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Theoretical study on rotating casson fluid in moving channel disk

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Abstract. The theoretical study of rotating Casson fluid in moving channel disk has been investigated in this article. The dimensional governing equation of proposed fluid is derived in the form of partial differential equation with initial and boundary conditions. The non-dimensional governing equation has been obtained by using the suitable non-dimensional variables. The expression of velocity for rotating Casson fluid has been obtained by using Laplace transform method. It is found that, they satisfied governing equation and conditions imposed. Computations are carried out and the results are analyzed and discussed in details.

1. Introduction

Recently, non-Newtonian fluids are widely used in industries such as chemicals, pharmaceuticals, food, oil, gas and cosmetic. For example, in the production of several chemicals, oil, gas, paint, syrup, juice, cleanser, deodorizer, in process of plastic sheets, glass fibre production, movement of lubricants and biological fluids [1-4]. The non-Newtonian fluid models show the non-linear relationship between the shear rate and shear stress which do not obey Newtons law of viscosity compared to Newtonian fluid. In Newtonian fluid, the relationship between the shear stress and the shear rate is assumed to be linear. However, not all shear stress and shear rate obey this relationship for certain cases. Thus, the study of non-Newtonian fluids is useful for the fundamental understanding of certain applications and to describe the fluids in industrial and other technological applications such as blood, soap, certain oils, paints, and many emulsions [3].

Some of non-Newtonian fluid models have been used to describe the complex behaviour of fluids in applications including power law [5], second grade [6], Jeffrey [7], Maxwell [8], viscoplastic [9], Bingham plastic [10], Brinkman type [11], Oldroyd-B [12] and Walters-B [13] models. One of the most famous non-Newtonian fluid models is Casson fluid model which has several applications in food processing, metal lurgy, drilling operations and bioengineering operations [14]. This fluid model was originally introduced by Casson in 1959 to analyze the prediction of the flow behaviour of pigment-oil suspensions [15]. Casson fluid model is the non-Newtonian fluid model which consists of yield stress. When the shear stress of Casson fluid

is less than yield stress, the fluid behaves like a solid. The examples of Casson fluid are jelly, tomato sauce, honey, soup, concentrated fruit juices [16].

Chen *et al.* [17] studied the unsteady state unidirectional MHD flow of Voigt fluids moving between two parallel surfaces under magnetic field effect using Laplace transform method. Seth *et al.* [18] investigated the effect of Hall current on the unsteady hydromagnetic Couette flow within a rotating porous parallel channel in the presence of an uniform transverse magnetic field when the magnetic field is fixed relative to the moving plate of the channel. Wang [19] studied the starting unsteady flow in the presence of an impulsive pressure gradient in a system of rotating parallel plate channel using Laplace Transform method. The author observed that as rotation is increased, the asymptotic (steady state) flow rate decreases. Jha *et al.* [20] investigated the unsteady hydromagnetic natural convection flow of an incompressible viscous electrically conducting fluid in the presence of transverse magnetic field in infinite vertical parallel plates through channel using Laplace Transform method. Seth *et al.* [21] studied the unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid in the presence of transverse magnetic field. The flow was considered through a porous medium in a rotating parallel plate channel with the effect of Hall current. VeeraKrishna *et al.* [22] investigated the unsteady convective flow of second grade fluid through a porous medium in a rotating parallel plate channel by considering the magnetohydrodynamic (MHD) effect. Mohamad *et al.* [16] analyzed the influence of thermal radiation on unsteady MHD free convection flow of Casson fluid over a vertical plate through a porous medium.

To the authors knowledge, no researchers attempted to study the unsteady flow of rotating Casson fluid in moving channel disk. From the previous literature, Casson fluid model is one of the important models that should be considered in the fluid flow. Thus, an attempt is made in this paper to investigate the convective flow by considering the Casson fluid. Motivated by the above studies, the present work focused on theoretical study on rotating Casson fluid in moving channel disk. The theoretical exact solution is obtained by using the Laplace transform method. The behavior of obtained physical parameters are plotted graphically and discussed in details.

2. Problem Formulation and Solution

Consider the unsteady flow of rotating incompressible Casson fluid in two infinite vertical disks separated by a distance h. The x^*-axis is taken along one of the disks in the vertically upward direction and the z^*-axis is taken normal to the disks. Initially, both the disks and fluid are at rest and at time $t^* = 0+$, the disk at left hand side starts to move in its plane with constant velocity U_0 and while the other disk (right hand side) is remains immovable. Due to that, the fluid starts solid body rotation with constant angular velocity Ω parallel to z^*-axis. The schematic diagram with a coordinate system is shown in Figure1. Therefore, the appropriate governing equation is given as

$$\frac{\partial F^*}{\partial t^*} + 2i\Omega F^* = \upsilon(1 + \frac{1}{\gamma})\frac{\partial^2 F^*}{\partial z^{*2}} \tag{1}$$

together with imposed initial and boundary conditions

$$\begin{aligned} F^*(z^*, 0) &= 0 \quad ; \quad 0 \le z^* \le h, \\ F^*(0, t^*) &= U_0 \; ; \quad t^* > 0, \\ F^*(h, t^*) &= 0 \quad ; \quad t^* > 0. \end{aligned} \tag{2}$$

where $F^* = F^*(z, t) = u^*(z^*, t^*) + iv^*(z^*, t^*)$ is a complex velocity, $u^*(z^*, t^*)$ is a primary velocity, $v^*(z^*, t^*)$ is a secondary velocity, i is an imaginary number, υ is a kinematic viscosity and γ is a Casson parameter. Introducing the suitable non-dimensional variables

$$z = \frac{z^*}{h}, \quad F = \frac{F^*}{U_0}, \quad t = \frac{t^*\upsilon}{h^2}. \tag{3}$$
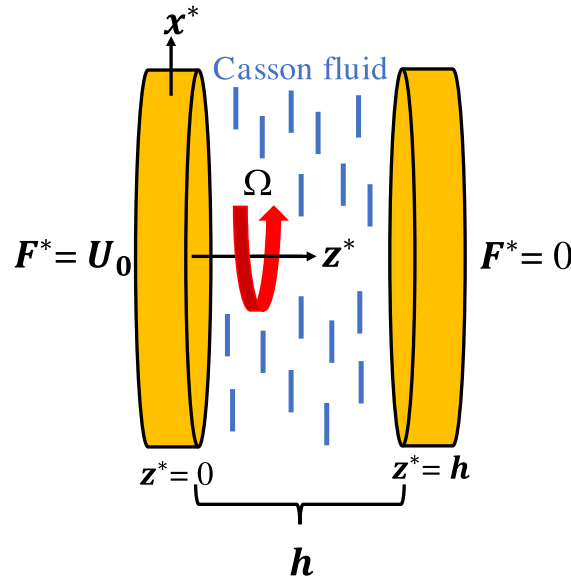
Figure 1. Schematic diagram with a coordinate system

Therefore, employing the non-dimensional variables (3) into Eq. (1) along with initial and boundary conditions (2), obtained the non-dimensional form as

$$\frac{\partial F}{\partial t} + 2i\omega F = A_1 \frac{\partial^2 F}{\partial z^2} \tag{4}$$

subjected to initial and boundary conditions

$$\begin{aligned}
F(z,0) &= 0 \quad ; \quad 0 \le z \le 1, \\
F(0,t) &= 1 \quad ; \quad t > 0, \\
F(1,t) &= 0 \quad ; \quad t > 0.
\end{aligned} \tag{5}$$

where $A_1 = 1 + \dfrac{1}{\gamma}$ is a constant parameter and $\omega = \dfrac{\Omega h^2}{v}$ is a rotation parameter. In order to obtain an exact solution of this problem, we use the Laplace transform method. Then, by applying the Laplace transform into **equation** (4) together with conditions (5), we obtained the transform equations as

$$\frac{d^2 \bar{F}(z,q)}{dz^2} - \left(\frac{q + 2i\omega}{A_1}\right)\bar{F}(z,q) = 0 \tag{6}$$

and

$$\begin{aligned}
\bar{F}(0,q) &= \frac{1}{q}, \\
F(1,q) &= 0,
\end{aligned} \tag{7}$$

which is q is a Laplace transform parameter. Using **equation** (7) into **equation** (6) leads to

$$\bar{F}(z,q) = \sum_{n=0}^{\infty} \left[\frac{1}{q}\mathrm{Exp}\left(-(2n+z)A_2\sqrt{q+2i\omega}\right) - \frac{1}{q}\mathrm{Exp}\left(-(2n+2-z)A_2\sqrt{q+2i\omega}\right) \right]. \tag{8}$$

Finally, the solution of velocity profile is obtained by using inverse Laplace transform and it can be written as

$$F(z,t) = \sum_{n=0}^{\infty} [A_1(z,t) + A_2(z,t)] \tag{9}$$

where

$$A_1(z,t) = \frac{1}{2}\text{Exp}\left(-a_1\sqrt{2i\omega}\right)\text{Erfc}\left(\frac{a_1}{2\sqrt{t}} - \sqrt{2i\omega t}\right) + \frac{1}{2}\text{Exp}\left(a_1\sqrt{2i\omega}\right)\text{Erfc}\left(\frac{a_1}{2\sqrt{t}} + \sqrt{2i\omega t}\right) \tag{10}$$

and

$$A_2(z,t) = \frac{1}{2}\text{Exp}\left(-a_2\sqrt{2i\omega}\right)\text{Erfc}\left(\frac{a_2}{2\sqrt{t}} - \sqrt{2i\omega t}\right) + \frac{1}{2}\text{Exp}\left(a_2\sqrt{2i\omega}\right)\text{Erfc}\left(\frac{a_2}{2\sqrt{t}} + \sqrt{2i\omega t}\right) \tag{11}$$

which is $a_1 = 2nA_2 + zA_2$ and $a_2 = 2nA_2 + 2A_2 - zA_2$.

3. Results and Discussions

This section is written to discuss the impact of flow parameters namely, Casson parameter γ, rotation parameter ω and time parameter t on velocity profiles which are displayed graphically in figure 2 until figure 4. The subsections (a) and (b) on plotting indicate the primary u and secondary v velocities, respectively. Here, Figure 2 shows the effect of Casson parameter γ on velocity profiles for primary and secondary parts. It is found that the velocity decrease in u but fluctuate behavior in v when the values of γ increase. This is due to increment of viscosity and yield stress in fluid flow which retard the movement of the fluid's velocity. Besides that, the effect of rotation parameter ω on velocity behavior is displayed in Figure 3. Obviously, it can be seen that, the velocity of the fluid will decrease in primary velocity but increase in secondary velocity. This is due to the Coriolis effect that exist in rotation fluid phenomena. This effect will reduce the force in primary velocity and enhance the formation of secondary velocity. In facts, the Coriolis force is defined as a deflection of moving objects in a frame rotating in the opposite direction. Lastly,the Figure 4 shows the behavior of velocity on time changing. It is found that, velocity increases when the value of t increases. During the change of time, the fluid gains an energy from the moving disk as an external force and increase the both velocities of fluid flow. In addition, the results obtained here satisfy all of the initial and boundary conditions (5).

4. Conclusion

The formulation and solution for the problem of rotating Casson fluid in moving channel disk have been obtained by using the Laplace transform method. The analysis results on embedded parameters for velocity profiles have been plotted graphically in figure 2 until figure 4. Therefore, the main conclusions from this study are

(i) Both primary and secondary velocities increase with increasing t.

(ii) Primary velocity decreases while secondary velocity first increases and then decreases when γ is increased.

(iii) For larger value of ω, the primary velocity is decreasing but increase for secondary velocity.

5. Acknowledgments

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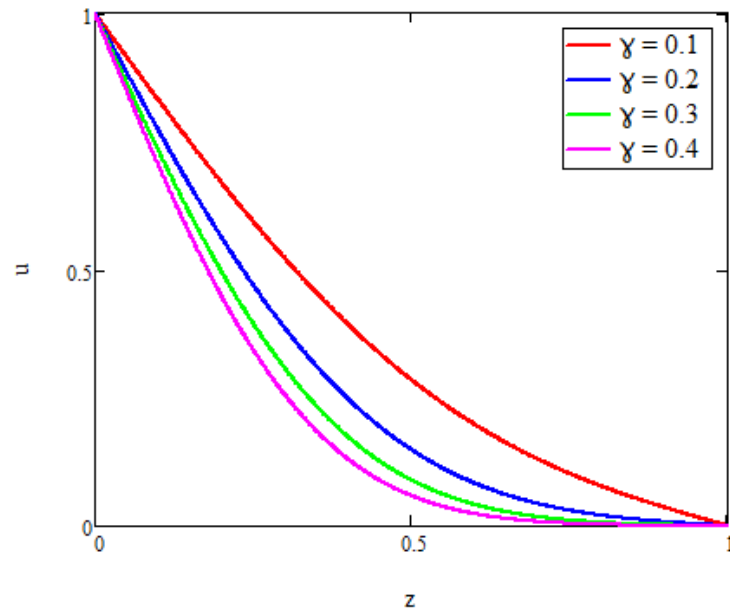
(a)
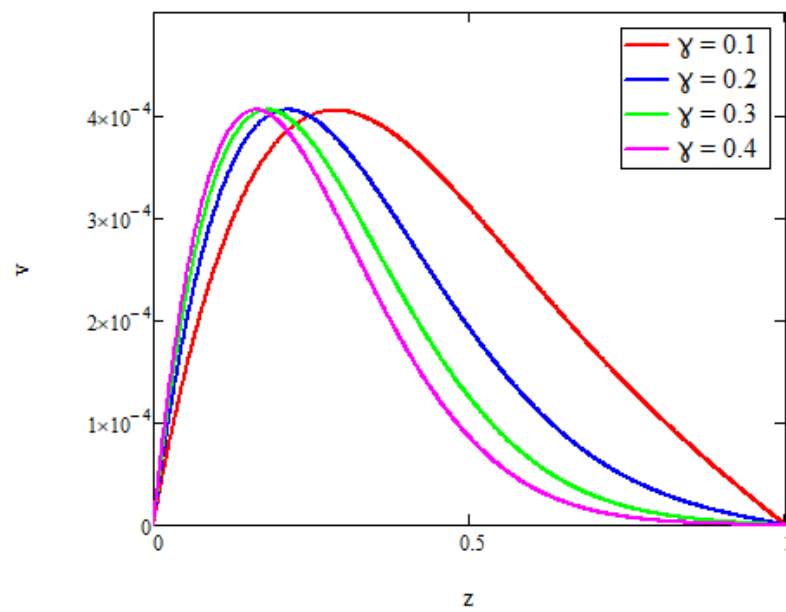
(b)

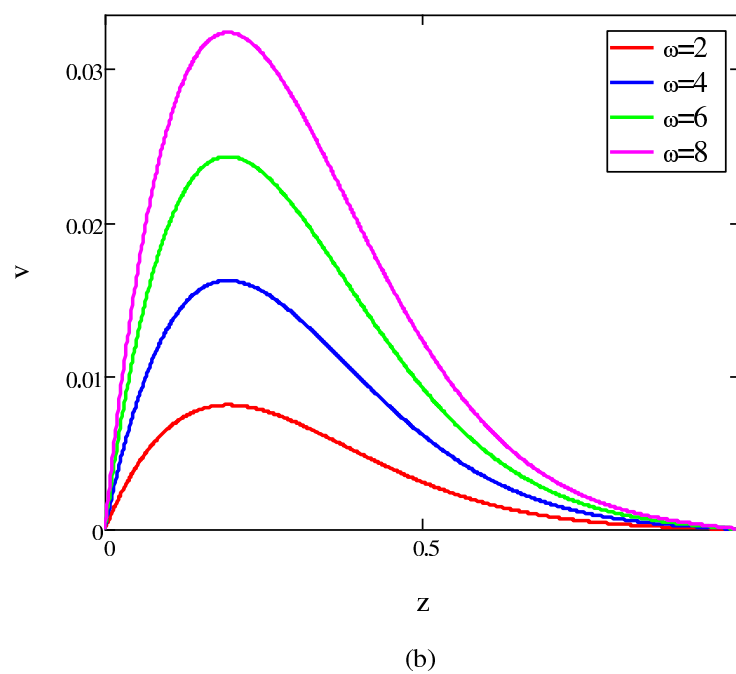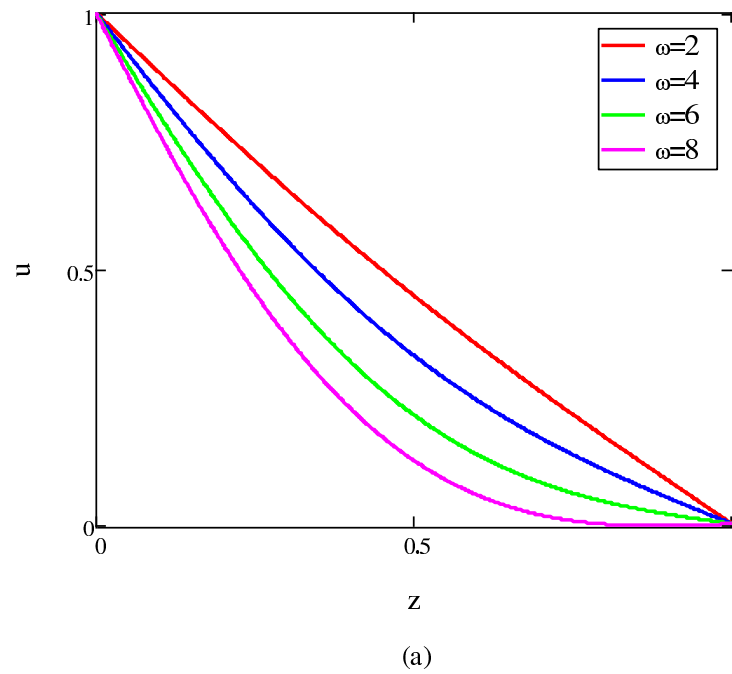**Figure 2.** Primary **(a)** and secondary **(b)** velocities profiles for different values of γ.

(a)

(b)

Figure 3. Primary **(a)** and secondary **(b)** velocities profiles for different values of ω.
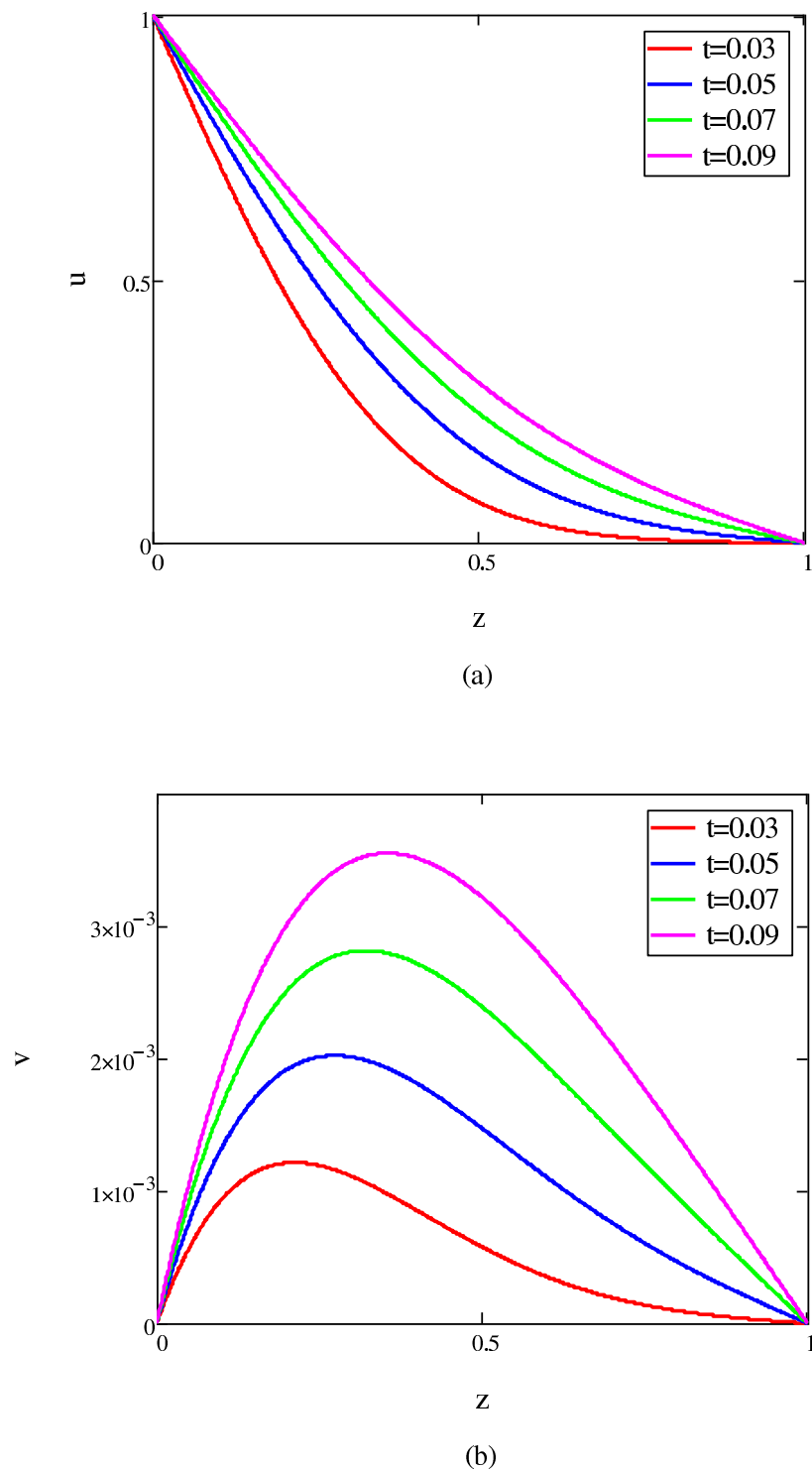
(a)

(b)

Figure 4. Primary **(a)** and secondary **(b)** velocities profiles for different values of t.