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Full Length Research Paper

A lower complexity k best algorithm for multiple input and multiple output detection

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This paper presents Multiple Input Multiple Output (MIMO) detection steps using tree search based method known as the 'K' best algorithm. This low complexity algorithm is based on probabilistic approach of sphere decoding with self adjustable capability depending on the levels (root, branch, leaf etc.) of a tree. While the tree was searched to estimate the transmitted symbols level by level, the algorithm took into account the effect of the undetected symbols in the search criteria. Simulation results showed that the proposed method reduced complexity (in terms of the average number of visited nodes) about 10% for higher (medium to high) signal to noise ratio (SNR) values without degrading the system BER performance.

Key words: Multiple input multiple output, sphere decoding, K best algorithm.

INTRODUCTION

The field of Multiple Input Multiple Output (MIMO) systems is, currently, an active research area where spectral efficiency can be enhanced without changing power or bandwidth. In a spatial multiplexing system, independent data streams are sent from the transmitter antennas. Spectral efficiency grows linearly with the minimum number of transmission and receive antennas, N_t and N_r, respectively, assuming a rich scattering wireless channel (Foschini and Gans, 1998; Burg et al., 2006). This performance improvement offered by MIMO systems comes at the price of higher computational complexity detectors. Great amount of research has been directed toward the design and implementation of less complex detectors within the constraints of various real applications. The most demanding maximum likelihood (ML) solution minimizes the bit error rate (BER) and makes use of the system's full available diversity order of N_r (Maurer et al., 2007); its complexity is exponentially proportional to the modulation constellation size and the number of the transmitter antennas. Many suboptimal solutions have been proposed in the literature such as V Blast (Wolniansky et al., 1998) and linear equalizers with quantization (Harold et al., 2003) with a performance inferior to that of the ML. ML solution can be obtained using the depth first tree search based detector, sphere decoder (Fincke and Pohst, 1985) at low complexity compared with the exhaustive search. Therefore, to reduce the detector complexity in MIMO systems while maintaining the BER is in demand. Hence, in this paper, focus is given to solve the aforementioned problem by proposing a lower complexity 'K' best algorithm for MIMO detection. The paper is organized as follows: subsequently, MIMO system model is introduced, the tree search algorithms as MIMO detectors are briefly reviewed; the probabilistic tree pruning and its application with K best search method is given, then simulation results is presented, lastly, the conclusion is presented.

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Abbreviations: $\mathbf{a}^H(\mathbf{A}^H)$, transpose with complex conjugation of the vector (matrix) a (A); **i.i.d,** independently and identically distributed; $p(\mathbf{a})$, probability density function (pdf) of a; $\|\mathbf{a}\|$, the second norm of a.

SYSTEM MODEL

The flat fading, complex valued, baseband equivalent MIMO system is represented by:

$$\mathbf{y}_c = \mathbf{H}_c\mathbf{s}_c + \mathbf{n}_c \qquad (1)$$

Where $\mathbf{y}_c$ is the $N_r \times 1$ received vector, $\mathbf{s}_t$ is the $N_t \times 1$ transmitted vector, $\mathbf{H}_c$ is the $N_r \times N_t$ ($N_r \geq N_t$) complex channel matrix, the element $H_c(i,j)$ is the propagation coefficient between R_x antenna *i* and transmit antenna *j* with mean value $\mu = 0$ and unit variance under the rich scattering channel assumption, $\mathbf{n}_c$ is the $N_r \times 1$ noise vector whose elements are i.i.d circularly symmetric complex Gaussian with zero mean and variance σ^2 and covariance matrix $E(\mathbf{n}_c\mathbf{n}_c^H) = \sigma^2 \mathbf{I}_{N_r \times N_r}$.

The elements of the vector $\mathbf{s}_c$ are independently drawn from a complex constellation of size $\mathcal{A} = 2^{M_c}$ where M_c is the number of bits per symbol whose average power is assumed to be unity or $E(\mathbf{s}_c\mathbf{s}_c^H) = \mathbf{I}_{N_t \times N_t}$ and I is the identity matrix with the indicated dimensions. E is the expectation operator, and $A(i,j)$ or $A_{i,j}$ is the element at the ith row and the jth column of the matrix A. Assuming the full knowledge of the channel matrix $\mathbf{H}_c$ at the receiver side, the optimal ML solution is to find the transmitted vector $\hat{\mathbf{s}}$ that maximizes the conditional posteriori probability $Pr(\mathbf{s}\,|\,\mathbf{y}_c, \mathbf{H}_c)$ (Hanzo and Keller, 2006):

$$\hat{\mathbf{s}} = \arg \max_{\mathbf{s}_c \in \mathcal{A}^{N_t}} Pr(\mathbf{s}_c\,|\,\mathbf{y}_c, \mathbf{H}_c) \qquad (2)$$

Now, $p(\mathbf{s}_c\,|\,\mathbf{y}_c, \mathbf{H}_c) = C \exp\left[-\frac{1}{\sigma^2}\|\mathbf{y}_c - \mathbf{H}_c\mathbf{s}_c\|^2\right]$ where C is an independent constant. The maximization in Equation 2 reduces to the minimization of the Euclidian distance:

$$\min_{\mathbf{s}_c \in \mathcal{A}^{N_t}} \|\mathbf{y}_c - \mathbf{H}_c\mathbf{s}_c\|^2$$

TREE SEARCH BASED DETECTORS

The complex model of Equation 1 can be converted into a real form as Damen et al. (2003):

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \qquad (3)$$

Where:

$$\mathbf{y} = \begin{bmatrix} \Re(\mathbf{y}_c) \\ \Im(\mathbf{y}_c) \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} \Re(\mathbf{s}_c) \\ \Im(\mathbf{s}_c) \end{bmatrix}, \quad \mathbf{n} = \begin{bmatrix} \Re(\mathbf{n}_c) \\ \Im(\mathbf{n}_c) \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} \Re(\mathbf{H}_c) & -\Im(\mathbf{H}_c) \\ \Im(\mathbf{H}_c) & \Re(\mathbf{H}_c) \end{bmatrix}$$

And $\Re(\ )$, $\Im(\ )$ stands for the real, imaginary parts of (), respectively. The Euclidian distance can now be rewritten as:

$$\min_{s \in \mathcal{D} \subset \mathbb{Z}^{2N_t}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2$$

In which: $\mathbf{y} \in \mathbb{R}^{2N_r}$, $\mathbf{H} \in \mathbb{R}^{2N_r \times 2N_t}$, s is $2N_t \times 1$ vector with integer entries, $\mathcal{D}$ is a finite subset of the infinite lattice $\mathbb{Z}^{2N_t}$ and $\mathbb{Z}$ is the set of integers (Hassibi and Vikalo, 2005). There are two main tree based search algorithms; the sphere decoder and the K best detector. The sphere decoder (Fincke and Pohst, 1985) restricts the search for the ML solution of $\min\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2$. It examines the lattice points $\mathbf{H}\mathbf{s}$ within the $2N_r$ dimensions hyper sphere of radius d centered at the received vector $\mathbf{y}$ as:

$$\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = \|\mathbf{n}\|^2 \leq d^2 \qquad (4)$$

The choice of the search radius, *d* is critical for the complexity of the detector, because for large *d*, there will be too many points to be examined, otherwise, there may be no points at all inside the sphere. There are many suggestions for that choice mentioned in Hassibi and Vikalo (2005) and Hochwald and Brink (2003). Assuming, for simplicity that: $2N_r = m$, $2N_t = v$ and performing the QR factorization on H:

$$\mathbf{H} = \mathbf{Q}\begin{bmatrix} \mathbf{R} \\ \mathbf{0}_{(m-v)\times v} \end{bmatrix}$$

Where R is a $v \times v$ upper triangular matrix, $\mathbf{Q} = [\mathbf{Q}_1 \quad \mathbf{Q}_2]$ is a $m \times m$ orthogonal matrix with sub matrices $\mathbf{Q}_1$ and $\mathbf{Q}_2$. Now, Equation 4 becomes:

$$\left\|\mathbf{y} - [\mathbf{Q}_1 \quad \mathbf{Q}_2]\begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix}\mathbf{s}\right\|^2 \leq d^2 \qquad (5)$$

$$\left\|\begin{bmatrix} \mathbf{Q}_1^T \\ \mathbf{Q}_2^T \end{bmatrix}\mathbf{y} - \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix}\mathbf{s}\right\|^2 \leq d^2$$

$$\|\mathbf{Q}_1^T\mathbf{y} - \mathbf{R}\mathbf{s}\|^2 \leq d^2 - \|\mathbf{Q}_2^T\mathbf{y}\|^2$$

$$\|\mathbf{y}' - \mathbf{R}\mathbf{s}\|^2 \leq d'^2 \qquad (6)$$

Where: $\mathbf{y}' = \mathbf{Q}_1^T\mathbf{y}$, $d'^2 = d^2 - \|\mathbf{Q}_2^T\mathbf{y}\|^2$ and $d'^2 = d^2$ if $m = v$.

Using the structure of matrix R in Equation 6:

$$\sum_{i=1}^{v}\left(y_i' - \sum_{j=i}^{v} R_{i,j}s_j\right)^2 \leq d'^2 \qquad (7)$$

Equation 7 is used to search over a tree with *v* levels: each level corresponds to one element of the vector s, where s_v and s_1 are the highest and lowest levels, respectively (Figure 1). The radius constraint becomes tighter as the search moves down the tree; at the top level *v*, the bounds for the symbol s_v are decided according to:

$$\left(y_v' - R_{v,v}s_v\right)^2 \leq d'^2 \qquad (8)$$

And for next lower level, $v-1$, s_{v-1} is estimated with:

$$\left(y_{v-1}' - \sum_{k=v-1}^{v} R_{v-1,k}\,s_k\right)^2 \leq d'^2 - \left(y_v' - R_{v,v}s_v\right)^2$$

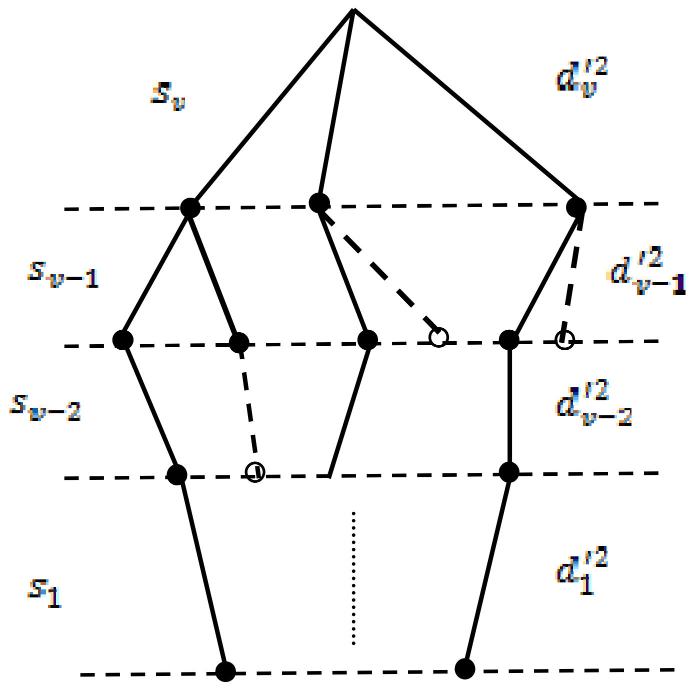

Figure 1. Levels of the depth first tree search.

In general, for the level $v-i$, $i=0,1,\cdots,v-1$, the bounds for s_{v-i} are derived on the basis of the inequality:

$$\left(y'_{v-i}-\sum_{k=v-i}^{v}R_{v-i,k}s_k\right)^2 \le d'^2-\sum_{j=v-i+1}^{v}\left(y'_j-\sum_{h=j}^{v}R_{j,h}s_h\right)^2 \qquad (8)$$

Here, the second term of the RHS corresponds to the group of i detected symbols from v down to the $(v-i+1)$ level: s_v ,...., s_{v-i+1}.

According to Figure 1, Equation 8 has two metrics at each level. The first is the branch metric, B_k, $(k=v-i)$, that originates from a certain node and depends on the symbol s_k for that branch and level:

$$B_k=\left(y'_k-\sum_{j=k}^{v}R_{k,j}s_j\right)^2 \qquad (9)$$

The second metric is the partial total or accumulated path metric, P^v_{k+1} . It is the right most term in Equation 8 and describes the sum of the branch metrics starting from the top level v to the level $k+1$. At each level, except the top, the branches that originate from the same node share the same parent path metric, P^v_{k+1}. Hence:

$$P^v_k=P^v_{k+1}+B_k \qquad (10)$$

The top level v with $P^v_{k+1}=0$, has equal branch and path metrics.

The search algorithm deals with one node at a time starting from the top level v, and if $P^v_k > d'^2$, then, the node k and all branches below it (dashed lines in Figure 1) will be pruned from the tree since all the terms in Equation 7 are positive. The full paths matrices (if there is any), starting from top to bottom level, that satisfy the radius constraint are considered as candidates and their minimum is considered as the ML solution. Each such candidate corresponds to a different path from the top of the tree (named as the root), to its bottom (known as the leaves). If no such candidates are found, then, the search radius is increased and the search is restarted again. Once a candidate that satisfies the radius constraint is found, the search radius is reduced to the path metric of that candidate and the search is continued (Viterbo and Boutros, 1999). To reduce complexity, the K best, breadth first tree search (Zhan and Nilsson, 2006), selects those K branches at each tree level with the lowest path metrics to be extended further down the tree and the candidate at the bottom level with the minimum path metric is considered as the solution. The total number of the considered candidates is $v\times K$ instead of $(\sqrt{\mathcal{A}})^v$ (in an exhaustive search with square QAM constellations). The factor K is chosen to tune the performance-complexity tradeoff with better performance and higher complexity for larger K values. Inherently, the K best method does not guarantee finding the ML vector candidate s that minimizes $\|\mathbf{y}-\mathbf{Hs}\|^2$. This is due to the fact that the solution minimizing the metric in Equation 9 for a certain branch and level, does not necessarily lead to minimization of the other branch metrics within the same path. Figure 2 shows a scenario of K best breadth algorithm with $N_t=2\ (v=4), K=4$ and 16 QAM.

SPHERE DECODING WITH PROBABILISTIC TREE PRUNING

In Byonghyo and Insung (2008), an aggressive pruning strategy

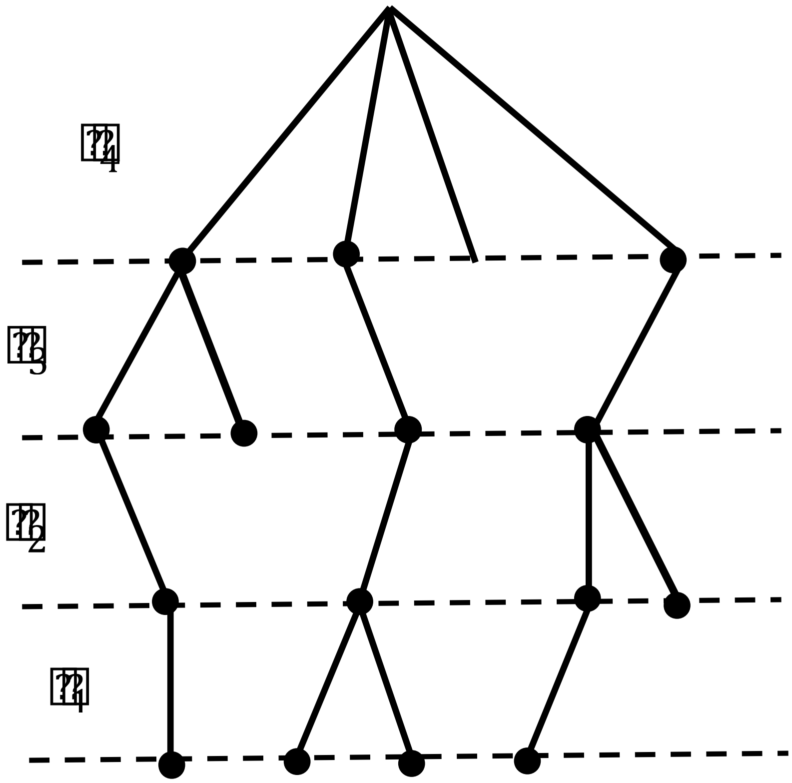

Figure 2. Breadth first, *K* best, tree search (four levels).

was suggested by reconsidering Equation 8 and taking into account the effect of the undetected symbols at each layer. The condition to be satisfied at the layer $v-i$ is:

$$P^{v}_{v-i} \le d'^{2}$$

Taking the effect of the $v-i-1$ undetected layers by considering their branch metrics of Equation 9:

$$P^{v}_{v-i} + \sum_{j=1}^{v-i-1} B_j \le d'^{2} \tag{11}$$

This will impose the desired effect of tightening the radius constraint at each layer by a variable amount according to the number of undetected layers which varies between $v-1$ at the top level and zero at the bottom. Since $\|\mathbf{y}-\mathbf{Hs}\|^2 = \|\mathbf{n}\|^2$, each of such metrics can be represented by the noise properties as:

$$P^{v}_{v-i} + \sum_{j=1}^{v-i-1} n_j^2 \le d'^{2} \tag{12}$$

The elements n_j are Gaussian random variable with zero mean and variance of $\frac{\sigma^2}{2}$ and hence the aforementioned sum of their squares is a chi-square variable with $v-i-1$ degrees of freedom:

chi_{v-i-1};

$$P^{v}_{v-i} + chi_{v-i-1} \le d'^{2}$$

The pruning probability ε is defined with:

$$Pr\left(chi_{v-i-1} \le d'^{2} - P^{v}_{v-i}\right) < \varepsilon \tag{13}$$

Any branch that satisfies Equation 13 is pruned from the tree and ε is a predefined value selected to adjust the complexity-performance tradeoff. Now, in terms of the cumulative distribution function (CDF), $F_{chi_{v-i-1}}$ of the chi-square random variable chi_{v-i-1}, the condition in Equation 13 becomes:

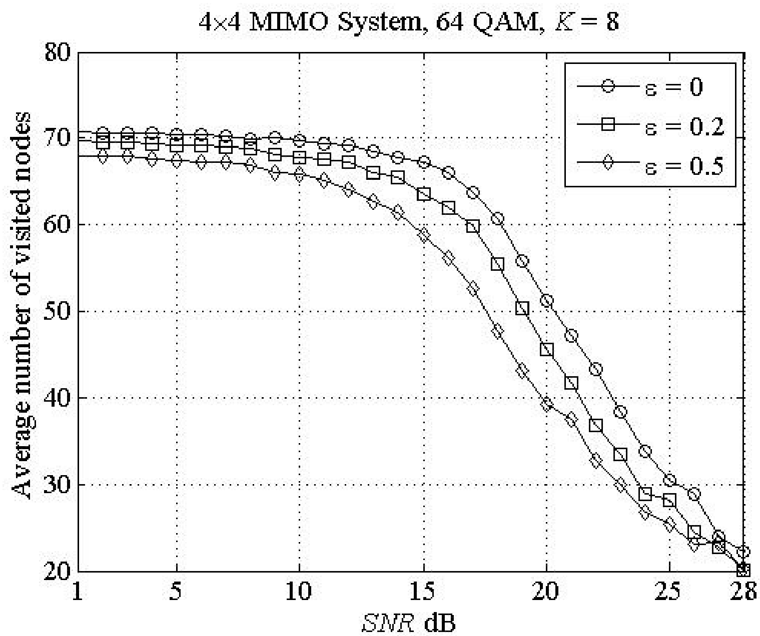

Figure 3. Effect of the radius constraint on complexity for a 4×4 MIMO system with 64 QAM.

$$\left(d'^2 - P^v_{v-i}\right) < F^{-1}_{chi_{v-i-1}}(\varepsilon)$$
$$P^v_{v-i} > d'^2 - F^{-1}_{chi_{v-i-1}}(\varepsilon) \quad (14)$$

This becomes the terms of the branch metric B_{v-i}:

$$B_{v-i} + P^v_{v-i+1} > d'^2 - F^{-1}_{chi_{v-i-1}}(\varepsilon)$$
$$B_{v-i} > d'^2 - P^v_{v-i+1} - F^{-1}_{chi_{v-i-1}}(\varepsilon) \quad (15)$$

Any branch that satisfies Equation 15 is pruned from the tree. We apply the probabilistic approach to the tree search using the *K* best algorithm: to do that, we have imposed a search radius constraint that should be satisfied by all path metrics at each level. The search radius is selected as: $d^2 = 2N_r c\left(\frac{\sigma^2}{2}\right)$ according to the noise variance σ^2 and N_r based on Equation 3 assuming equal number of transmit and receive antenna with $c \geq 1$. This radius selection is suggested in Hochwald and Brink (2003) and it is the mean value of the $2N_r$ degrees of freedom, chi-square random variable described by $\|\mathbf{n}\|^2$. The complexity reduction is due to the fact that, with this added constraint, the selected branches at each level for further extension towards the bottom of the tree may be less than *K*.

RESULTS AND DISCUSSION

We used Matlab 7.7.0 for our simulations. The channel was assumed to have a block fading model; it remains unchanged within each block consisting of a number of transmitted vectors and then, changes independently from one block to another. The values of $F^{-1}_{chi_{v-i-1}}(\varepsilon)$ in Equation 15 were computed offline for each value of *SNR* with degrees of freedom ranging between 1 and $2m-1$.

For a tree search algorithm, it is usual to describe complexity in terms of the number of visited nodes (Byonghyo and Insung, 2008; Burg et al., 2005). The effect of the radius constraint on the complexity of the *K* best algorithm ($\varepsilon = 0$) is shown in Figure 3 and the corresponding *BER* curves are shown in Figure 4, for the 4×4 MIMO system with 64 QAM and the indicated values of *K*. Without having the radius constraint, the number of visited nodes usually remains constant ($2mK$), regardless of *SNR* value, which is 8*K* (that is, 64, 56, 48 nodes for $K = 8, 7$ and 6, respectively) in this case ($\mathbf{s}$ has double the size of $\mathbf{s}_c$). On the other hand, with radius constraint, the search radius is selected according to the noise variance and number of receiver antennas. The search usually restarts if there are no possible candidates, at any of the tree levels that satisfy the radius constraint. The number of visited nodes during

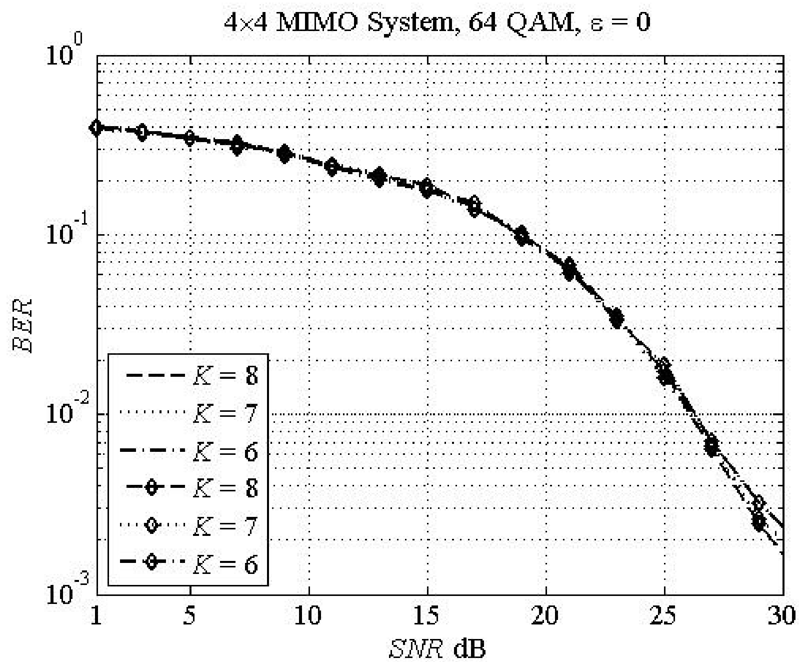

Figure 4. *BER* of the 4×4 MIMO system with 64 QAM; without (unmarked) and with radius constraint (marked).

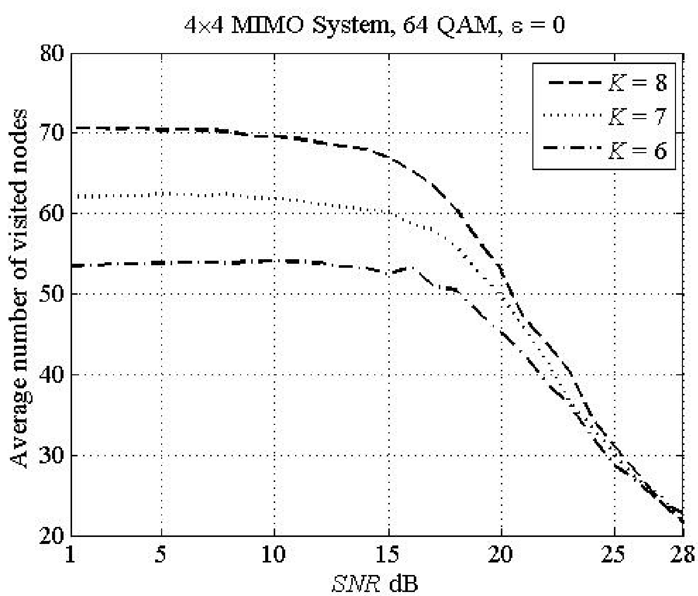

Figure 5. Number of visited nodes vs. *SNR* for a MIMO 4×4 system with 64 QAM and *K* = 8.

such uncompleted trials is included in complexity measurements. Figure 3 shows that the inclusion of such nodes is behind the larger number of visited nodes for $SNR \leq 15$ dB. It also shows a lower complexity in terms of the average number of visited nodes of 57, 52 and 46 compared to the case without radius constraint with percentage savings of about 10, 7 and 3 for *K* values of 8, 7 and 6, respectively. The *BER* performances are the same as that without radius constraint for the same values of *K* as shown in Figure 4. For the same system, Figure 5 shows the effect of ε value on the complexity with *K* = 8 as an example. As ε increases from 0 (which

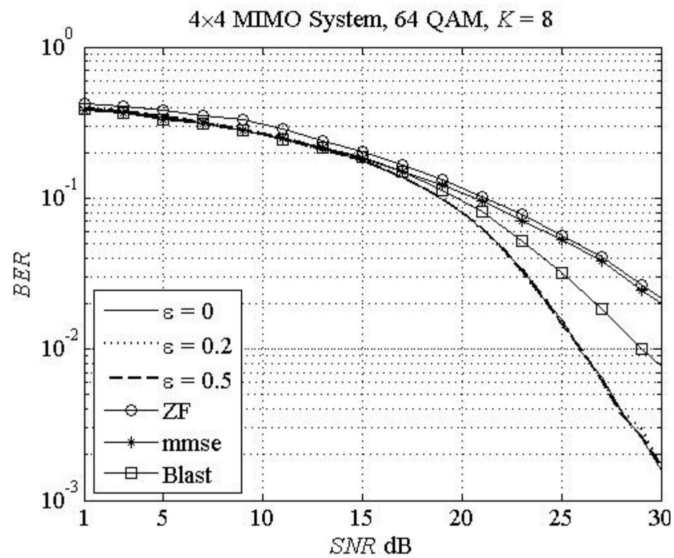

Figure 6. *BER* vs. *SNR* for a 4×4 MIMO system with 64 QAM and *K* = 8 compared to that of the ZF, mmse and Blast detectors.

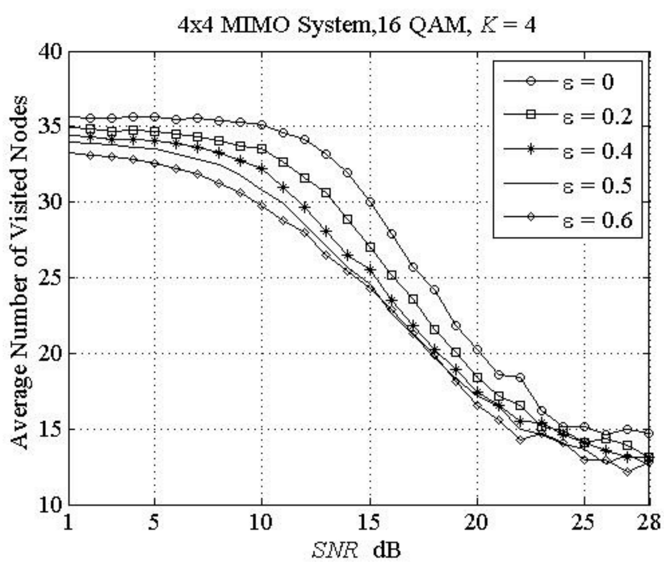

Figure 7. Average Number of visited nodes vs. *SNR* for a MIMO 4×4 system with 16 QAM and *K* = 4.

corresponds to the uppermost curve in Figure 3), the average number of visited nodes is further decreased to 55 and 51 with percentage saving of 15 and 19, that is, an additional saving of complexity about 5 and 9% compared to the case of *K* = 8, and ε = 0 (Figure 3) by setting the pruning probability parameter ε to 0.2 and 0.5 respectively. But the *BER* values are almost the same for all ε as shown in Figure 6 along with those of the zero forcing (ZF), minimum mean square (mmse) (Harold et al., 2003) and the mmse based successive interference cancellation (Blast) (Wolniansky et al., 1998) detectors. This shows, the *BER* is unaffected and remains essentially the same due to the change of ε and using radius constraint while it is improved compared to ZF, mmse and Blast.

A similar comparison is made for the 4×4 MIMO system with 16 QAM in Figures 7and 8 with the indicated values of *K* and ε. In Figure 7, for the case of $\varepsilon = 0$, the

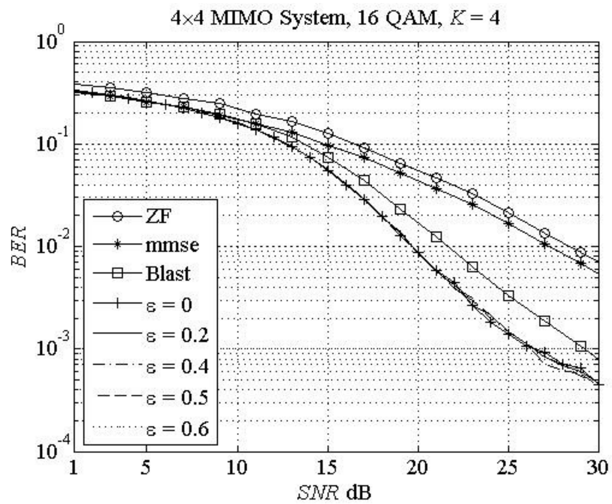

Figure 8. *BER* vs. *SNR* for a 4×4 MIMO system with 16 QAM and K = 4 compared to that of the ZF, mmse and Blast detectors.

average number of visited nodes is 27 and the complexity reduction is around 15% compared to that without radius constraint (where the average number of visited nodes is 32). The figure shows that the average numbers of visited nodes are 26, 25, 24 and 23 implying an additional complexity savings of around 5, 8, 9 and 12% using ε = 0.2, 0.4, 0.5 and 0.6, respectively. Figure 8 shows the *BER* for this case reduced significantly compared to that of the ZF, mmse and Blast detectors.

Conclusion

A lower complexity K best detector, in terms of the average number of the visited nodes has been proposed for MIMO systems. According to the suitable K values and modulation scheme in order to maintain low *BER*, it is possible to reduce complexity around 10 to 12% for a 4×4 MIMO system using 'radius constraint' and by adjusting the value of a probability threshold (ε) that takes into account noise statistics. It is shown that for the 4×4 MIMO systems with 16 and 64 QAM cases, this complexity reduction is achievable while maintaining the *BER* performance.

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