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ACCURACY OF CONVENTIONAL COMPUTATIONS IN ASSESSING VOLUME OF SMALL DIAMETER LOGS OF *HOPEA ODORATA*

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The accuracy of volumetric computation of small diameter logs based on mathematical formulae was evaluated. Plantation logs of 13-year-old *Hopea odorata* with diameter measurements of 20 to 30 cm were assessed using Huber's, Smalian's, Newton's and truncated cone formulae. The reference volume of each log was determined using the fundamental method of water-displacement. Results of average values showed that all measurements were higher, compared to reference volume. Empirical evidence indicated that the truncated cone formula was the most precise measurement compared to other formulae. In the analyses of the separated sections, strong linear correlations between mathematical computations and reference volume were observed in middle logs. In contrast, evaluation of bottom and top logs showed lower r^2 values. A correction factor was developed and proposed to estimate a more accurate volume of bottom logs. Based on standard geometrical profile of the logs, other plantation species fit for similar computation were suggested.

Keywords: Plantation timber, log geometry, precision engineering, volumetric computation

INTRODUCTION

The volumetric assessment of sawlogs requires precise geometrical measurements as the sizes become smaller. For instance, the logs of matured *Hopea odorata* were recorded of having 100 cm in diameter at breast height (Choo et al. 2001), whereas plantation logs of 13-year-old trees ranged merely 20 to 30 cm. Clearly, inaccurate geometrical measurements of small diameter logs will greatly affect the magnitude of Hoppus volume, per waste volume, compared to large-matured logs. The expected marketable value of the plantation output will be erroneous, especially on a large commercial scale. A sawing study of 20-year-old and 16-year-old *Acacia mangium* logs, with corresponding average diameters of 37.9 cm and 32.5 cm, was conducted (Khairul et al. 2011). The two measurements resulted in a substantial difference of recovery values of 41.3% and 35.3% respectively.

Likewise, log geometry provides important information regarding the input masses of sawmilling operation. The productivity of sawmill is evaluated based on the ratio of output per input variables. Thus, sawmill efficiency is highly dependent on the precision of geometrical measurements of logs. A study of *Hevea brasiliensis* mill indicated that with an average gross recovery of 46%, the production rate was 1.3 m^3 $hour^{-1}$ (Lopez et al. 1980).

A customary method for volumetric assessment of logs is based on Smalian's formula. It is the easiest and most applied technique for determining the bulk quantity of timber material in a log. The mathematical computation requires measurements of cross-sectional area at both ends of a log, based on the formula:

$$V = \frac{L}{2}(A_1 + A_2)$$

where V = volume of log (m^3), L = length of log (m), A_1 = cross-sectional area at bottom end (m^2) and A_2 = cross-sectional area at top end (m^2). However, the method based on paraboloid principle is considered as the least accurate measurement of the volume of logs, especially of flared or tapered shapes. Besides, butt logs normally formed neiloid-type contours, thus the paraboloid calculation is rarely true.

Geometrically, Newton's formula is recognised as the most accurate calculation for volumetric determination of logs regardless of the profile (i.e. cylinder, conoid, paraboloid or neiloid). The method gives allowance for three consecutive cross-sectional measurements along the length as opposed to Smalian's open-end diameters, given by the formula:

$$V = \frac{L}{6}(A_1 + 4_m + A_2)$$

where V = volume of log (m^3), L = length of log (m), A_1 = cross-sectional area at bottom end (m^2), A_m = cross-sectional area at midpoint (m^2) and A_2 = cross-sectional area at top end (m^2). Nevertheless, at commercial scale, this exercise is very expensive and time-consuming. Besides, it is impractical to measure the midpoint diameter when the logs are in piles. Hence, the method is generally restricted to research purposes.

Primary assessments of sawing recovery of plantation logs were based on truncated cone formula (Lopez et al. 1980, Ho & Rokiah 1983). However, some investigators preferred the standard measurement of Smalian's formula (Sim 1989). In recent work, a new set of volumetric formula, based on mean cumulative diameter, was introduced for all plantation species (Tan et al. 2010). It appears that each study has a unique technique of measuring log volume, but the accuracy of one method compared to another has not been investigated. Neglecting the consistency of the volumetric representation of logs can either mean overestimation or reduction of the recovery rate, and thus data comparison will be inappropriate. Despite many methods being used to determine the volume of a log, the results were allegedly accepted an equated without any scientific validation.

A quantitative study was conducted to evaluate the volume of small logs using the customary measurement of Huber's, Smalian's, Newton's and truncated cone formulae, based on plantation outputs of *Hopea odorata*. The specific objectives of the study were:

1) To determine the most accurate volumetric computation of small diameter logs of *Hopea odorata* for better engineering tolerance.
2) To recommend other plantation species that fit for similar computation based on the standard geometrical profile of the logs.

MATERIALS AND METHODS

Preparation of samples

Trees of *Hopea odorata,* locally known as merawan siput jantan, 13-year-old, were randomly selected from a plantation site in Segamat, Johor. Each tree was cut into logs of 2 meters giving most trees with bottom, middle and top sections. A total of 82 logs, bottom (28), middle (32) and top (22) were prepared. The bottom and middle logs were obtained from clear bole section. The top logs were obtained from crown bole section (Figure 1).

Diameter and length measurements

The cross-sectional surfaces were categorised into 5 geometrical profiles, i.e. round, elliptic, triangular, squared and irregular. Two cross-sectional measurements were made through the pith, namely d_1 and d_2 (Figure 2). The measurement of diameter was made using a log caliper with ± 1 mm accuracy, inclusive of the bark. A measuring tape of similar accuracy was used to measure the actual length of each log. The length of the log was measured based on the distance of pith point of each cross-sectional surface.

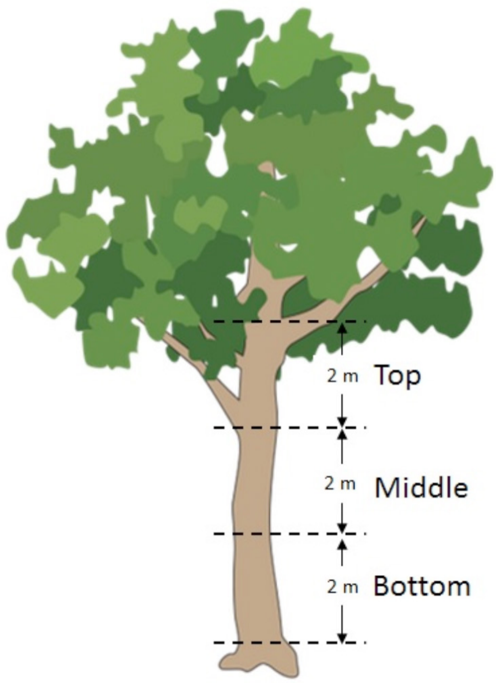


Figure 1 Sampling of the logs

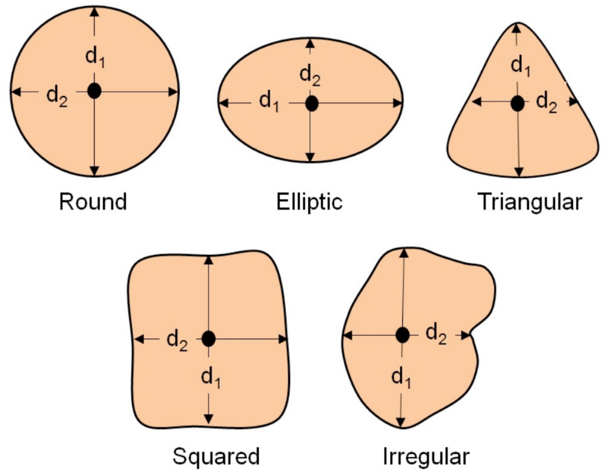


Figure 2 Diameter measurements of various geometrical profiles

Smalian's and truncated cone formulae

The volumetric computation of log using Smalian's, V_S (m^3) and truncated cone, V_{tc} (m^3) measurements were based on the formulae:

$$V_S = \frac{\pi L}{8}\left[\left(d_1 \times d_2\right) + \left(d_3 \times d_4\right)\right]$$

$$V_{tc} = \frac{\pi L}{12}\left[\left[\left(\frac{d_1+d_2}{2}\right)+\left(\frac{d_3+d_4}{2}\right)\right]^2 - \left[\left(\frac{d_1+d_2}{2}\right)\left(\frac{d_3+d_4}{2}\right)\right]\right]$$

where L = length of log (m), d_1 = diameter 1 of end 1 (m), d_2 = diameter 2 of end 1 perpendicular to d_1 (m), d_3 = diameter 1 of end 2 (m) and d_4 = diameter 2 of end 2 perpendicular to d_3 (m) (Figure 3).

Huber's and Newton's formulae

The volumetric computation of log using Huber's, V_H (m^3) and Newton's, V_N (m^3) measurements were based on the formulae:

$$V_H = \frac{\pi L}{4}(dc_1 \times dc_2)$$

$$V_N = \frac{\pi L}{24}\left[\left(d_1 \times d_2\right) + \left(4 \times dc_1 \times dc_2\right) + \left(d_3 \times d_4\right)\right]$$

where L = length of log (m), d_1 = diameter 1 of end 1 (m), d_2 = diameter 2 of end 1 perpendicular to d_1 (m), dc_1 = diameter 1 at centre (m), dc_2 = diameter 2 at centre perpendicular to dc_1 (m), d_3 = diameter 1 of end 2 (m) and d_4 = diameter 2 of end 2 perpendicular to d_3 (m) (Figure 4).

Reference volume

The reference volume of every log was determined using the fundamental measurement of water-displacement (WD). Each log was fully immersed under water to measure the increase in the water level of the tank. The reference volume, V_r (m^3) was calculated using the formula:

$$V_r = \left(h_i - h_o\right) \times w \times l$$

where h_i = height of water level with submerged log (m), h_o = initial height of water level without log (m), w = width of the water tank (m) and l = length of the water tank (m) (Figure 5). Water penetration through the surfaces of the logs is negligible since the samples were in green condition, the density of timber was fairly high and the immersing time of one log was merely in a few minutes.

RESULTS AND DISCUSSION

The actual measurement of the length of 82 sample logs of *H. odorata* varied from 1.990 to 2.410 m. All logs were generally in straight form. The diameter measurements of the logs varied from 0.178 to 0.401 m (bottom), 0.128 to 0.278 m (middle) and 0.115 to 0.254 m (top). The bottom, middle and top logs were characterised by particular shapes of the cross-sectional surfaces. The bottom logs were found to have round, elliptic, triangular, squared and irregular cross-sectional surfaces. The non-round shapes were mainly found from the basal part of the boles due to the buttress profile. Logs of middle section were mostly characterised by round and elliptic cross-sectional surfaces. The top logs were characterised by round, elliptic and irregular cross-sectional surfaces.

The average volumes calculated using five methods of volumetric measurements (i.e. Huber's, Smalian's, Newton's, truncated cone and water-displacement), are summarised in Tables 1 and 2. Based on the basic principle of physics, results of the water-displacement method were considered as the most accurate presentation of the volumetric amount and denoted as the reference volume. The deviations of average values from reference volume using Huber's, Smalian's, Newton's and truncated cone formulae were 1.4%, 4.1%, 2.7% and 2.7% respectively. All computations of average volume

Figure 3 Geometrical measurements of Smalian's formula

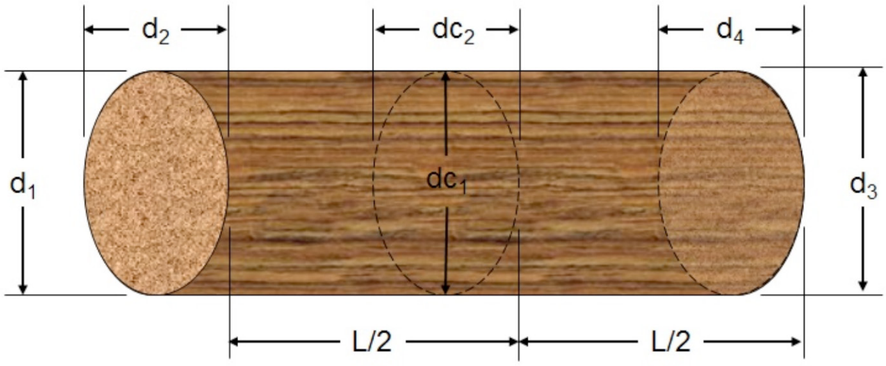


Figure 4 Geometrical measurements of Newton's formula

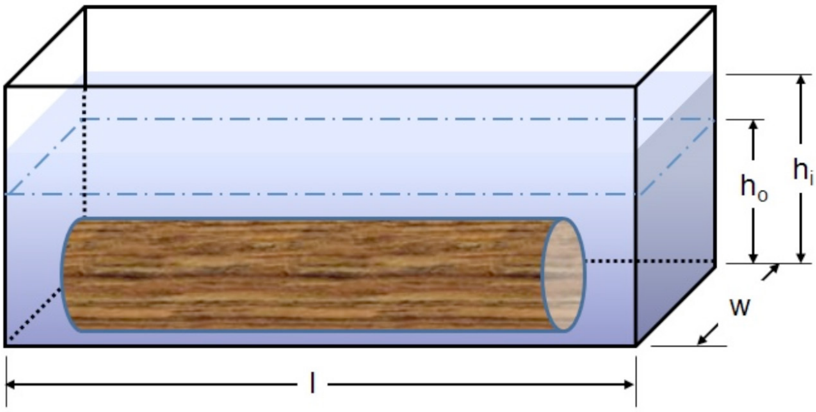


Figure 5 Test arrangement for the determination of reference volume

using mathematical formulae resulted in higher quantity compared to reference volume, except for the evaluation of top logs using Smalian's and truncated cone formulae. Only the valuation of top logs using Newton's formula showed a similar average quantity with reference volume. The computation of bottom logs based on Smalian's formula showed the highest deviation of 11.7% over reference volume. The second highest deviation was the computation of bottom logs using truncated cone formula with 10.6% over reference volume. Other measurements deviated in the range of -5.6 to 4.3% from the reference volume.

The t-test analysis was conducted to evaluate whether the average calculated volumes were statistically different from average reference volume. The probability level was set at 0.05 indicating that 5 times out of a hundred, a statistically significant difference between the average values will be found. Resultantly, the t-test values of Huber's, Smalian's, Newton's and truncated cone formulae were 0.54, 0.82, 0.65 and 0.72. Thus as the tested values were lower than the t-value table (i.e. 1.96), it was concluded that the average values of all formulae were comparable with the average reference volume.

A graph of individual measurement was plotted to evaluate the correlations between mathematically computed versus reference volumes (Figure 6). The values of coefficient of determination (r^2) were derived using Microsoft Excel. Huber's, Smalian's, Newton's and truncated cone computations were correlated to reference volume with r^2 of 0.620, 0.746, 0.694 and 0.751 respectively. Thus, based on the fitted regressions, the truncated cone formula demonstrated the most precise measurement, irrespective of log positioning.

Separated sections

In order to determine the factors that contributed to the measurement deviations, the separated plots of the bottom, middle and top logs were analysed (Figures 7a to 7c). Evidently, the graphs of the three divisions resulted in distinctive r^2 values. Strong linear correlations between mathematical formulae and reference volume were observed in the middle logs. Relative to the reference volume, Huber's, Smalian's, Newton's and truncated cone formulae showed r^2 of 0.813, 0.790, 0.824 and 0.786 respectively (Figure 7b). The precision of volumetric measurements of middle logs (plus the consistency of r^2 values between different methods) was clearly the results of round and elliptical shapes of the cross-sectional surfaces, regardless of the formula used.

In contrast, evaluation of bottom logs showed lower r^2 values. Huber's, Smalian's, Newton's and truncated cone computations of bottom logs were correlated to reference volume with r^2 of 0.596, 0.662, 0.660 and 0.687 respectively (Figure 7a). The correlation of measurements was more conflicting for top logs – the same formulae were correlated to reference volume with r^2 of 0.010, 0.154, 0.036 and 0.157 respectively (Figure 7c).

The inconsistency of valuation of top logs was noticeably influenced by the cross-sectional variation along the length. The masses of the protruding branch collars were explicitly accounted in the water displacement measurement, however not by the mathematical computations. This explained the negative percentage (-5.6%) of the average volume of Smalian's and truncated cone formulae, relative to reference volume (Table 1). The two formulae were based on the surface areas of end 1 and 2,

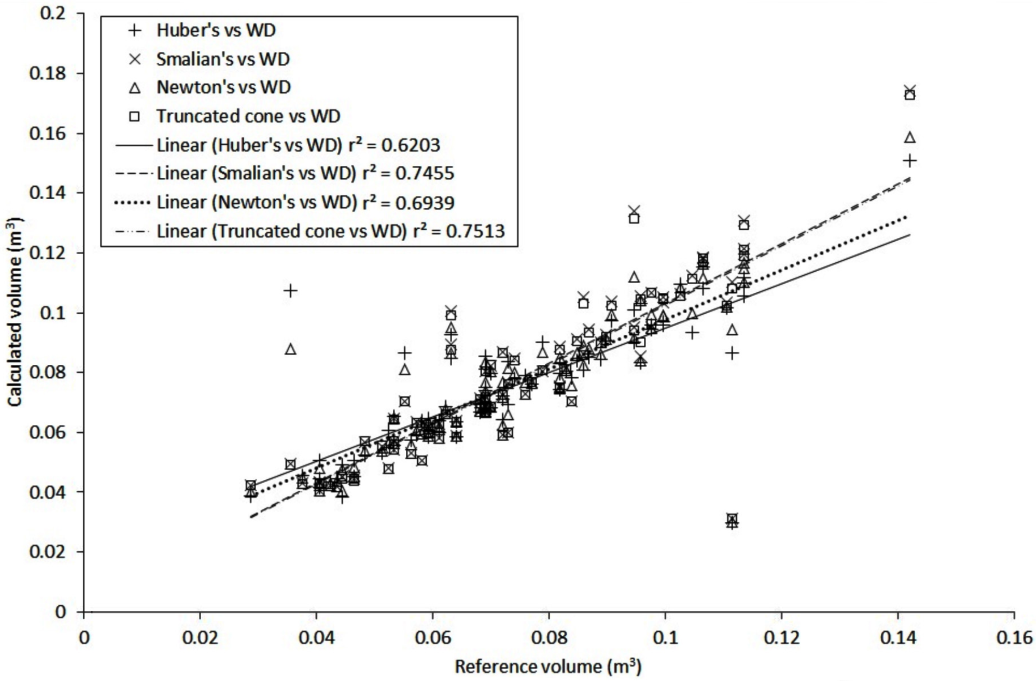
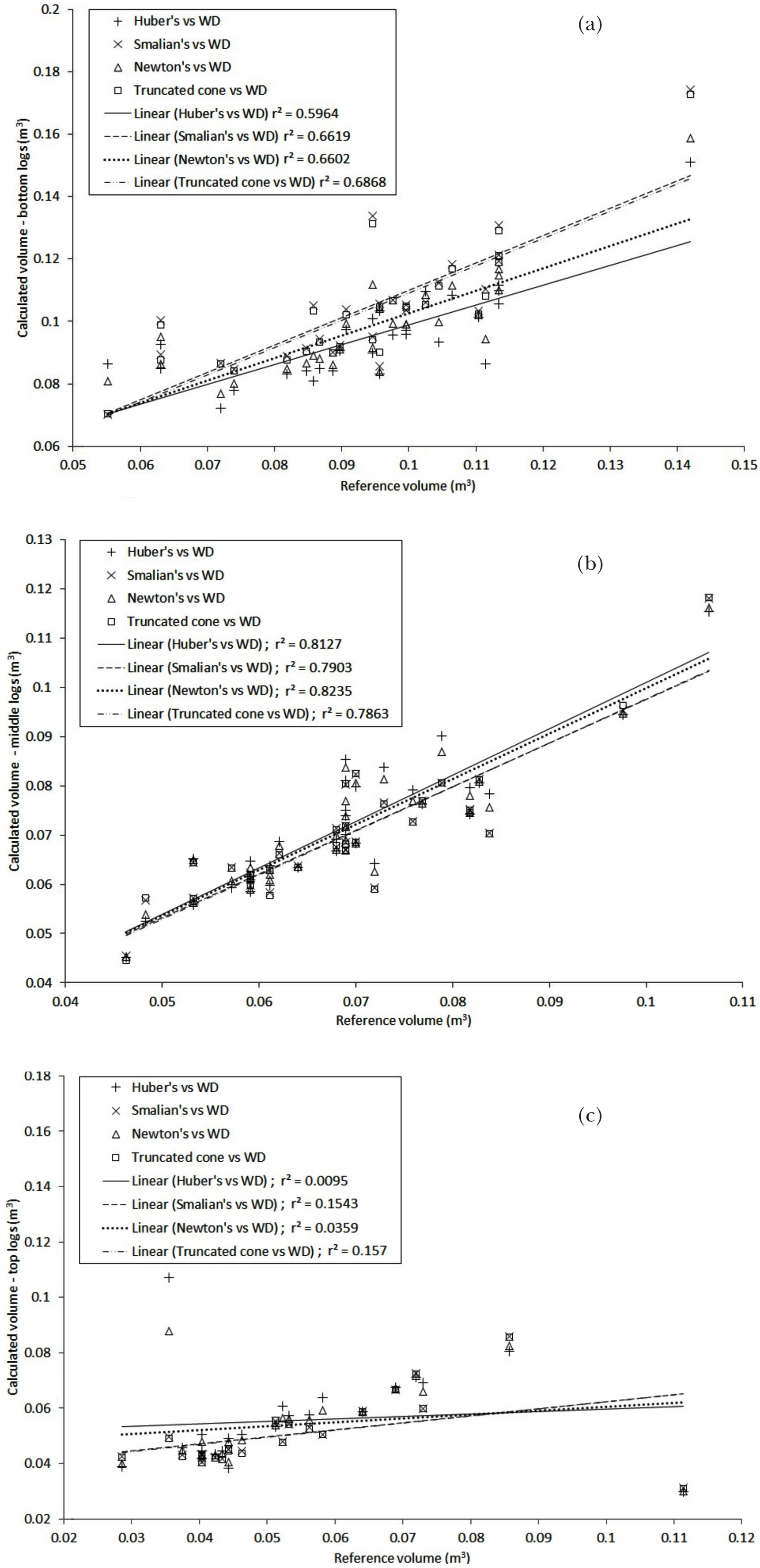


Figure 6 Plot of volumetric measurements between Huber's, Smalian's, Newton's and truncated cone formulae versus reference volume



Figures 7 Linearity evaluations of volumetric measurements based on bottom (a), middle (b) and top (c) logs

Table 1 Average volume of *Hopea odorata* logs using water-displacement method

Measurement method	Position of log	Min. reference volume (m^3)	Max. reference volume (m^3)	Total reference volume (m^3)	Average volume (m^3)	Standard deviation (m^3)
Water-displacement	Total (82 logs)	0.029	0.142	6.036	0.074	0.023
	Bottom (28 logs)	0.055	0.142	2.630	0.094	0.018
	Middle (32 logs)	0.046	0.106	2.216	0.069	0.013
	Top (22 logs)	0.029	0.111	1.190	0.054	0.019

thus measuring only the effective volume of the bole without consideration of the branch collars. On the contrary, that was not the case for Huber's and Newton's formulae since the centre cross-sectional area was accounted into the volumetric computation, which means the formulae involved consideration of the branch collars to a certain degree. Nevertheless, the exercise of Smalian's or truncated cone formula were recommended for assessment of top logs by considering the useful volume of the bole. Although they did not depict the actual volumetric amount, the formulae were suitable to determine the effective masses of the logs that can be successfully processed into planks.

Based on the evaluation of bottom logs, the r^2 of Huber's, Smalian's, Newton's and truncated cone formulae were more or less equivalent (Figure 7a). However, Smalian's and truncated cone formulae demonstrated highest deviations from the reference volume, 11.7% and 10.6% respectively. The results were reasonable since both formulae did not account for the curvature profile along the length. Most bottom logs were having the shape of the frustum of a neiloid due to the presence of buttress roots. Even with Newton's formula, which supposedly the most accurate method for measuring neiloid-form log, the deviation from reference volume was relatively high with 4.3%.

Overall, it was summarised that, based on volumetric assessment of *H. odorata* logs, the mathematical computation of truncated cone formula demonstrated the most precise and practical method. The volumetric measurement of the total logs using the truncated cone computation correlated to reference volume with r^2 of 0.751. Based on the same formula, the separated bottom and middle logs correlated to the reference volume with r^2 of 0.687 and 0.786 respectively. Although the formula did not show a good coefficient of determination for top logs, for practicality and economic reasons, it was considered as the best method. Besides, at the industrial scale, most practices normally ignored the sectioning of a log in a tree.

Correction factor

Nevertheless, if bottom logs could be distinguished, the method of data conversion was proposed, using a correction factor for the more accurate presentation of volumetric quantity. Based on the total and average volumes of the truncated cone formula (Table 2), the percentages of deviation versus the number of samples of bottom logs derived the equation:

$$P_n = 10.6426 - 0.0026n$$

where P_n = percentage of deviation from the reference volume (%) for n number of logs and n = number of logs ($n \le 4000$ as p = 0 at n = 4000), and

$$V_{tc,\,n} = V_{ref,\,n} + \left(V_{ref,\,n} \times {}^{p_n}\!/_{100}\right)$$

Therefore, by incorporating these equations, the adjusted volume of bottom log ($V_{adj,\,n}$) is given by the formula:

$$V_{adj,\,n} = \frac{100V_{tc,\,n}}{(110.6426\text{-}0.0026n)}$$

where $V_{tc,n}$ = volumetric measurement of n number of logs (truncated cone formula) and n = number of logs (≤ 4000). For the evaluation of more than 4000 samples, the ultimate volumetric amount can be directly taken from the truncated cone formula since p = 0 at n = 4000.

To validate the newly developed correction factor, the results of the present study were reassessed and the adjusted values are shown

Table 2 Average volume of *Hopea odorata* logs using mathematical formulae

Mathematical formula	Position of log	Min. reference volume	Max. reference volume	Total volume	Average volume	Standard deviation	Comparison of average volume with reference volume
		(m^3)	(m^3)	(m^3)	(m^3)	(m^3)	(%)
Huber's	Total	0.030	0.151	6.190	0.075	0.022	1.4
	Bottom	0.072	0.151	2.662	0.095	0.015	1.1
	Middle	0.045	0.115	2.303	0.072	0.014	4.3
	Top	0.030	0.107	1.225	0.056	0.017	3.7
Smalian's	Total	0.031	0.174	6.295	0.077	0.027	4.1
	Bottom	0.070	0.174	2.930	0.105	0.020	11.7
	Middle	0.045	0.118	2.248	0.070	0.013	1.4
	Top	0.031	0.086	1.117	0.051	0.012	-5.6
Newton's	Total	0.030	0.159	6.225	0.076	0.023	2.7
	Bottom	0.077	0.159	2.751	0.098	0.016	4.3
	Middle	0.045	0.116	2.285	0.071	0.013	2.9
	Top	0.030	0.088	1.189	0.054	0.014	0
Truncated cone	Total	0.031	0.173	6.263	0.076	0.026	2.7
	Bottom	0.070	0.173	2.908	0.104	0.019	10.6
	Middle	0.045	0.118	2.244	0.070	0.013	1.4
	Top	0.031	0.086	1.112	0.051	0.012	-5.6

in Table 3. Based on random sampling and a random number of samples, the adjusted values were clearly more approximated to reference volume. For instance, a random sample of 20 logs gave the volume of 1.928 m^3 when calculated using the truncated cone formula. Applying the correction formula, $V_{adj.n}$ to the value, resulted in 1.743 m^3 which was closer to reference volume of 1.715 m^3. For the best conversion results, the applied volumetric limits were within 0.03 to 0.12 m^3 of each log. For the assessment of logs having smaller or larger diameter outwith the present work; further investigation is recommended to verify the precision of conversion results. The adjustment equation is also applicable for timber species with logs of similar geometrical profiles.

A list of plantation timbers with similar log geometrical profile to *H. odorata* is provided (Table 4). The suggestion was based on the overall resemblance of buttress profile, the shape of the trunk and crown characteristics. For example, *H. odorata* had fairly round and straight boles. The buttresses were not spread far away from the trunk. Stilt root was not present in juvenile trees but probably developed in matured trees. Crown shapes were mainly round and oval. The forks and branches developed in the crown section. Thus, based on similar features, tree species listed in Table 4 were recommended for volumetric assessment of log using truncated cone formula. If the bottom log can be distinguished, the use of the aforementioned correction factor was proposed.

CONCLUSIONS

The deviations of the average volumetric amount from reference volume using Huber's, Smalian's, Newton's and truncated cone formulae were 1.4%, 4.1%, 2.7% and 2.7% respectively. T-test analysis demonstrated that the average values of all formulae were comparable with the average reference volume. The exercise of the truncated cone and Smalian's formulae for volumetric assessment of small plantation logs was relatively practical than the Huber's and Newton's formulae. In general, the most precise volumetric measurement was obtained from the truncated cone formula. In order to estimate the more accurate volume of the bottom log, a correction factor was developed. Thus, the truncated cone formula and the correction factor were recommended for the volumetric assessment of other plantation species with similar log profiles.

Table 3 Adjusted volume of *Hopea odorata* logs (bottom section) using the correction factor

Random sampling (Number of logs)	Volume of truncated cone formula (m^3)	Reference volume (m^3)	Adjusted volume (m^3)
1	0.070	0.055	0.064
1	0.088	0.063	0.079
1	0.099	0.063	0.089
1	0.086	0.072	0.078
1	0.084	0.074	0.076
10	0.892	0.755	0.806
10	1.036	0.960	0.936
20	1.928	1.715	1.743
28	2.908	2.630	2.630

Table 4 Plantation timbers having similar log geometrical profile with *Hopea odorata*

No.	Plantation species	Botanical name
1	Batai	*Paraserianthes falcataria*
2	Binuang	*Octomeles sumatrana*
3	Jelutong	*Dyera costulata*
4	Kapur	*Dryobalanops aromatica*
5	Khaya	*Khaya ivorensis*
6	Laran / Kelempayan	*Neolamarckia cadamba*
7	Pelong	*Pentaspadon motleyi*
8	Rubberwood	*Hevea brasiliensis*
9	Sentang	*Azadirachta excelsa*
10	Sesendok	*Endospermum diadenum*

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