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MHD Effect on Nano-Maxwell fluid with Cobalt nanoparticles passing over a Plate with Porous medium

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Abstract: This article presents the study on the flow of Nano-Maxwell fluids with cobalt nanoparticles over a vertical plate in a porous medium. Free convection flow and magnetic effects are influencing the flow. The present problem is modelled in terms of PDE's with

boundary conditions. Non-dimensional variables are interposed to transform the governing equations into dimensionless form. The resulting equations are tackled via Laplace transform technique. Exact solutions are evaluated for velocity and temperature. These solutions are significantly controlled by some parameters involved. The present outcomes are compared with

previous published results and briefly explained. Increasing the amount of cobalt nanoparticles in the Maxwell fluid causes enhancement in the thermal conductivity, the viscosity forces tend to strengthen as the fluids becomes denser.

1. Introduction

Nanofluids are employed as heat transfer fluid due to their favorable thermal properties and hence are the source of attention for the researchers. Nanofluids are used in electric devices, power generators, Bio-medical industry for sensing and imaging purposes, transportation and medical treatments of many diseases. A very useful application of nanofluids is there in cancer imaging and drug delivery. Iron-based nanofluids are used to pass the drug particles to the tumor cells. Re-engineering the nanoparticles properties, they can be applied in almost any kind of disease treatment. Magnetite and Diamond particles are promising to destroy the undesired tissues in Nano-Cyrosurgery using freezing technique [1]. A review on nanofluids was carried out by Verma and Tiwari [2] regarding the progress of nanofluids implementations in solar collectors. They surveyed the literature of nanofluids on the basis of thermal conductivity enhancement, variation of specific heat capacity and viscosity with varying factors of nanoparticles such as size shapes and volume fractions. Nanofluids heat transfer flow for non-Newtonian fluids researches can be found in [3-6].

Exact solution for such problems was acquired by many researchers for different fluids and nanoparticles via Laplace transform technique [7]. Turkyilmazoglu [8] obtained the exact solution (Unique and double) for the problem of slip flow in nanofluids over a shrinking sheet.

Besides the analytical studies in the above area, a plenty of researches are embedded in the literature on numerical analysis of nanofluids via different techniques [9-13]. Flow of ferrofluid in a square cavity was investigated by Javed et al. [14] via Finite element method. They analysed that the strength of streamline circulation was maximized with Prandtl and Darcy numbers.

The above literature and our previous problem on Maxwell nanofluid Aman et al. [15] needs to be extended for oscillatory flow, porous media, magnetic effect and radiation effect. For the current analysis, the exact solution for oscillatory flow of Maxwell nanofluid with cobalt nanoparticles and kerosene oil is the main focus. The problem is tackled via Laplace transform method and Zakian technique for numerical Inverse Laplace transform. The results are plotted graphically using MathCAD and discussed.

2. Mathematical Formulation

Consider kerosene based Maxwell nanofluid with cobalt nanoparticles at rest over a vertical flat plate in $(x, y)-$ plane under free convection. At first, having constant wall temperature T_∞ both the plate and fluid are static. At time $t = 0^+$, the temperature of the plate is upgraded to a constant value T_w and touches a value T_∞. The fluid flows due to temperature gradient which produces buoyancy force. The governing equations are:

$$\rho_{nf}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=\mu_{nf}\frac{\partial^2 u}{\partial y^2}-\sigma_{nf}B_0^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)u(y,t)$$
$$-\frac{\mu_{nf}\phi}{k}u(y,t)+\left(1+\lambda_1\frac{\partial}{\partial t}\right)(\rho\beta)_{nf}\,g\left(T-T_\infty\right), \tag{1}$$

$$(\rho c_p)_{nf}\frac{\partial T}{\partial t}=k_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

The BC's are:

$$u(y,0)=0,\ u(0,t)=U_0\cos\omega t,\ u(\infty,t)=0,$$
$$T(y,0)=T_\infty,\ T(0,t)=T_w,\ T(\infty,t)=T_\infty. \tag{3}$$

Interpolating the non-dimensional elements:

$$u^*=\frac{u}{U_0},\ y^*=\frac{yU_0}{\nu},\ t^*=\frac{tU_0^2}{\nu},\quad \tau^*=\frac{\nu\tau}{\mu U_0^2},\ \theta=\frac{T-T_\infty}{T_\infty}, \tag{4}$$

After acquiring the non-dimensional partial differential equations, we employ the Laplace transform to equations (1)-(3):

$$A_1(1+\lambda q)q\bar{u}(y,q)=A_2\frac{\partial^2\bar{u}(y,q)}{\partial y^2}-A_3(1+\lambda q)M\,\bar{u}(y,q)$$
$$-A_2k\,\bar{u}(y,q)+A_4(1+\lambda q)Gr\,\bar{\theta}(y,q), \tag{5}$$
$$\bar{u}(0,q)=\frac{q}{q^2+\omega^2},\ \bar{u}(y,q)\to 0, y\to\infty.$$

$$A_6\frac{\partial^2\bar{\theta}(y,q)}{\partial y^2}-A_5\Pr q\bar{\theta}(y,q)+Q\bar{\theta}(y,q)=0.$$
$$\bar{\theta}(0,q)=\frac{1}{q},\ \bar{\theta}(y,q)\to 0\ at\ y\to\infty. \tag{6}$$

Where

$$\lambda=\frac{\lambda_1 U^2{}_0}{\nu_f},\ Gr=\frac{\nu_f g\beta_f(T_w-T_\infty)}{U^3{}_0},\ \Pr=\frac{(\mu c_p)_f}{k_f},\ M=\frac{\sigma_f B_0^2\nu}{\rho_f U^2{}_0},$$
$$\gamma=\frac{\mu_f\varphi\nu}{\rho_f k_1 U^2{}_0},\ A_1=(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right),\ A_2=\frac{1}{(1-\phi)^{2.5}},$$
$$A_3=\sigma_f\left[1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)}\right], A_4=(1-\phi)+\phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right), \tag{7}$$
$$A_5=(1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f},\ A_6=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s-2k_f)+\phi(k_f-k_s)},$$

The solutions of the equations (5) and (6) using transformed boundary conditions are:

$$\bar{\theta}(y,q)=\frac{1}{q}\exp\left(-y\sqrt{b_0}\sqrt{q-b_2}\right), \tag{8}$$

$$\bar{u}(y,q)=\frac{q}{q^2+\omega^2}\exp\left(-y\sqrt{\{(\lambda q+1)(\beta_0 q+\beta_1 M)+k\}}\right)$$
$$+\frac{\beta_2(1+\lambda q)Gr}{q\left[b_1q+\{(\lambda q+1)(\beta_0 q+\beta_1 M)+k\}\right]}\exp\left(-y\sqrt{\{(\lambda q+1)(\beta_0 q+\beta_1 M)+k\}}\right) \tag{9}$$
$$-\frac{\beta_2(1+\lambda q)Gr}{q\left[b_1q+\{(\lambda q+1)(\beta_0 q+\beta_1 M)+k\}\right]}\exp\left(-y\sqrt{b_0}\sqrt{q-b_2}\right).$$

Where

$$\beta_0=\frac{A_1}{A_2},\ \beta_1=\frac{A_3}{A_2},\ \beta_2=\frac{A_4}{A_2},\ b_0=\frac{A_5Pr}{A_6},\ b_1^2=\frac{QPr}{A_6},\ b_2=\frac{b_1^2}{b_0}. \tag{10}$$

Using the inverse Laplace transform and convolution theorem, equations (8) and (9) emits:

$$\theta(y,t)=erfc\left(\frac{y\sqrt{b_0}}{2\sqrt{t}}\right)\exp(b_0t). \tag{11}$$

$$u(y,t)=-\int_0^t \cos\omega\left(t-\left(\beta_0+\lambda\beta_1 M\right)\right)\left[\begin{array}{l}\exp\left[-y\left(\dfrac{\beta_0+\lambda\beta_1 M}{\sqrt{\lambda\beta_0}}\right)\right]+\\ \dfrac{y\sqrt{\lambda\beta_0^2}\sqrt{\left(\beta_0+\lambda\beta_1 M\right)^2}}{2\lambda\beta_0}\exp\left(-\dfrac{\beta_0+\lambda\beta_1 M}{\lambda\beta_0^2}(t-s)\right)\end{array}\right]ds \tag{12}$$
$$+\beta_2 Gr\int_0^t\left[\cos\omega\left(t-\left(\beta_0+\lambda\beta_1 M\right)\right)\right] erfc\left(\frac{y\sqrt{b_0}\exp\left(b_2(t-s)\right)}{2\sqrt{(t-s)}}\right)ds.$$

3. Nusselt number

The Nusselt number is acquired as in [3]:

$$Nu=-\frac{k_{nf}}{k_f}\frac{\partial\theta}{\partial y}\bigg|_{y=0}=A_6 k_f Re_x\left[\frac{\sqrt{b_0}e^{b_2 t}}{\sqrt{\pi t}}+\sqrt{b_0}\,erfc\left(\sqrt{b_2 t}\right)\right]. \tag{13}$$

Where Re_x is the Reynold's number: $Re_x=\dfrac{xU_0}{\nu_f}$.

4. Limiting Case

The limiting case for the present problem is acquired by taking $M=0,\ Q=0$ and $k=0$, we get the problem reduced to the previous published result in Aman et al. [15]:

$$A_1(1+\lambda q)q\bar{u}(y,q)=A_2\frac{\partial^2\bar{u}(y,q)}{\partial y^2}+A_4(1+\lambda q)Gr\bar{\theta}(y,q), \tag{14}$$

$$\bar{u}(y,q)=\frac{-b_1^2(\lambda q+1)}{q^2\left[(\beta_0\lambda q)+(\beta_0-b_1)\right]}\exp\left(-y\sqrt{\beta_0 q(\lambda q+1)}\right)+\frac{b_1^2(\lambda q+1)}{q^2\left[(\beta_0\lambda q)+(\beta_0-b_1)\right]}\exp\left(-y\sqrt{b_1 q}\right), \tag{15}$$

$$u(y,q)=-\frac{b_1^2}{\beta_0\lambda}\int_0^t h_1(t-s)f(y,s)ds+\frac{b_1^2}{\beta_0\lambda}\int_0^t h(t-s)g(y,s)ds. \tag{16}$$

Where

$$F(y,q)=\exp\left(-y\sqrt{\beta_0 q\left(\lambda q+1\right)}\right)=\exp\left(-y\sqrt{\beta_0\lambda}\sqrt{\left(q+\frac{1}{2\lambda}\right)^2-\left(\frac{1}{2\lambda}\right)^2}\right),$$
$$H(y,q)=\exp\left(-y\sqrt{\beta_0\lambda}\sqrt{q}\right),\ G(q)=\frac{(\lambda q+1)}{q^2\left(q+\dfrac{\beta_0-b_1}{\beta_0\lambda}\right)}. \tag{17}$$

And their Laplace Inverses are given as

$$h_1(y,t)=L^{-1}\left\{H(y,q)\right\}=\begin{cases}\dfrac{y\sqrt{\beta_0\lambda}\exp\left(-\dfrac{y^2\lambda\beta_0}{4t}\right)}{2t\sqrt{\pi t}}; & y>0,\\ \delta(t); & y=0\end{cases}$$

$$\begin{aligned}f_1(y,t)&=\left[h_1(y,t)+\frac{1}{2\lambda}\int_0^t h_1(y,z)\frac{z}{\sqrt{t^2-z^2}}I_1\left(\frac{1}{2\lambda}\sqrt{t^2-z^2}\right)dz\right]\exp\left(-\frac{1}{2\lambda}t\right)\\&=\left(\begin{aligned}&\frac{y\sqrt{\beta_0\lambda}}{2t\sqrt{\pi t}}\exp\left(-\frac{y^2\beta_0\lambda}{4t}-\frac{1}{2\lambda}t\right)\\&+\frac{1}{2\lambda}\exp\left(-\frac{1}{2\lambda}t\right)\int_0^t\frac{y\sqrt{\beta_0\lambda}}{2z\sqrt{\pi z}}\exp\left(-\frac{y^2\beta_0\lambda}{4z}\right)\frac{z}{\sqrt{t^2-z^2}}I_1\left(\frac{1}{2\lambda}\sqrt{t^2-z^2}\right)dz\end{aligned}\right),\end{aligned}\tag{18}$$

$$\begin{aligned}&f(y,t)=L^{-1}\left\{F(y,q)\right\}=\begin{cases}f_1(y,t); & y>0\\ \delta(t); & y=0\end{cases},\\&g(t)=\frac{a\lambda-1}{a^2}H(t)+\frac{1}{a}t+\frac{1-a\lambda}{a^2}\exp(-at),\ \text{where}\quad a=\frac{\beta_0-b_1}{\beta_0\lambda}.\end{aligned}\tag{19}$$

$$h(y,t)=L^{-1}\left\{\exp\left(-y\sqrt{b_1}\sqrt{q}\right)\right\}=\begin{cases}\dfrac{y\sqrt{b_1}\exp\left(-\dfrac{y^2b_1}{4t}\right)}{2t\sqrt{\pi t}} & ;\ y>0,\\ \delta(t); & ;\ y=0\end{cases}\tag{20}$$

5. Results and discussion

The problem of magnetic effects on the flow of cobalt Nano-Maxwell fluid has been explored and tackled for various volume fraction values using Laplace transform technique.

The thermo-physical properties of cobalt nanoparticles and Kerosene oil are given in Table 1 as in Javed et al. [14]. These values have been used in coding while plotting the graphs. The quantity of physical interest of Nusselt number is established. The effect of Maxwell fluid parameter, λ and volume fraction, ϕ on velocity is depicted in Fig. 1.

Fig. 1 depicts the impact of volume fraction, ϕ of cobalt particles on velocity for range $0.01 \le \phi \le 0.04$. The velocity is found to be a decreasing function of volume fraction, ϕ. Increasing the amount of cobalt nanoparticles in the Maxwell fluid causes enhancement in the thermal conductivity, the viscosity forces tend to strengthen as the fluids becomes denser. Thus, the fluid velocity shows decreasing plots.

The variation of velocity in this case due to the varying values of Maxwell fluid parameter, λ has a tremendous importance as it effects the entire physical structure of the fluid. Fig. 2 shows the alteration of the flow of Maxwell nanofluid with λ. Gradually with a consistent behavior, the velocity increases for increasing various values of λ. This plot is mapped for the range, $0.01 \le \lambda \le 0.06$ and other ingrained constant parameters, $Gr = 1.6, M = 0.02, \phi = 0.03, k = 0.2$.

For the verification of present acquired results, the velocity equation [12] plot is compared with that of [15]. The result is found to be in consistency with previously established result which is shown in Fig. 3. This comparison is carried out for velocity solution with $M = 0, k = 0.00001$.

Table 1. Thermo-physical properties of cobalt and Kerosene oil. (Javed et al. [14]).

Model	$\rho(kg/m^3)$	$c_p(kg^{-1}/k^{-1})$	$\beta(k^{-1})\times10^{-5}$	$k(Wm^{-1}k^{-1})$
Cobalt	8900	420	21	100
Kerosene oil	997.1	4179	1.3	0.163

6. Conclusions

In this research paper, exact solution for flow of cobalt based Maxwell nanofluid is acquired using Laplace transform method. Increasing magnetic effects tends to decrease the flow velocity with a significant variation. Increasing values of Maxwell fluid parameter enhances the fluid velocity, the variation is more significant at $y = 1$. The results are in good agreement with previous results [15].

Figures

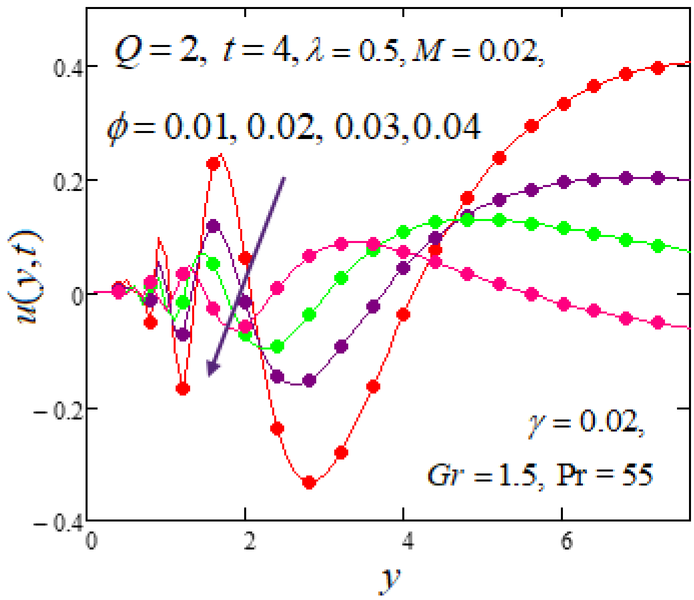

Figure 1: Velocity profiles for different values of volume fraction.

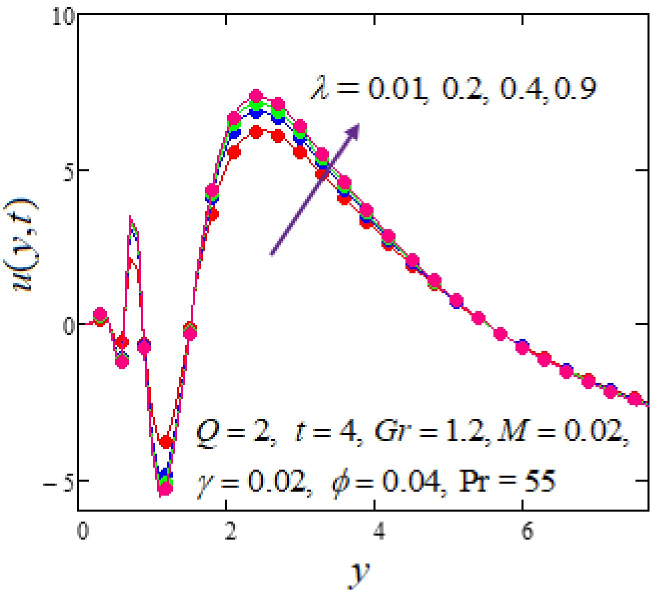

Figure 2: Velocity profiles for different values of Maxwell parameter.

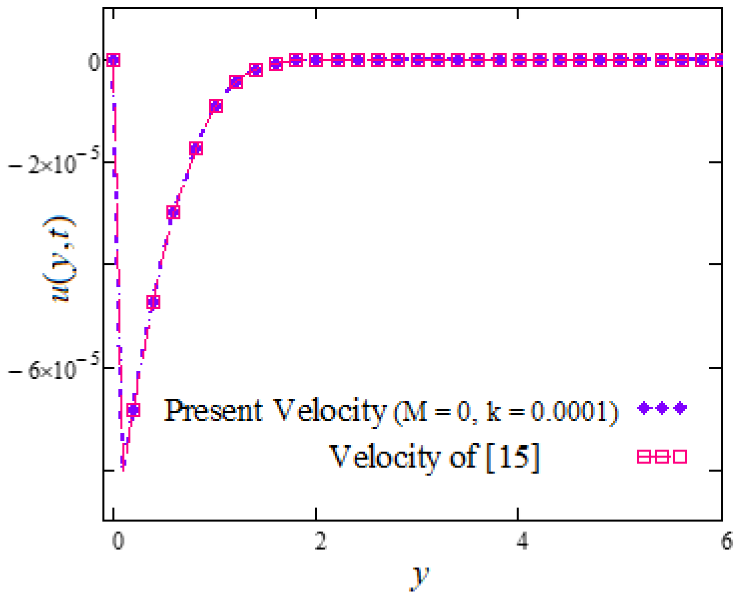

Figure 3: Comparison of present results with previous result [15].

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