experiments were conducted as presented in the experimental results and analysis section.

The remaining sections of this paper are organized as follows. The original HS and its variants. Then GWO algorithm and modified OBL are investigated. The proposed algorithm is described after that. Then, a section will provide the results and discussion. Finally, a conclusion is provided, and possible future improvements are provided.

II. HS and its Variants

In this part, we will comprehensively describe HS, and different variants were created to overcome the HS variable selection and improve its performance. Some researchers utilized fuzzy logic to automatically update the HS parameters [40]. Mahdavi et al. [21], created a modified variant of HS by adding new functions to modify the HMCR and PAR values throughout the search process. Other researchers, such as Omran et al. [22], modified the search process, which he borrowed from Particle Swarm Optimization [41].

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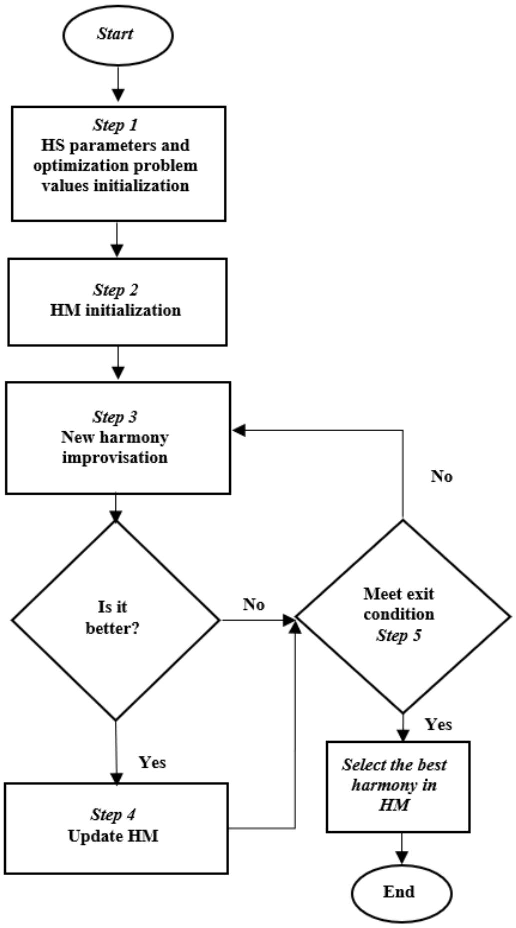

FIGURE 1. HS Process

A. HS ALGORITHM

The HS algorithm process contains five main steps, as shown in Figure 1:

Step 1: Creating initial values of HS parameters: BW, PAR, HMCR, number of iterations (NI), and HMS. The optimization objective function will be determined in this step either by using the maximum or minimum objective function *f(x)*, which are the benchmark functions used in this paper. X_i is the prospect solution vector from N (all possible solution vectors of X_i, and the X_i value is within (lower and upper boundaries) for all the decision variables.

Step 2: In this step, HM will be initialized within the upper and lower boundary ranges, as shown in the next equation, and X_1 is a random value between 0 and 1.

$$X_i = LB + r_1 \times (UB - LB) \quad (4)$$

Step 3: In this step, the improvisation of new harmony will be performed using a combination of three major parameters, namely, HMCR, PAR, and BW, according to line 9 in Algorithm 1. First, random number X_2 generated between 0 and 1; if X_2 is larger than HMCR, then a new value X_j will be created using Equation 1; otherwise, a random value of X_i will be chosen from HM. Afterward, another random value r_3 will be generated between 0 and 1; if it is smaller than or equal to PAR, then X_i will be modified using Equation 2, as follows:

$$X_i{}' = X_i{}' \pm BW \times rnd \quad (5)$$

Step 4: If the newly generated vector $X_i{}'$ is better than the worst vector in the harmony memory, then the worst vector will be replaced with the new vector $X_i{}'$ because of the objective function.

Step 5: The stopping criteria, such as the maximum number of improvisations, should be checked after every improvisation. A detailed description of the HS algorithm is presented in the following pseudocode:

Algorithm1: *Harmony Search algorithm improvisation*

```
1.  while (t < Max number of iterations)
2.  for (j = 1 to D) = {D: number of dimensions}
3.   If (R2) ≤ HMCR {Memory consideration}
4.     x_i' = x_{i,j}  {i is a random integer (1, ... HMS)}
5.     if (R3 ≤ PAR){Pitch adjustment}
6.       x_j' = x_j' ± R4 × bw
7.     end if
8.     else
9.       x_j' = LB + R5 × (UB − LB))
10.   end if
11.  end for
12.  Update HM:
13.   if (x_j' better than worst x_j {x_j ∈ HM})
14.   x_j = x_j'
15.   t = t + 1
16.   End while
17.   return best harmony
```

B. EXPLORATORY POWER OF THE HARMONY SEARCH ALGORITHM: ANALYSIS AND IMPROVEMENTS FOR GLOBAL NUMERICAL OPTIMIZATION (EHS; 2011)

To improve HS performance, Das et al. [42] conducted a theoretical study of the HS algorithm; another variant of the HS algorithm was introduced. The new variant is compared with other variants of HS and other state-of-the-art optimization algorithms. The new variant shows competitive results. The new variant has the same steps as the original HS except for the BW value, which is updated based on the following equations:

$$BW = k\sqrt{Var(x)} \quad (6)$$

$$Var(x) = \frac{1}{\mathrm{m}}\sum_{k=1}^{m}(x_i - \bar{x})^2 = x_i^2 - \bar{x}^{\,2} \quad (7)$$

For the benchmark function, the author suggests using (k = 1 .17); meanwhile, m = HMS, and X is the population average.

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TABLE 1
BENCHMARK FUNCTIONS (GOV: GLOBAL OPTIMUM VALUE).

Function	Function Formula	Type	Range	GOV
F1: Sphere	$\sum_{i=1}^{n} x_i^2$	UM	-100, 100	0
F2: Schwefel's 2.22	$\sum_{i=1}^{D} \lvert X_i\rvert + \Pi_{i=1}^{D} = \lvert X_i\rvert$	UM	-10, 10	0
F3: Step	$\sum_{i=1}^{D} (\lvert X_i + 0.5\rvert)^2$	UM	-100, 100	0
F4: Rosenbrock	$\sum_{i=1}^{D} 100 \times (X_i - X_{i-1}^2)^2 + (x_{i-1} - 1)^2$	UM	-30, 30	0
F5: Schwefel's 2.26	$-\sum_{i=1}^{n} \left[x_i \sin\left(\sqrt{\lvert x_i\rvert}\right)\right]$	UM	-500, 500	-12569.5
F6: Rastrigin	$\sum_{i=1}^{D} (X_i^2 - 10\cos(2\pi x_i) + 10)$	M	-5.12, 5.12	0
F7: Ackleys	$-20\exp\left(-0.2\sqrt{\frac{1}{30}\sum_{i=1}^{D} x^2}\right) - \exp\left(\sqrt{\frac{1}{30}\sum_{i=1}^{D} \cos 2x^2}\right) + 20 + e$	M	-32, 32	0
F8: Griewank	$\frac{1}{4000}\sum_{i=1}^{D} x^2 - \Pi_{i=1}^{D} \cos\frac{x_i}{\sqrt{i}} + 1$	M	-600, 600	0
F9: Rotated hyper-ellipsoid	$\sum_{i=1}^{n} (\sum_{j=1}^{j=i} x_j)^2$	UM	-100, 100	0
F10: Schaffer	$0.5 + \frac{\sin^2\left(\sqrt{(x_1^2 + x_2^2)}\right) - 0.5}{\lvert 1 + 0.001(x_1^2 + x_2^2)\rvert^2}$	M	-100, 100	0
F11: Zakharov	$\sum_{i=1}^{n} x_i^2 + \left(\sum_{i=1}^{n} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{n} 0.5ix_i\right)^4$	M	-5, 10	0
F12: Alpine	$\sum_{i=1}^{n} \lvert x_i . \sin(x_i) + 0.1x_i\rvert$	M	-10, 10	0
F13: Inverted Cosine Wave	$-\sum_{i=1}^{n-1} e^{\left(\frac{-(x_i^2 + x_{i+1}^2 + 0.5x_ix_{i+1})}{8}\right)} \cos 4 \times \sqrt{x_i^2 + x_{i+1}^2 + 0.5x_ix_{i+1}}$	M	-1, 1	0
F14: Dixon price	$(x_1 - 1)^2 + \sum_{i=1}^{n} i(2x_i^2 - x_i - 1)^2$	UM	-10, 10	0
F15: Axis parallel hyper-ellipsoid 2.2	$\sum_{i=1}^{D} i \times X_i^2$	UM	-5.12, 5.12	0
F16: Sum of a different power 2.8	$\sum_{\{i=1\}}^{\{D\}} X_i^{\{1+i\}}$	UM	-1, 1	0

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F17: Levy	$sin^2(\pi\omega_1) + \sum_{i=1}^{D-1}(\omega_i - 1)^2[1 + 10\backslash sin^{2(\pi\omega_i + 1)}] + (\omega_D - 1)^2[1 + \backslash sin^{2(2\pi\omega_D)}]$	M	-10, 10	0
F18: Salomon's 2.8	$1 - cos(2\pi \lvert x \rvert) + 0.1\lvert x \rvert, \quad \lvert x \rvert = \sqrt{\sum_{i=1}^{n} x_i^2}$	M	-100, 100	0
F19: Pathologic	$\sum_{i=1}^{n-1}[0.5 +]\frac{sin^2\left(\sqrt{\{100x_i^2 + x_{\{i+1\}}^2\}}\right) - 0.5}{1 + 0.001\left(x_i^2 - 2x_{ix_{\{i+1\}}} + x_{\{i+1\}}^2\right)^2}$	M	-100, 100	0
F20: Whitley's	$\sum_{i=1}^{n}\sum_{i=1}^{n}\left(\frac{\left(100(x_i^2 - x_j)^2 + (1 - x_j)^2\right)^2}{4000} - cos\left(100(x_i^2 - x_j)^2 + (1 - x_j)^2\right) + 1\right)$	M	-10, 10	0
F21: Schwefel's problem 2.21	$max_i\{\lvert x_i \rvert, 1 \le i \le n\}$	UM	-100, 100	0
F22: Quartic	$\sum_{i=0}^{n} i\, x_i^4 + random(0,1)$	UM	-1.28, 1.28	0
F23: Penalized 1	$\frac{\pi}{n} \times \{10 \times sin^2(\pi y_1) +\} \sum_{i=1}^{n-1}(y_1 - 1)^2[1 + 10\, sin^2(\pi y_1 + 1)] + (y_n - 1)^2 + \sum_{i=1}^{n} u(x_i, \text{a, k, m})$	UM	-50, 50	0
F24: Penalized 2	$\frac{\pi}{n} \times \{10 \times sin^2(\pi y_1) +\} \sum_{i=1}^{n-1}(y_1 - 1)^2[1 + 10\, sin^2(\pi y_1 + 1)] + (y_n - 1)^2 + \sum_{i=1}^{n} u(x_i, \text{a, k, m})$	UM	-50, 50	0

C. AN IMPROVED GLOBAL-BEST HARMONY SEARCH ALGORITHM (IGHS; 2013)

El-Abd [24] developed as an improved variant of GHS [22] by focusing on the explorative range at the beginning, and then on the exploitative range at the end of a search. To accomplish this, the author used Gaussian distribution to select the random pitch adjustment, as described in the next Equation:

$$X_j' = HM_d^r + Gauss(0,1) \times BW \tag{8}$$

Where HM_d^r is a randomly selected value from HM, and Gauss is a random number with a mean of 0 and a standard deviation of 1. For pitch adjustment, the next equation is used as follows:

$$X_j' = HM_d^{best} + \emptyset \times BW \tag{9}$$

Where HM_d^{best} is the best value in HM based on the objective function evaluation f(x). The value φ is a random number that is uniformly distributed within the range "-1 to 1". PAR value is decreased within the iterations to achieve great exploitation, as described by [43]. For BW, the author borrowed its formula from the IHS [21] variant. The algorithm was compared with seven previous HS-variants using the CEC 2005 benchmark function.

D. DIFFERENTIAL-BASED HARMONY SEARCH ALGORITHM FOR THE OPTIMIZATION OF CONTINUOUS PROBLEMS (DH/BEST; 2016)

Hosein et al.[25] introduced a new HS-variant by modifying two aspects of the original HS. The first modification is applied to the initialization of HS by using a new method to initiate feasible solutions with less randomness. The second modification involves replacing pitch adjustment with the applied to the initialization of HS by using a new method to initiate feasible solutions with less randomness. The second modification involves replacing pitch adjustment with the updated version inspired by the differential evolution (DE) mutation strategy and excluding the BW parameter. The following algorithm describes the new initialization processes, which is implemented by replacing the random value with a new calculation based on HMS:

Algorithm4: DH/best Initialization (Hosein 2016)

$for(j = 1\ to D)\ \{D = dimensions\}$
$for(i = 1\ to\ HMS)$
$temp_i = LB + ((i - \frac{0.5}{HMS})) \times (UB - LB)$
$\quad end\ for$
$Shuffle\ the\ temporary\ array$
$for(i = 1\ to\ HMS)$
$\quad HM = temp_i$
$end\ for$
$end\ for$

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Where UB and LB are the upper and lower bounds of the decision variables. The new variant eliminates the requirement of setting BW, and pitches are adjusted based on the distances between the pitches in HM by using DE/best/1 mutation, as described in the following Pseudo-code:

Algorithm5: DH/best Improvisation (Hosein 2016)

$for\ (i = 1\ to\ D)$
  $if\ (r(0{\sim}1)\ \leq HMCR)$
    $X_i' = X_{ij}\ (i\ is\ random\ integer\ from\ 1..HMS)$
    $if(\ r(0{\sim}1) \leq PAR)$
      $X_i' = X_{best} + r(0{\sim}1) \times (X_{r1,J} - X_{r2,J})$
      $if(\ X_j' < LB\ or\ X_j' > UB)$
        $X_j' = r(0{\sim}1) \times (UB - LB) + LB$
      $end\ if$
    $end\ if$
  $else$
    $X_j' = r(0{\sim}1) \times (UB - LB) + LB$
  $end\ if$
$end\ for$

where UB and LB are the upper and lower bounds of the decision variables, $r(0-1)$ is the random value between 0 and 1, X_{best} is the best X_i in HM based on the objective function, and $X_{r1,J}$ and $X_{r2,J}$ are two random values in the jth dimension.

E. A HYBRID HARMONY SEARCH AND SIMULATED ANNEALING (HS-SA; 2018)

New hybrid HS algorithm and SA algorithm were presented by Assad et al. [26], the temperature parameter in SA has been introduced inside the HS algorithm. The new hybrid algorithm adopts a similar process to the original HS, except that it has been updated to accept the poor results of the improvisation process via the probability of the temperature parameter. The temperature starts with a high value to provide high exploration, and it then decreases at each iteration to focus on exploitation through the search process. The new hybrid algorithm provided better results in comparison with the original HS and SA.

III. GWO ALGORITHM

GWO algorithm is a new metaheuristic algorithm developed by Mirjalili et al. [28], GWO has been presented as a swarm-based algorithm that simulates the natural driving life of grey wolves[30, 31]. The GWO algorithm shows high performance in many optimization problems [32-35].

The GWO algorithm divides the population into four groups, namely alpha α, beta β, Delta δ, and Omega ω.

Firstly, random populations of wolves are created. The wolves change their location through the optimization phase on the basis of the fittest wolves, which is α. Consequently, the second and third best solutions are named β, and δ, ω will be guided through the search by those wolves. In order to attack the prey, wolves will encircle the prey as described in the following equations:

$$\vec{D} = |\ \vec{C}\,.\,\vec{X}_p(t) - \vec{X}(t)\ | \tag{10}$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A}\,.\,\vec{D} \tag{11}$$

$\vec{X}_p$ marks the location vector of the prey, and $\vec{X}$ marks the location vector of the grey wolf. $\vec{C}$ and $\vec{A}$ represent the coefficient vectors, whereas t indicates the current iteration value. $\vec{C}$ and $\vec{A}$ values are calculated using the following equations:

$$\vec{A} = 2\vec{A}\,.\,\vec{r}_1 - \vec{a} \tag{8}$$

$$\vec{C} = 2.\,\vec{r}_2 \tag{9}$$

where $\vec{r}_1$ and $\vec{r}_2$ are random vectors in (0,1), and $\vec{a}$ decreased from 2 to 0 through iterations.

The α, β, and δ values will be the best solution acquired thus far. Then, all the other values (wolves) are considered as ω and will be relocated with respect to α, β, and δ. The updated value of the wolves is based on the following equations:

$$\vec{D}_\alpha = |\ \vec{C}_1\,.\,\vec{X}_\alpha - \vec{X}\ | \tag{12}$$

$$\vec{D}_\beta = |\ \vec{C}_2\,.\,\vec{X}_\beta - \vec{X}\ | \tag{13}$$

$$\vec{D}_\delta = |\ \vec{C}_3\,.\,\vec{X}_\delta - \vec{X}\ | \tag{14}$$

Where $\vec{X}$ is the location of the current solution; $\vec{X}_\alpha$, $\vec{X}_\beta$, and $\vec{X}_\delta$ are the α, β, δ locations, respectively; $\vec{C}_1$, $\vec{C}_2$, and $\vec{C}_3$ are random vectors between (0 to 2); and $\vec{X}_\alpha$, $\vec{X}_\beta$, and $\vec{X}_\delta$, represent the distance between the current solution and α, β, and δ, respectively. Afterward, the final location of the current solution is calculated using the following equations:

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1\,.\,(\vec{D}_\alpha) \tag{15}$$

$$\vec{X}_2 = \vec{X}_\beta - \vec{A}_2\,.\,(\vec{D}_\beta) \tag{16}$$

$$\vec{X}_3 = \vec{X}_\delta - \vec{A}_3\,.\,(\vec{D}_\delta) \tag{17}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{18}$$

Where $\vec{A}_1$, $\vec{A}_2$, $\vec{A}_3$ are random vectors between {-2a, 2a}, where a decreased from 2 to 0, within the course of iteration (t).

The final location will be calculated using Equations (10 to 12). Finally, $\vec{A}$ and $\vec{C}$ assist the exploration and exploitation as random and adaptive vectors, respectively. The entire process is described in algorithm 2.

IV. Modified opposition-based learning technique

The original OBL introduced by Tizhoosh [29], and many variants of OBL developed after that and used by different research areas [36]. Many HS variants and hybridizations utilized the OBL and its variants in the literature [37-39].

In this article we applied a modified version of the original OBL within the HS updating process, to improve the HS exploration, as described in Algorithm 3.

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Algorithm2: *Grey wolf algorithm*

1. Initialize grey wolf population within the boundaries $x_i(i = 1,2, \ldots., n)$
2. Initialize A, a and C
3. Calculate the fitness of each search agent
4. $x_\alpha = best\ search\ agent$
5. $x_\beta = second - best\ search\ agent$
6. $x_\delta = third - best\ search\ agent$
7. $while\ (t < Max\ number\ of\ iterations)$do
8. $for\ (each\ search\ agent)$
9. $update\ the\ current\ search\ agent\ possition\ by\ eq$ 18
10. End for
11. Update A, a, and C
12. Calculate the fitness of all search agents
13. Update x_α, x_β and x_δ
14. $t = t + 1$
15. $return\ X_a$

In algorithm 3, $x\{d\}$ represents the new improvisation vector, r is a random value between 0, and 1, d is the number of dimensions, and x_i is the modified opposition value. Once the improvisation process of HS creates a new value x_j, the modified opposition will be applied on the new improvisation value x_j in the update section and will replace it if it is better on the basis of the objective function f.

Algorithm3: Modified opposition

$x\{d\} = \{x_1, x_2, \ldots\ldots x_d\}$
$r = random\ value\ between\ (0\ , 1)$
$for\ (i = 1\ to\ d\ )do$
$\overline{X_\iota} = -1 \times x\{i\} \times r;$
$if\big(f(\bar{x}) < f(x)\big)$
$\mathrm{x} = \bar{x}$

V. PROPOSED HYBRID ALGORITHM

A hybrid algorithm is an algorithm that merges two or more algorithms to solve a problem. The goal of this algorithm is to create a new algorithm that combines advantages from these algorithms. The main purpose of this paper is to design, implement, and evaluate a new hybrid algorithm of HS and GWO with a self-adaptive parameter selection, where the benchmark functions are the case studies to evaluate the new proposed algorithm.

Given that the PAR and BW have a high effect on the efficiency of HS [22, 44], we utilize the GWO algorithm to find the right values of PAR and BW through the search process. We use a modified version of the original OBL technique [29] to improve improvisation results because HS suffers from bad exploration, especially if one or more of its vectors are near the local optimum. Meanwhile, we use the static values of 5 and 0.99 for HMS and HMCR, respectively. The new algorithm was tested on the benchmark function and proves the superior performance compared with the previous HS variants and other well-known metaheuristics. Figure 6 presents the general process of the hybrid algorithm, which is described as follows:

1. Hybrid algorithm parameter and population initialization:
   a. Hybrid parameters will be initialized, as described in Table 2: HMCR, HMS, the minimum and maximum value of PAR and BW, number of iterations of HS (HS-NI), GWO number of iterations (GWO-NI), and the number of GWO search agents.
   b. The GWO population will be initialized for PAR and BW within their upper and lower boundaries and represented as two dimensions.
   c. The HS population vectors (for the benchmark functions in this paper) will be initialized using HS initialization process. These vectors will be used as HM through the whole process of the hybrid algorithm.
2. Improvisation process:
   a. In the HS-improvisation process, the HM vectors will be optimized using the objective function (benchmark functions in this paper).
   b. A modified OBL was used to improve the obtained result, from HS improvisation process, within the updating phase of HS, which is described in Algorithm 3. The final result is sent as a fitness function value of GWO optimization process.
   c. The GWO improvisation process, as described in Algorithm 2, will be used to improvise the PAR and BW values. The fitness function (as included in line 3 in Algorithm 2) value will be the result of HS improvisation process in every GWO improvisation.
3. Results: The best results of the hybrid algorithm will be presented in this phase.

Algorithm 6: Hybrid algorithm GWO-HS

Define the objective function f(x)
Initialize HS and GWO Parameters (HMS, HMCR, GWO-Number-of-Agents, HS-NI, GWO-NI)
Initialize GWO population (PARi; BWi)
Initialize HS population (Xi)
$while(it < GWO\ max\ iteration)do$
$\quad while(i < search\ agents)do$
$\quad\quad while(d < 2)do\ (for\ PAR\ and\ BW)$
$\quad\quad\quad fitnes = HS()$*(HS-improvisation)*
$\quad\quad\quad Improvise\ new\ PAR\ and\ BW(using\ GWO)$
$Update\ Alpha, Beta, and\ Delta$
$Improvise\ new\ PAR\ and\ BW\ (using\ GWO\ improv$
$process)$
$Return\ best\ harmony$

The values of *PARi*, *BWi* in Algorithm 6 are random values of *PAR* and *BW* within their lower and upper bounds. Possible solutions for x_i for HS initialization are the random values between the objective function boundaries.

To conclude the whole process, the GOW-initialization will be used to create *PAR* and *BW* possible values (as search agents). HS initialization will be used to initialize the benchmark functions possible solution vectors (as HM). In every iteration of GWO, the GWO-fitness function will be the result of HS optimization using the *PAR* and *BW* values from GWO-memory. HS improvisation will improvise HM values to find

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possible solutions to the benchmark functions. Finally, we included a modified version of OBL technique as part of our hybrid algorithm through HS updating. The modified OBL will improve the exploration of HS and help the algorithm avoid falling in local optima. Figure 6 presents the general structure of the hybrid algorithm process. The pseudo code of Algorithm 6 describes the hybrid algorithm.

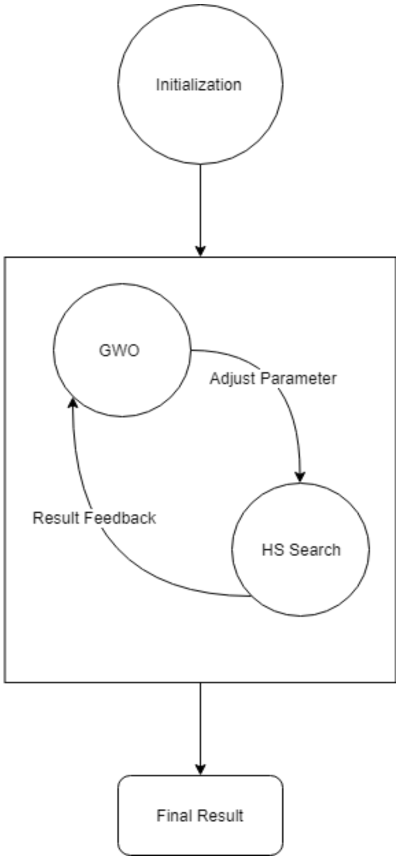

FIGURE 6. The general process of GWO/HS hybrid algorithm.

VI. EXPERIMENT RESULTS AND ANALYSIS

In the first section, we investigate HMCR and HMS parameter best values for the hybrid algorithm using the first 15 classical benchmark functions from Table 1. In the second and third sections, we apply the hybrid algorithm to minimize a set of 24 classical benchmark functions, as described in Table 1 and 30 state-of-the-art test cases from CEC2014 [45]. The classical test functions contain unimodal and multimodal functions to provide insight into the hybrid algorithm capabilities to cover different types of problems. The CEC2014 is also a well-known experimental test for single objective optimization problems that contain shifted, rotated, hybrid, and composition optimization test cases. Friedman test and Wilcoxon nonparametric test at $\alpha = 5\%$ significance level were conducted to evaluate the overall performance of the new hybrid algorithm. All experiments are performed on Microsoft Windows 10 Education in a computer with Intel Core i7 Quad CPU 4702MQ processor 2.2 GHz with 240 GB SSD hard drive and 16GB DDR3 RAM. All algorithms are coded in Java. The best results obtained from the experiments are highlighted in bold.

A. EFFECTS OF HMS AND HMCR ON THE HYBRID ALGORITHM

To determine the best values of the static parameters of the hybrid algorithm, we investigate the different values of the static parameters, namely, HMS and HMCR. Other parameters of the hybrid algorithm for these experiments are the same as those shown in Table 2. We used the first 15 benchmark functions as described in Table 1 to determine the best values of HMS and HMCR as static values in this article. The total number of improvisations is set to 10^4 for all experiments in this article, except for CEC2014 experiments in which we used10^6. The mean and SD are calculated for 30 runs of each function with 30 dimensions. Table 4 presents the results of using different HMS values (i.e., 5, 30, 50, and 100). Meanwhile, *f* presents function.

TABLE 2 PARAMETERS SETTING GWO-HS

Algorithm	Parameters	Value
Harmony search	HMS	5
	HMCR	0.99
	PAR minimum value	0.1
	PAR maximum value	0.4
	BW minimum value	0.1
	BW maximum value	0.4
	HS iteration	100
Grey wolf optimizer	Number of search agents	10
	iteration	100
	number of dimensions	2

TABLE 3
PARAMETERS SETTING FOR COMPARED ALGORITHMS

Algorithm	Parameters	Value
ACS2013	N	5
	GLOBAL MINIMUM	1.0E+20
	PP	0.1
MULTIVERSE2016	N	5
	BEST UNIVERSE INFLATION RATE	1.0E+20
ABC2005	N	5
	LIMIT2	800
DE1997	N	5
	F	0.9
	CR	0.5

TABLE 4
PARAMETERS SETTING FOR HS VARIANTS

Algorithm	HMS	HMCR	PAR	BW	Other
EHS2011	5	0.99	PAR = 0.33	$BW = k.\sqrt{Var}(x)$	
IGHS2013	5	0.9	$PAR_{min} = 0.01$ $PAR_{max} = 0.99$	$BW_{min} = 0.0001$ $BW_{max}=0.06$	
DHBest2016	5	0.99	0.9	-	CR=0.5
HS-SA2018	5	0.9	0.3	0.001	α =0.99

TABLE 5
EFFECTS OF HMS ON THE GWO-HS PERFORMANCE (HMCR = 0.99).

F	**Index**	**HMS**			
		5	**30**	**50**	**100**
F1	**Mean**	**0.0**	0.0	4.7E-147	2.1E-157
	SD	**0.0**	0.0	4.7E-147	2.1E-157
F2	**Mean**	**0.0**	0.0	6.3E-161	2.0E-74
	SD	**0.0**	0.0	6.3E-161	2.0E-74
F3	**Mean**	**0.0**	0.0	0.0	0.0
	SD	**0.0**	0.0	0.0	0.0
F4	**Mean**	**27.6**	27.729	27.738	**27.73**
	SD	**27.6**	27.729	27.738	**27.73**
F5	**Mean**	**-12528**	-12500	-12494	-12454
	SD	**12528**	12500	-2494	12454
F6	**Mean**	**0.0**	0.0	0.0	0.0
	SD	**0.0**	0.0	0.0	0.0
F7	**Mean**	**4.4E-16**	4.4e-16	4.4e-16	4.4e-16
	SD	**4.4E-16**	4.4e-16	4.4e-16	4.4e-16
F8	**Mean**	**0.0**	0.0	0.0	0.0
	SD	**0.0**	0.0	0.0	0.0
F9	**Mean**	**0.0**	0.0	0.0	0.0
	SD	**0.0**	0.0	0.0	0.0
F10	**Mean**	0.06	0.049	**0.009**	0.06
	SD	0.06	0.049	**0.009**	0.06
F11	**Mean**	**3.8e-14**	4.0e-8	4.0e-2	1.59
	SD	**3.8e-14**	4.0e-8	4.0e-2	1.59
F12	**Mean**	**1.5e-53**	3.2e-107	**4.5e-145**	0.45
	SD	**1.5e-53**	3.2e-107	**4.5 e-145**	0.45
F13	**Mean**	**-26.836**	-26.79	26.783	-26.87
	SD	**26.836**	26.79	26.783	-26.87
F14	**Mean**	**0.666**	0.667	0.666	0.67
	SD	**0.666**	0.667	0.666	0.67
F15	**Mean**	**0.0**	0.0	1.3E-241	1.3E-148
	SD	**0.0**	0.0	1.3E-241	1.3E-148

TABLE 6
EFFECTS OF HMCR ON THE GWO-HS PERFORMANCE (HMS = 5).

F	**Index**	**HMCR**			
		0.7	**0.8**	**0.9**	**0.99**
F1	**Mean**	7.0E-24	1.4E-37	3.9E-76	**0.0**
	SD	7.0E-24	1.4E-37	3.9E-76	**0.0**
F2	**Mean**	1.1E-1	9.6E-15	4.3E-70	**0.0**
	SD	1.1E-1	9.6E-15	4.3E-70	**0.0**
F3	**Mean**	0.0	0.0	0.0	**0.0**
	SD	0.0	0.0	0.0	**0.0**
F4	**Mean**	28.05	27.91	27.5	**27.6**
	SD	28.05	27.91	27.5	**27.6**
F5	**Mean**	-10081	-12091	-12552	**-12528**
	SD	-10081	-12091	-12552	**12528**
F6	**Mean**	0.0	0.0	0.0	**0.0**
	SD	0.0	0.0	0.0	**0.0**
F7	**Mean**	3.6E-12	8.3E-13	4.4E-16	**4.4E-16**
	SD	3.6E-12	8.3E-13	4.4E-16	**4.4E-16**
F8	**Mean**	0.0	0.0	0.0	**0.0**
	SD	0.0	0.0	0.0	**0.0**
F9	**Mean**	6.6E-20	4.0E-34	7.1E-15	**0.0**
	SD	6.6E-20	4.0E-34	7.1E-15	**0.0**
F10	**Mean**	8.4	0.79	0.029	**0.009**
	SD	8.4	0.79	0.029	**0.009**
F11	**Mean**	2.59	2.60	7.7E-6	**3.8e-14**
	SD	2.59	2.60	7.7E-6	**3.8e-14**
F12	**Mean**	0.49	0.45	**0.062**	**1.5e-53**
	SD	0.49	0.45	**0.062**	**1.5e-53**
F13	**Mean**	-26.44	-26.73	**-26.87**	**-26.836**
	SD	-26.44	-26.73	-26.87	**26.836**
F14	**Mean**	3.08	2.6	0.84	**0.666**
	SD	3.08	2.6	0.84	**0.666**
F15	**Mean**	1.1E-23	1.0E-30	0.0	**0.0**
	SD	1.1E-23	1.0E-30	0.0	**0.0**

TABLE 7
MEAN AND SD OF THE ERRORS OF HS VARIANTS FOR (D = 30).

F	Index	Algorithms				
		EHS2011	**IGHS 2013**	**DHBest 2016**	**HS-SA2018**	**GWO-HS**
F1	Mean	2.235 e-60	14.613	0.0	10.242	**0.0**
	SD	2.235 e-60	14.61	0.0	10.242	**0.0**
F2	Mean	3.484 e-35	0.179	0.0	0.851	**0.0**
	SD	3.484 e-35	0.179	0.0	0.851	**0.0**
F3	Mean	0.0	20.0	0.0	11.766	**0.0**
	SD	0.0	20.0	0.0	11.766	**0.0**
F4	Mean	28.712	393.048	28.767	553.709	**27.766**
	SD	28.712	393.048	28.767	553.709	**27.766**
F5	Mean	-10238.560	-12539.117	**-12565.425**	-12542.17	-12540.709
	SD	10238.560	12539.117	**12565.425**	12542.17	12540.709
F6	Mean	0.0	3.152	0.335	1.449	**0.0**
	SD	0.0	3.152	0.335	1.449	**0.0**
F7	Mean	5.417 e-15	1.841	4.440 e-16	1.610	**4.440 e-16**
	SD	5.417 e-15	1.841	4.440 e-16	1.610	**4.440 e-16**
F8	Mean	8.924 e-4	1.050	0.0	1.103	**0.0**
	SD	8.924 e-4	1.050	0.0	1.103	**0.0**
F9	Mean	11.881	70.978	0.0	92.409	**0.0**
	SD	11.881	70.978	0.0	92.409	**0.0**
F10	Mean	0.016	0.441	0.155	0.405	**0.009**
	SD	0.016	0.441	0.155	0.405	**0.009**
F11	Mean	9.206 e-5	975.251	57.17	24.633	**1.002 e-5**
	SD	9.206 e-5	975.251	57.17	24.633	**1.002 e-5**
F12	Mean	5.954 e-4	0.189	0.032	0.068	**1.153 e-62**
	SD	5.954 e-4	0.189	0.032	0.068	**1.153 e-62**
F13	Mean	**-26.530**	-26.875	-26.786	-26.842	-26.753
	SD	**26.530**	26.875	26.786	26.842	26.753
F14	Mean	0.697	4.555	10.520	7.625	**0.666**
	SD	0.697	4.555	10.520	7.625	**0.666**
F15	Mean	0.032	1.45 e-5	0.033	0.10	**0.0**
	SD	0.032	1.45 e-5	0.033	0.10	**0.0**
F16	Mean	3.250 e-10	4.692 e-14	1.785 e-8	1.70E-16	**0.0**
	SD	3.250 e-10	4.692 e-14	1.785 e-8	1.70E-16	**0.0**
F17	Mean	1.587	0.785	2.869	**0.043**	0.305
	SD	1.587	0.785	2.869	**0.043**	0.305
F18	Mean	0.103	3.506	0.0	1.867	**0.0**
	SD	0.103	3.506	0.0	1.867	**0.0**
F19	Mean	1.22	2.623	0.0	1.674	**0.0**
	SD	1.22	2.623	0.0	1.674	**0.0**
F20	Mean	372.07	947.823	411.16	394.573	**362.217**
	SD	372.07	947.823	411.16	394.573	**362.217**
F21	Mean	-2835.156	-2985.634	**-2129.06**	-3076.838	-2928.403
	SD	2835.156	2985.634	**2129.06**	3076.838	2928.403
F22	Mean	4.574	8.894	6.747	8.116	**2.80**
	SD	4.574	8.894 2	6.747	8.116	**2.80**
F23	Mean	0.330	2.175	1.592	**0.054**	0.398
	SD	0.330	2.175	1.592	**0.054**	0.398
F24	Mean	2.086	7.155	2.938	**0.448**	1.976
	SD	2.086	7.155	2.938	**0.448**	1.976

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TABLE 8
MEAN AND STANDARD DEVIATION (SD) OF THE ERRORS OF HS VARIANTS FOR (D = 50).

F	Index	Algorithms **EHS2011**	**IGHS 2013**	**DHBest 2016**	**HS-SA 2018**	**GWO-HS**
F1	Mean	3.958 e-6	382.791	0.0	524.218	**0.0**
	SD	3.958 e-6	382.791	0.0	524.218	**0.0**
F2	Mean	2.08 e-5	8.310	0.0	10.119	**0.0**
	SD	2.08 e-5	8.310	0.0	10.119	**0.0**
F3	Mean	0.0	280	0.0	535.9	**0.0**
	SD	0.0	280	0.0	535.9	**0.0**
F4	Mean	48.869	18234	48.644	30394	**47.718**
	SD	48.869	18234	48.644	30394	**47.718**
F5	Mean	**-12372.787**	-20216	-20929	-20093	-20750
	SD	**12372.787**	20216	20929	20093	20750
F6	Mean	2.677	41.536	1.519	45.022	**0.0**
	SD	2.677	41.536	1.519	45.022	**0.0**
F7	Mean	1.678 e-4	4.366	4.440 e-16	5.711	**4.440 e-16**
	SD	1.678 e-4	4.366	4.440 e-16	5.711	**4.440 e-16**
F8	Mean	0.054	2.476	0.0	5.788	**0.0**
	SD	0.054	2.476	0.0	5.788	**0.0**
F9	Mean	54.862	4507.177	0.0	8509	**0.0**
	SD	54.862	4507.177	0.0	8509	**0.0**
F10	Mean	0.057	0.471	0.306	0.488	**0.037**
	SD	0.057	0.471	0.306	0.488	**0.037**
F11	Mean	3.165	7410.696	175.885	131.789	**0.036**
	SD	3.165	7410.696	175.885	131.789	**0.036**
F12	Mean	0.103	1.08	0.051	2.329	**2.528 e-68**
	SD	0.103	1.08	0.051	2.329	**2.528 e-68**
F13	Mean	-42.985	-45.301	-45.144	**-45.094**	-45.370
	SD	42.985	45.301	45.144	**45.094**	45.370
F14	Mean	0.724	270.194	25.759	360.223	**0.666**
	SD	0.724	270.194	25.759	360.223	**0.666**
F15	Mean	0.195	17.166	0.169	23.059	**0.0**
	SD	0.195	17.166	0.169	23.059	**0.0**
F16	Mean	2.50 e-9	3.774 e-12	2.799 e-7	6.07 E-13	**7.591 e-19**
	SD	2.50 e-9	3.774 e-12	2.799 e-7	6.07 E-13	**7.591 e-19**
F17	Mean	3.319	7.999	1.813	**1.729**	2.954
	SD	3.319	7.999	1.813	**1.729**	2.954
F18	Mean	0.129	7.635	0.0	5.176	**0.0**
	SD	0.129	7.635	0.0	5.176	**0.0**
F19	Mean	3.835	5.289	0.0	4.437	**0.0**
	SD	3.835	5.289	0.0	4.437	**0.0**
F20	Mean	1051.242	682667.321	1096.557	4199.406	**1032.046**
	SD	1051.242	682667.321	1096.557	4199.406	**1032.046**
F21	Mean	**-4094.937**	4539.074	-4936.221	-4894.840	-4399.1614
	SD	**4094.937**	4539.074	4936.221	4894.840	4399.1614
F22	Mean	11.247	23.853	12.479	21.414	**8.30**
	SD	11.247	23.8532	12.479	21.414	**8.30**
F23	Mean	**0.527**	25.025	0.652	2.710	0.579
	SD	**0.527**	25.025	0.652	2.710	0.579
F24	Mean	4.004	173.633	**3.528**	22.184	3.887
	SD	4.004	173.633	**3.528**	22.184	3.887

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TABLE 9
MEAN AND STANDARD DEVIATION (SD) OF THE ERRORS FOR THE EXISTING OPTIMIZATION ALGORITHMS FOR (D = 30).

F	**Index**	**Algorithms**				
		ACS 2013	**Multiverse 2016**	**ABC2005**	**DE 1997**	**GWO-HS**
F1	Mean	3.356 e-34	0.0039	1.674	173.066	**0.0**
	SD	3.356 e-34	0.0039	1.674	173.066	**0.0**
F2	Mean	5.936 e-20	0.024	0.158	0.623	**0.0**
	SD	5.936 e-20	0.024	0.158	0.623	**0.0**
F3	Mean	0.0	0.666	0.0	775.266	**0.0**
	SD	0.0	0.666	0.0	775.266	**0.0**
F4	Mean	33.133	28.816	2232289.712	161316.322	**27.766**
	SD	33.133	28.816	2232289.712	161316.322	**27.766**
F5	Mean	**-12542.508**	-6745.606	-11701.463	-10417.237	-12540.709
	SD	**12542.508**	6745.606	11701.463	10417.237	12540.709
F6	Mean	0.8622	2.026	7.07	44.736	**0.0**
	SD	0.8622	2.026	7.07	44.736	**0.0**
F7	Mean	0.098	0.051	2.545	6.379	**4.440 e-16**
	SD	0.098	0.051	2.545	6.379	**4.440 e-16**
F8	Mean	0.001	0.009	0.348	3.338	**0.0**
	SD	0.001	0.009	0.348	3.338	**0.0**
F9	Mean	8.252 e-34	0.582	0.0	2137.225	**0.0**
	SD	8.252 e-34	0.582	0.0	2137.225	**0.0**
F10	Mean	0.195	0.009	0.459	0.347	**0.009**
	SD	0.195	0.009	0.459	0.347	**0.009**
F11	Mean	0.871	0.001	335.010	4.624	**1.002 e-5**
	SD	0.871	0.001	335.010	4.624	**1.002 e-5**
F12	Mean	1.221 e-6	0.017	0.014	0.567	**1.153 e-62**
	SD	1.221 e-6	0.017	0.014	0.567	**1.153 e-62**
F13	Mean	-26.864	-26.850	**-26.553**	-26.833	-26.753
	SD	26.864	26.850	**26.553**	26.833	26.753
F14	Mean	0.746	0.741	3366.446	1036.294	**0.666**
	SD	0.746	0.741	3366.446	1036.294	**0.666**
F15	Mean	1.186 e-36	0.001	0.230	6.354	**0.0**
	SD	1.186 e-36	0.001	0.230	6.354	**0.0**
F16	Mean	2.913 e-148	9.550 e-12	1.131 e-16	1.439 e-4	**0.0**
	SD	2.913 e-148	9.550 e-12	1.131 e-16	1.439 e-4	**0.0**
F17	Mean	3.349	3.349	3.349	3.349	**0.305**
	SD	3.349	3.349	3.349	3.349	**0.305**
F18	Mean	0.0	0.0	0.0	0.0	**0.0**
	SD	0.0	0.0	0.0	0.0	**0.0**
F19	Mean	0.0	3.250 e-10	0.0	0.0	**0.0**
	SD	0.0	3.250 e-10	0.0	0.0	**0.0**
F20	Mean	413.952	413.952	413.952	413.952	**362.217**
	SD	413.952	413.952	413.952	413.952	**362.217**
F21	Mean	-3099.99	-2971.234	-3088.953	-3075.226	**-2928.403**
	SD	3099.99	2971.234	3088.953	3075.226	**-2928.403**
F22	Mean	7.013	3.324	13.676	17.238	**2.80**
	SD	7.013	3.324 2	13.676	17.238	**2.80**
F23	Mean	1.668	1.668	1.668	1.668	**0.398**
	SD	1.668	1.668	1.668	1.668	**0.398**
F24	Mean	3.0	3.0	3.0	3.0	**1.976**
	SD	3.0	3.0	3.0	3.0	**1.976**

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TABLE 10
MEAN AND STANDARD DEVIATION (SD) OF THE ERRORS EXISTING OPTIMIZATION ALGORITHMS FOR (D = 50).

F	**Index**	**Algorithms**				
		ACS 2013	**Multiverse 2016**	**ABC2005**	DE 1997	**GWO-HS**
F1	Mean	5.212 e-19	0.027	1689.224	459.970	**0.0**
	SD	5.212 e-19	0.027	1689.224	459.970	**0.0**
F2	Mean	5.496 e-12	0.065	6.481	1.352	**0.0**
	SD	5.496 e-12	0.065	6.481	1.352	**0.0**
F3	Mean	1.4	1.633	0.3	2854.266	**0.0**
	SD	1.4	1.633	0.3	2854.266	**0.0**
F4	Mean	96.360	50.836	2050687.392	2232815.458	**47.718**
	SD	96.360	50.836	2050687.392	2232815.458	**47.718**
F5	Mean	-20842.549	-10662.148	-17807.783	**-16560.174**	-20750.237
	SD	20842.549	10662.148	17807.783	**16560.174**	20750.237
F6	Mean	4.123	5.1942	50.508	95.855	**0.0**
	SD	4.123	5.1942	50.508	95.855	**0.0**
F7	Mean	0.271	0.097	9.991	9.743	**4.440 e-16**
	SD	0.271	0.097	9.991	9.743	**4.440 e-16**
F8	Mean	0.004	0.057	13.013	10.318	**0.0**
	SD	0.004	0.057	13.013	10.318	**0.0**
F9	Mean	3.679 e-18	18.199	26232.197	14722.357	**0.0**
	SD	3.679 e-18	18.199	26232.197	14722.357	**0.0**
F10	Mean	0.384	0.042	0.497	0.477	**0.037**
	SD	0.384	0.042	0.497	0.477	**0.037**
F11	Mean	24.058	0.034	667.084	144.123	**0.036**
	SD	24.058	0.034	667.084	144.123	**0.036**
F12	Mean	1.573 e-4	0.114	0.619	1.804	**2.528 e-68**
	SD	1.573 e-4	0.114	0.619	1.804	**2.528 e-68**
F13	Mean	-45.313	-45.385	-44.409	**-45.283**	-45.370
	SD	45.313	45.385	44.409	**45.283**	45.370
F14	Mean	3.910	1.022	43610.849	23618.905	**0.666**
	SD	3.910	1.022	43610.849	23618.905	**0.666**
F15	Mean	7.768 e-21	0.016	88.173	41.068	**0.0**
	SD	7.768 e-21	0.016	88.173	41.068	**0.0**
F16	Mean	**2.123 e-124**	6.993 e-12	8.463 e-5	2.128 e-4	7.591 e-19
	SD	**2.123 e-124**	6.993 e-12	8.463 e-5	2.128 e-4	7.591 e-19
F17	Mean	5.166	5.166	5.166	5.166	**2.954**
	SD	5.166	5.166	5.166	5.166	**2.954**
F18	Mean	0.0	0.0	0.0	0.0	**0.0**
	SD	0.0	0.0	0.0	0.0	**0.0**
F19	Mean	0.0	0.0	0.0	0.0	**0.0**
	SD	0.0	0.0	0.0	0.0	**0.0**
F20	Mean	1149.869	1149.869	1149.869	1149.869	**1032.046**
	SD	1149.869	1149.869	1149.869	1149.869	**1032.046**
F21	Mean	-5099.996	-4682.634	-4891.577	-5011.839	**-4399.161**
	SD	5099.996	4682.634	4891.5774	5011.839	**4399.161**
F22	Mean	15.965	9.513	34.917	31.775	**8.30**
	SD	15.965	9.513	34.917	31.775	**8.30**
F23	Mean	1.472	1.472	1.472	1.472	**0.579**
	SD	1.472	1.472	1.472	1.472	**0.579**
F24	Mean	5.0	5.0	5.0	5.0	**3.887**
	SD	5.0	5.0	5.0	5.0	**3.887**

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TABLE 11
MEAN AND STANDARD DEVIATION (SD) OF THE ERRORS FOR HS VARIANTS USING THE CEC2014 (D = 30).

F	Index	Algorithms EHS2011	IGHS 2013	DHBest 2016	HS-SA 2018	GWO-HS
F1	Mean	4.78 E7	2.44 E7	2.17 E7	1.64 E7	**413113**
	SD	4.78 E7	2.44 E7	2.17 E7	1.64 E7	**413113**
	Time	35	36	402.25	**32.483**	60.163
F2	Mean	1.016 E9	2799924	1.62 E8	1180792	**18904**
	SD	1.016 E9	2799924	1.62 E8	1180792	**18904**
	Time	20	18	219.85	**17.855**	20.395
F3	Mean	9247	12394	15863.40	13003	**5051**
	SD	9247	12394	15863.40	13003	**5051**
	Time	**20**	21	239.87	20.542	25.632
F4	Mean	644	533	530.31	538	**439**
	SD	644	533	530.31	538	**439**
	Time	21	**20.45**	256.36	20.926	25.033
F5	Mean	520.67	519.99	520.08	520.04	**520.00**
	SD	520.67	519.99	520.08	520.04	**520.00**
	Time	33.023	**31.048**	405.066	32.749	49.116
F6	Mean	619.58	616.68	**614.96**	616	620.245
	SD	619.58	616.68	**614.96**	616	620.245
	Time	**3305**	3499.66	39014	4057.221	6312.374
F7	Mean	708.46	700.95	708.03	700.52	**700.01**
	SD	708.46	700.95	708.03	700.52	**700.01**
	Time	41	**40.83**	485	41.882	52.229
F8	Mean	863.71	800.20	803.37	800.09	**800**
	SD	863.71	800.20	803.37	800.09	**800**
	Time	29	27.35	335	27.826	**27.31**
F9	Mean	1047.49	977.72	989.04	**971.87**	1037
	SD	1047.49	977.72	989.04	**971.87**	1037
	Time	**36**	38.59	460	39.245	37.85
F10	Mean	1995.28	1001.11	1048.71	1001.26	**1001**
	SD	1995.28	1001.11	1048.71	1001.26	**1001**
	Time	58	**56.51**	13273	60.757	63.43
F11	Mean	4479.43	3333.98	3282.25	**3220.57**	3793.02
	SD	4479.43	3333.98	3282.25	**3220.57**	3793.02
	Time	**67**	82.37	726.23	69.857	97.10
F12	Mean	1200.71	**1200.16**	1200.17	1200.22	1200.18
	SD	1200.71	**1200.16**	1200.17	1200.22	1200.18
	Time	**690**	708.27	8221	739.323	1083.54
F13	Mean	1300.64	1300.57	1300.62	1300.59	**1300.55**
	SD	1300.64	1300.57	1300.62	1300.59	**1300.55**
	Time	28	26.45	360.72	26.858	**17.811**
F14	Mean	1686.38	1660.91	1686.04	1685.41	**1685.40**
	SD	1686.38	1660.91	1686.04	1685.41	**1685.40**
	Time	29	26.40	370	26.07	**16.18**
F15	Mean	1541.48	1519.03	2321.51	**1515.24**	1537.52
	SD	1541.48	1519.03	2321.51	**1515.24**	1537.52
	Time	43	40.17	492	41.091	**32.40**
F16	Mean	1611.25	1610.12	**1610.06**	1610.07	1610.51
	SD	1611.25	1610.12	**1610.06**	1610.07	1610.51
	Time	42	39.22	497	41.409	**31.95**
F17	Mean	2699841	3011342	2894964	3716289.87	**58210.62**
	SD	2699841	3011342	2894964	3716289.87	**58210.62**
	Time	52	51.86	639	54.856	**24.30**
F18	Mean	12719	7058.42	445339	5989.24	**3523.70**

	SD	12719	7058.42	445339	5989.24	**3523.70**
	Time	37	34.15	437	37.037	**24.30**
F19	Mean	2539.23	**2296.31**	2534	2538.38	2539
	SD	2539.23	**2296.31**	2534	2538.38	2539
	Time	834	**798.49**	9895	947.406	1046
F20	Mean	**14549.12**	15983.76	28188	17269.85	15049.83
	SD	**14549.12**	15983.76	28188	17269.85	15049.83
	Time	37	36.81	448	30.658	**49.09**
F21	Mean	748419	585489	1097997	774049.91	**61828**
	SD	748419	585489	1097997	774049.91	**61828**
	Time	49	47.32	856	**40.798**	63
F22	Mean	2758	2727.08	2844	**2703.86**	2813
	SD	2758	2727.08	2844	**2703.86**	2813
	Time	115	114.450	1696	**109.921**	139.4
F23	Mean	2617	2616.48	2620.21	2616.43	**2500**
	SD	2617	2616.48	2620.21	2616.43	**2500**
	Time	139	139.79	1976	**134.524**	173.45
F24	Mean	2600	2635.87	2603	2634.43	**2600**
	SD	2600	2635.87	2603	2634.43	**2600**
	Time	**107**	111.17	1503	118.136	127.97
F25	Mean	2707	2710.26	2700.32	2709.29	**2700**
	SD	2707	2710.26	2700.32	2709.29	**2700**
	Time	**140**	142.09	1846	155.969	172.26
F26	Mean	2782	**2740.74**	2800.04	2766.17	2798.04
	SD	2782	**2740.74**	2800.04	2766.17	2798.04
	Time	4118	4186.60	50167	**3616.477**	11220.31
F27	Mean	3443	3431.52	3273.93	3401.17	**2900**
	SD	3443	3431.52	3273.93	3401.17	**2900**
	Time	4382	4398	45428	**2723.65**	5419.98
F28	Mean	4495	3925.84	4050.87	3870.42	**3000**
	SD	4495	3925.84	4050.87	3870.42	**3000**
	Time	299	295.40	2768	**186.109**	344.37
F29	Mean	16215	4177.09	2015087	4362.35	**3100**
	SD	16215	4177.09	2015087	4362.35	**3100**
	Time	926	**825**	9245	646.452	1190.62
F30	Mean	14983	11741.60.0	17050.01	11977.69	**3200**
	SD	14983	11741.60	17050.01	11977.69	**3200**
	Time	208	178.75	1773	**126.545**	369.84

TABLE 12
MEAN AND STANDARD DEVIATION (SD) OF THE ERRORS EXISTING OPTIMIZATION ALGORITHMS USING THE CEC2014 (D = 30).

F	**Index**	**Algorithms**				
		ACS 2013	**MultiVerse 2016**	**ABC2005**	**DE 1997**	**GWO-HS**
F1	Mean	**68849**	2463618	2.34 E7	3889379	413113
	SD	**68849**	2463618	2.34 E7	3889379	413113
	Time	273	174	113	449	**60.163**
F2	Mean	**200**	1908	993	4.76 E8	18904
	SD	**200**	1908	993	4.76 E8	18904
	Time	147	97	32	163	**20.395**
F3	Mean	**300**	374	1600.05	5684	5051
	SD	**300**	374	1600.05	5684	5051
	Time	161	95	38	197	**25.632**
F4	Mean	**400.42**	470	500.03	512	439
	SD	**400.42**	470	500.03	512	439
	Time	172	99	43	198	**25.033**
F5	Mean	520.01	520	520.01	520.81	**520.00**
	SD	520.01	520	520.01	520.81	**520.00**

	Time	228	139	89	389	**49.116**
F6	Mean	610.23	**604.01**	617.59	613	620.245
	SD	610.23	**604.01**	617.59	613	620.245
	Time	37222	16555	17323	84542	**6312.374**
F7	Mean	**700**	700.01	700.11	706.74	700.01
	SD	**700**	700.01	700.11	706.74	700.01
	Time	235	207	110	451	**52.229**
F8	Mean	800.91	868	800	856.48	**800**
	SD	800.91	868	800	856.48	**800**
	Time	162	165	12749	856.48	**27.31**
F9	Mean	**961**	971	1039.01	993	1037
	SD	**961**	971	1039.01	993	1037
	Time	231	192	85	527	37.85
F10	Mean	**1000.49**	2649.50	1000.08	1743	1001
	SD	**1000.49**	2649.50	1000.08	1743	1001
	Time	337	316	158	881	**63.43**
F11	Mean	2899.09	4168.99	3536.66	6115	**3793.02**
	SD	2899.09	4168.99	3536.66	6115	**3793.02**
	Time	423	360	221	1103	**97.10**
F12	Mean	**1200.10**	1200.16	1200.14	1201.50	1200.18
	SD	**1200.10**	1200.16	1200.14	1201.50	1200.18
	Time	4105	3451	3011	11534	**1083.54**
F13	Mean	1300.29	1300.26	**1300.22**	1300.40	1300.55
	SD	1300.29	1300.26	**1300.22**	1300.40	1300.55
	Time	160	167	56	215	**17.811**
F14	Mean	1685.39	1691	**1685.39**	1686.15	1685.40
	SD	1685.39	1691	**1685.39**	1686.15	1685.40
	Time	169	169	67	227	**16.18**
F15	Mean	**1505.86**	1508.67	1516.61	1612.67	1537.52
	SD	**1505.86**	1508.67	1516.61	1612.67	1537.52
	Time	243	216	116	476	**32.40**
F16	Mean	**1609.12**	1610.97	1610.11	1611.78	1610.51
	SD	**1609.12**	1610.97	1610.11	1611.78	1610.51
	Time	247	218	127	497	**31.95**
F17	Mean	15909.63	68155	3051313	440195.74	**58210.62**
	SD	15909.63	68155	3051313	440195.74	**58210.62**
	Time	348	267	203	701	**24.30**
F18	Mean	1970.31	**3693**	7320	3.09 E7	**3523.70**
	SD	1970.31	**3693**	7320	3.09 E7	**3523.70**
	Time	277	205	112	406	**24.30**
F19	Mean	**2538.04**	2547.94	2538.38	2538.97	2539
	SD	**2538.04**	2547.94	2538.38	2538.97	2539
	Time	6847	3787	3944	14593	**1046**
F20	Mean	2799	**2227**	22185.48	5931	15049.83
	SD	2799	**2227**	22185.48	5931	15049.83
	Time	246	221	118	434	**49.09**
F21	Mean	**7125.01**	34062	658713	125970	61828
	SD	**7125.01**	34062	658713	125970	61828
	Time	314	275	160	593	**63**
F22	Mean	2550.16	**2475.32**	2657.33	2486	2813
	SD	2550.16	**2475.32**	2657.33	2486	2813
	Time	721	611	503	1809	**139.4**
F23	Mean	2615.24	2502	2617.83	2617.13	**2500**
	SD	2615.24	2502	2617.83	2617.13	**2500**
	Time	999	715	662	2406	**173.45**
F24	Mean	2627	2600.60	2630	2646.36	**2600**
	SD	2627	2600.60	2630	2646.36	**2600**

	Time	781	703	500	1761	**127.97**
F25	Mean	2707.99	2700.08	2712	2711.38	**2700**
	SD	2707.99	2700.08	2712	2711.38	**2700**
	Time	1059	919	633	2227	**172.26**
F26	Mean	2715	**2700.34**	2721	2725.87	2798.04
	SD	2715	**2700.34**	2721	2725.87	2798.04
	Time	26476	21762	21399	61044	**11220.31**
F27	Mean	3124	2903.90	3168	3297.27	**2900**
	SD	3124	2903.90	3168	3297.27	**2900**
	Time	25270	21615	20556	48621	**5419.98**
F28	Mean	3753	3017.05	4751	3913.60	**3000**
	SD	3753	3017.05	4751	3913.60	**3000**
	Time	1776	1831	1339	2895	**344.37**
F29	Mean	3626.38	19816.49	4931	59318.770	**3100**
	SD	3626.38	19816.49	4931	59318.770	**3100**
	Time	6063	4775	4515	9249	**1190.62**
F30	Mean	5500.63	5142.61	7907	7762.58	**3200**
	SD	5500.63	5142.61	7907	7762.58	**3200**
	Time	1403	1232	916	1876	**369.84**

TABLE 13
WILCOXON SIGNED-RANK TEST RESULTS GWO-HS VS HS VARIANTS 30D.

Algorithms	P-value	R+	R-	n/h/l/s
GWO-HS vs EHS2011	0.0001	221	-52	24/21/1/2
GWO-HS vs IGHS 2013	0.00022	276	0	24/24/0/0
GWO-HS vs DHBest 2016	0.00782	217	-23	24/14/2/8
GWO-HS vs HS-SA 2018	0.00614	254	-22	24/21/3/0

TABLE 14
WILCOXON SIGNED-RANK TEST RESULTS GWO-HS VS HS VARIANTS 50D.

Algorithms	P-value	R+	R-	n/h/l/s
GWO-HS vs EHS2011	0.0001	208	-2	24/20/3/1
GWO-HS vs IGHS 2013	0.0001	251	-21	24/22/2/0
GWO-HS vs DHBest 2016	0. 02642	188	-29	24/13/3/8
GWO-HS vs HS-SA 2018	0.0002	247	-6	24/21/3/0

TABLE 15
WILCOXON SIGNED-RANK TEST RESULTS GWO-HS VS OTHER METAHEURISTICS 30D.

Algorithms	P-value	R+	R-	n/h/l/s
GWO-HS vs ACS 2013	0. 00374	225	0	24/21/0/3
GWO-HS vs MultiVerse 2016	0. 00124	228	0	24/22/0/2
GWO-HS vs ABC2005	0. 00078	213	-8	24/19/1/4
GWO-HS vs DE 1997	0. 00044	228	0	24/22/0/2

TABLE 16
WILCOXON SIGNED-RANK TEST RESULTS GWO-HS VS OTHER METAHEURISTICS 50D.

Algorithms	P-value	R+	R-	n/h/l/s
GWO-HS vs ACS 2013	0. 00214	231	-15	24/20/2/2
GWO-HS vs MultiVerse 2016	0. 00086	240	-4	24/21/1/2
GWO-HS vs ABC2005	0. 00034	249	-8	24/20/2/2
GWO-HS vs DE 1997	0. 00038	256	-4	24/20/2/2

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TABLE 17
WILCOXON SIGNED-RANK TEST RESULTS GWO-HS VS HS VARIANTS CEC2014 30D.

	P-value	R+	R-	n/h/l/s
GWO-HS vs EHS2011	0.0002	414	-50	30/25/4/1
GWO-HS vs IGHS 2013	0.01778	347	-117	30/19/-11/0
GWO-HS vs DHBest 2016	0.00044	403	-62	30/24/6/0
GWO-HS vs HS-SA 2018	0.00782	359	-103	30/22/8/0

TABLE 18
WILCOXON SIGNED-RANK TEST RESULTS GWO-HS VS OTHER METAHEURISTICS CEC 2014 30D.

Algorithms	P-value	R+	R-	n/h/l/s
GWO-HS vs ACS 2013	0. 03662	362	-103	30/22/8/0
GWO-HS vs Multiverse 2016	0. 4965	267	-195	30/17/11/2
GWO-HS vs ABC2005	0. 14706	300	-163	30/16/13/1
GWO-HS vs DE 1997	0. 00128	389	-76	30/23/7/0

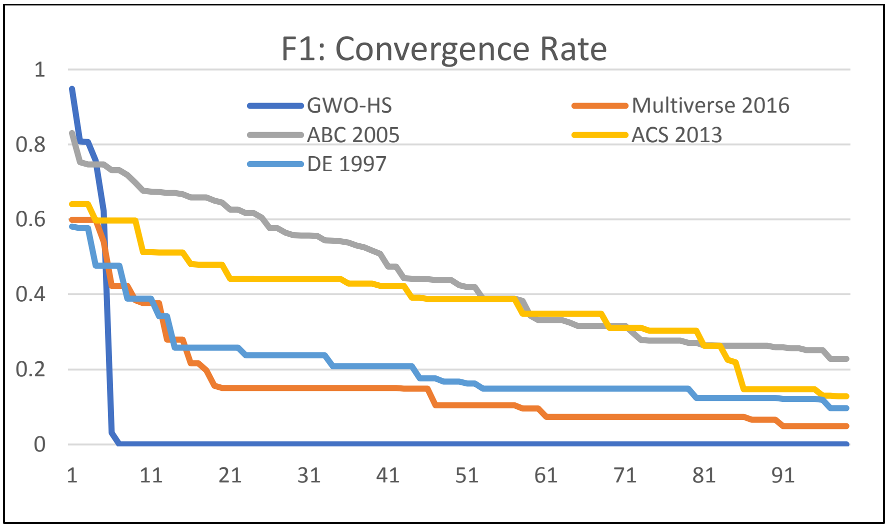

FIGURE 2. CONVERGENCE CURVE FOR $F1$

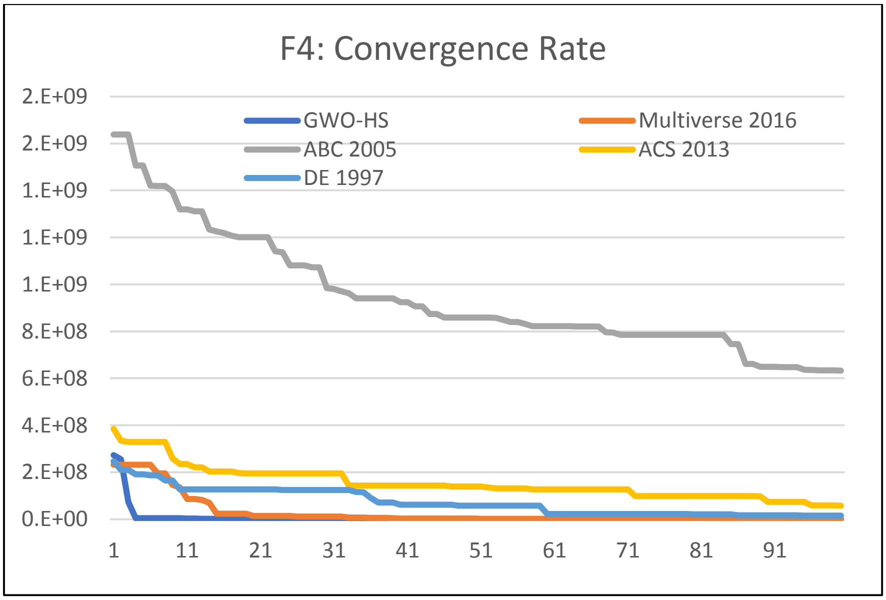

FIGURE 3. CONVERGENCE CURVE FOR F4

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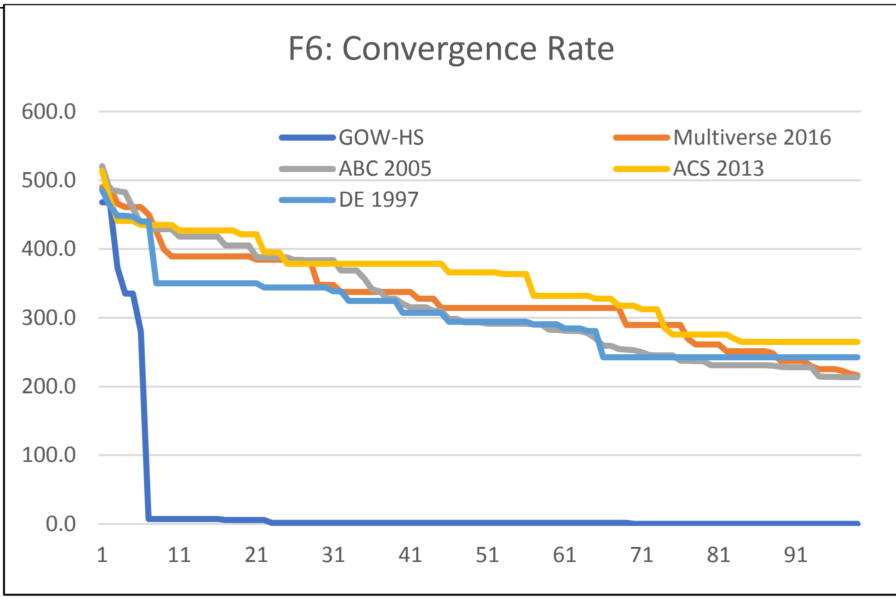

FIGURE 4. CONVERGENCE CURVE FOR *F*6

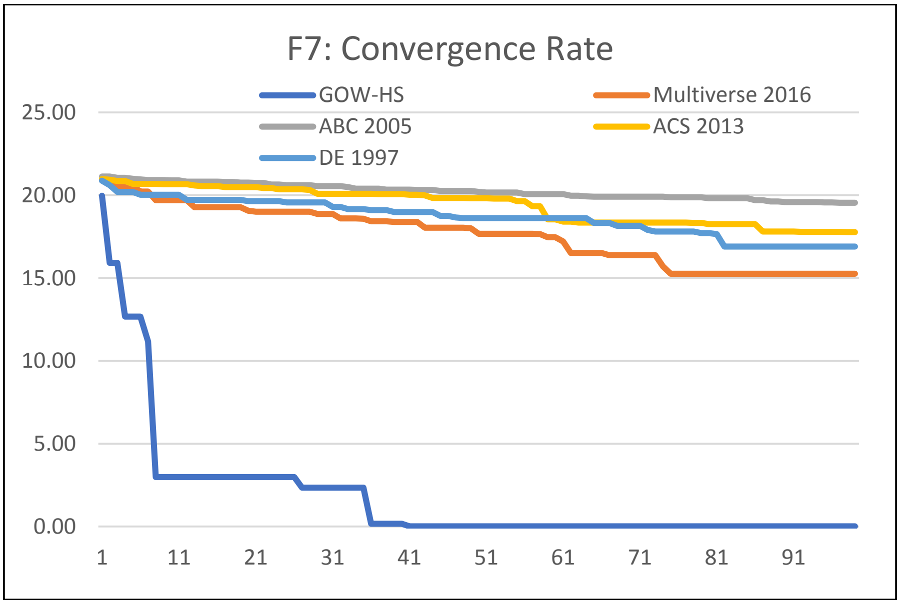

FIGURE 5. CONVERGENCE CURVE FOR F7

TABLE 19
FRIEDMAN TEST RESULTS GWO-HS VS HS VARIANTS.

Algorithms	Classical 30D	Classical 50D	CEC2014 30D
EHS2011	2.8542	2.9583	4.0833
IGHS 2013	4.1250	4.2083	2.6333
DHBest 2016	2.9792	2.1875	3.6000
HS-SA 2018	3.3750	4.1667	2.7000
GWO-HS	1.6667	1.4792	1.9833

TABLE 20
FRIEDMAN TEST RESULTS GWO-HS VS OTHER METAHEURISTICS.

Algorithms	Classical 30D	Classical 50D	CEC2014 30D
ACS 2013	2.4792	2.5417	1.8667
Multiverse 2016	3.1458	2.8750	2.6667
ABC2005	3.6250	2.5417	3.4500
DE1997	4.1875	3.9583	4.2333
GWO-HS	1.5625	1.5000	2.7833

Table 4 shows that the best results for the hybrid algorithm are obtained using HMS = 5 and shows the fastest results obtained in most functions. Table 4 shows that increasing HMS does not improve the performance in most algorithms. Thus, a small HMS improves the update rate in HM for most cases. Table 5 presents the results of running the hybrid algorithm with different HMCR values (i.e., 0.7, 0.8, 0.9, and 0.99). The obtained results show that the HMCR value has a high influence on the HS performance. A large HMCR value provides improved results. The best results are obtained using HMCR = 0.99 for most benchmark functions with the fastest convergence rate. Through the experiment of HMS values, we use HMCR = 0.99 and HMS = 5 to run the HMCR value experiment.

B. EXPERIMENT 1

In this part, we will analyze the experiment of the new hybrid algorithm compared with four recent HS variants and one hybrid algorithm (i.e., EHS 2011, IGHS 2013, DH/best 2016 and HS-SA 2018). The parameter configurations for these variants are described in Table 4. The parameter values for the hybrid algorithm are the same as those listed in Table 2. First, we examine the hybrid algorithm together with four HS variants using 24 benchmark functions with 30 and 50 dimensions, as described in Table 1.

For both dimensions, as presented in Tables 7 and 8, the hybrid algorithm provides better results than the other HS variants in most cases. Second, we compare the hybrid algorithm with the recent variants of HS using 30 state-of-the-art CEC benchmark functions [45], with 30 dimensions. The results presented in Table 11 show that the new hybrid algorithm outperforms the recent variants in 20 out of the 30 test cases and provides highly competitive results. In terms of speed, the algorithm only outperforms the other variants in seven functions, but it provides high speed in all cases.

Wilcoxon's rank test was applied to the mean results of Tables 7, 8, and 11 presented in Tables 13, 14, and 17 respectively. The p-value shows the significance of the results and performance improvement in comparison with other variants. A low p-value means high improvement. R+ presents the total ranks whenever the hybrid algorithm provides better results than the other variants, whereas R- provides the total ranks of lower results than the other variants. N is the total number of benchmark functions, l, h, and s indicate the total number of functions with higher, lower, or similar results of the hybrid algorithm compared with other variants. As presented in Tables 13, 14, and 17, the new hybrid algorithm outperforms all variants of HS with improved performance. Finally, to establish a comparative assessment, Friedman statistical test has been conducted based on the mean results of Tables 7, 8, and 11. The results presented in Table 19 confirm that the new hybrid algorithm outperforms all previous variants of HS because it provides the highest ranking. These results obtained the lowest value on the Friedman test, which shows a high ranking. The results contain classical 30D as classical benchmark functions with 30 dimensions, and classical 50D as classical benchmark functions with 50 dimensions, and finally the CEC2014 test cases with 30 dimensions.

C. EXPERIMENT 2

To investigate the capability of the hybrid algorithm, we evaluate it with other state-of-the-art metaheuristic algorithms from different families, as follows: artificial cooperative search (ACS 2013) [46], (multi-verse 2016) [47], artificial bee colony (ABC 2005)[48], and differential evolution (DE 1997) [49]. The parameter characteristics of these algorithms are shown in Table 3 as used in this experiment.

In Table 9, we compare the hybrid algorithm with other metaheuristics using classical benchmark functions as described in Table 1. These functions have 30 dimensions. The hybrid algorithm provides the best results in all test functions, except for F5 and F13. The hybrid algorithm provides the second-best results. Table 10 presents the mean, and the SD of the hybrid algorithm with other metaheuristics by using 50 dimensions for the classical benchmark functions. The hybrid algorithm outperforms other metaheuristics in all test functions, except for F5, F13, and F16; the hybrid algorithm provides the second-best result. Finally, we compare the hybrid algorithm with other metaheuristics in Table 12 using 30 state-of-the-art benchmark functions from CEC 2014. The results of mean and standard deviations and running time show that the hybrid algorithm provides the highest speed in all the 30 cases. Moreover, this algorithm outperforms all other metaheuristics in 12 cases, as presented in Table 12. Overall, according to the results shown in Tables 9 and 10, the hybrid algorithm provides a competitive result compared to other metaheuristic algorithms in terms of efficiency.

Similar to the previous section, we conducted Wilcoxon's rank test and Friedman statistical test based on the mean results of Tables 9, 10, and 12. The Wilcoxon's rank test results presented in Tables 15, 16, and 18 were derived from 30, 50 dimensions of classical benchmark functions, and CEC2014 test cases, respectively.

As shown in Tables 15 and 16, the hybrid algorithm provides very small p-values. Therefore, it outperforms all other metaheuristic algorithms and provides very high significant improvement. Table 18 shows the results based on CEC2014 experiment results presented in Table 12. The hybrid algorithm provides high significance results against two algorithms, namely, ACS and DE.

For the Friedman test, Table 20 presents a full overview of the classical benchmark functions with two dimensions, 30 and 50, and the CEC test cases. As seen in the classical experiments, the hybrid algorithm has the lowest value on Friedman test, which means it has the highest ranking among other metaheuristics. For the CEC2014 experiment, the hybrid algorithm has the second raking following ACS algorithm.

To provide insight into the hybrid algorithm convergence rate, we run experiment using four benchmark functions. Two functions with unimodal optimum (F1, F4), and two functions with multimodal optimum (F6, F7). Figures (2 - 5) illustrate the best score obtained so far of the hybrid algorithm and other HS variants versus the iteration.

VII. CONCLUSION

This paper presents a new hybrid algorithm of the HS algorithm with the GWO algorithm called GWO-HS algorithm for the global continuous optimization problem. The new hybrid algorithm solves the parameter selection problem of the HS algorithm by using another algorithm, namely, GWO, to modify the values of the PAR and BW parameters as a self-adaptive process. Another modification is performed to harmonize search by applying the modified opposition technique to the search result and improving the obtained results. The GWO-HS convergence is very high compared to the existing HS variants due to the opposition technique, and GWO-HS can reach the optimum results with less iterations. The new hybrid algorithm can cover different types of problems with the same parameter setting, which makes it a better version of HS than the original one. Two groups of evaluation tests are used to examine the new algorithm performance. First, we compare the hybrid algorithm with the recent variants of the HS algorithm using different types of optimization functions, namely, 24 classical and 30 CEC2014 benchmark functions. The results show that the hybrid algorithm is better than the previous variants in terms of accuracy and provides competitive time consumption. Additionally, the algorithm has been evaluated with well-known metaheuristics from different families. The hybrid algorithm shows improved results and speed compared with these algorithms. The new hybrid algorithm shows high performance, which is essential in solving real-world optimization. Therefore, we recommend using a new algorithm to solve real-world problems. The current experiment focuses on continuous benchmark functions. Future work could utilize the new hybrid algorithm in discrete optimization problems.

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