PROBABILISTIC ANALYSIS OF THE HUMAN VERTEBRA UNDER COMPRESSIVE LOADING

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Abstract—The objective of this study is to determine the probability of injury of human crack vertebra condition subjected to compressive loading. The model has been used in this study was reconstructed from image processing and develop using SolidWorks. Three dimensional finite element model of human lumbar was conducted using Ansys software. In this work, all the model components were meshed using the tetrahedral solid element (SOLID186). In order to simplify the model, all the spinal components were modeled as an isotropic, elastic material and symmetry model. The model failure was occurred when the stress intensity factor (SIF) of the bone exceeds the fracture toughness. Biological structures, as well as a vertebra, exists a lot of related uncertainties and should not be solved by deterministic analysis. A Monte Carlo Simulation (MCS) technique was performed to conduct the probabilistic analysis using the built-in ANSYS parametric design language (APDL) modules. The results observed that the highest stress was found 2% on the adjacent pedicle to create the weakness area and probability of failure for cracked condition. Therefore, pedicle was become the most critical area to be emphasize. Despite, any flaws exist on the model such as crack will give a huge effect to the results especially fracture. Hence, the current study was very useful to investigate how the bone toughness and bone characteristics capable of sustained compressive loading in terms of the probabilistic approach.

Keywords:Probabilistic, Finite element model, Lumbar spine, Monte Carlo, Crack

I. INTRODUCTION (HEADING 1)

Biological structure such as a vertebra has many aspects of biomechanics and orthopedics parameters which variable in nature. These uncertainties are usually described by randomness, fuzziness, and intervals [1]. Normal bones in humans and animals are known to exist small cracks in vivo [2, 3]. Due to this discrepancy, chances of bone failure and fracture are higher. Obviously, the prediction of crack shape is based on the SIF solution [4]. The uncertainties of fracture response parameters are including the crack geometry, loading distribution, material properties and clinical outcomes [5]. Any uncertainty in these parameters is then accounted for by a safety factor. Conversely, this approach may be unacceptable for structures due to the maximum potential of the structure is not realized, and reliability of the structure is never quantified [6]. Moreover, significant uncertainty in the response of the system due to the inherent variability of the parameters and the degree of uncertainty increases as more parameters are considered [7].

In the finite-element analysis (FEA), the stresses of the structure are determined using fixed values for the fracture response parameters that control the behaviour of the structure [7]. This deterministic analysis was developed and validated to neglect the existence uncertainty in the systems. However, a purely deterministic approach provides an incomplete picture of the reality [8]. Therefore, probabilistic analysis was proposed to account for the uncertainty of fracture response parameters. This probabilistic approach can be also considered an extension of previous deterministic studies [9, 10].

The most common and traditional probability method is Monte Carlo simulation (MCS). Probabilistic analysis allows the effects of uncertainty in the parameters to be included explicitly in the analysis and the resulting statistical variation in the system response. Probabilistic analysis considers the potential interaction effects between parameters by perturbing multiple input parameters in each trial [5]. The analysis is modeled accurately, the effect of structure integrity can be observed in actual behavior [11]. Each random variable is sampled underlying its distribution and probability of failure is determined by repeating the deterministic analysis.

The ideal of surface crack geometry in bone is approximately semi-ellipse [12]. However, some element constraint such as irregular cracks, vertebra, and material properties need to modify the following crack shape [4]. Obviously, the prediction of crack shape is based on the stress intensity factor (SIF) [11].

The aims of the present study were to determine the probability of failure based on SIF of crack shape of a vertebra under compressive loading. Sensitivity analysis becomes a

significant tool to determine which crucial parameters relative to failure condition.

II. METHODOLOGY

A. Finite element model

The finite element model was created using with 20-nodes tetrahedral elements (SOLID186). This element is a higher order 3D type that exhibit quadratic displacement behavior and well suit to modeling irregular meshes.

In nature, bone is a nonlinear, inhomogeneous and anisotropic material and varies among the boundary regions between cortical and cancellous bone [13, 14]. However, most studies performed in this area were based on the assumption that bone material was isotropic and inhomogeneous distribution of material properties due to its simplicity [15]. Therefore, this study was conducted on linear isotropic and assume that the whole vertebra considered as cortical bone properties. Material properties that have been used in this study were summarized in Table 1.

TABLE I. MATERIAL PROPERTIES

Description	Parameters	Mean	COV[a]	Distribution
Young Modulus	YOUNG	12GPa	0.21	Lognormal
Poisson ratio	PSSNRAT	0.3	±0.017	Uniform
Body force	FORBDY	414 N	0.1	Normal
Facet force	FORFCT	46 N	0.1	Normal
Body area	AREBDY	1298 mm2	0.1	Lognormal
Facet area	AREFCT	166 mm2	0.1	Lognormal
Crack radius	R	3.0 mm	0.1	Normal
Fracture	K	1.46 MPa.mm2	0.19	Lognormal

a. coefficient of variation

The vertebrae are comprised by six components. There are vertebral body, spinous process, transverse process, lamina, pedicle, and facet joints. The boundary condition of vertebra anatomy was shown in Figure 1 while the unilateral crack was developed on the left pedicle. Pressure force of loading was subjected to the superior at the vertebral body whereas fixed in the inferior side.
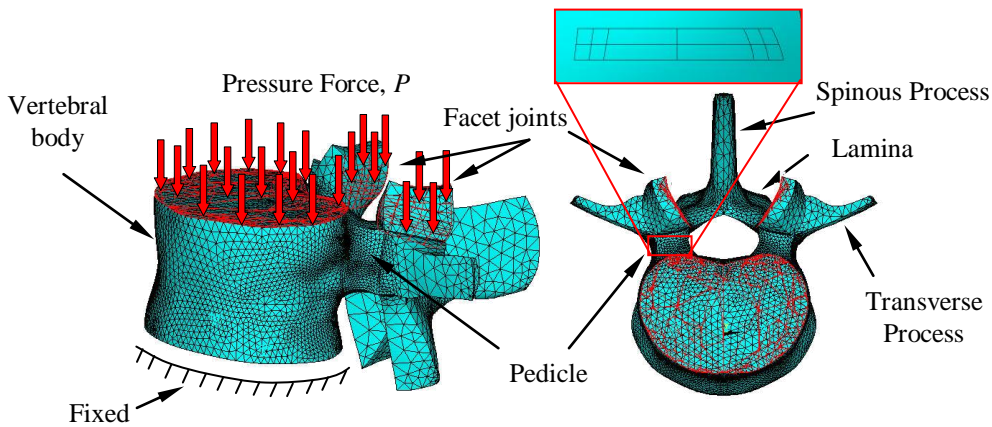
Superposition principle was employed to estimate the SIF, in order to avoid modeling cracks on the model. This technique required the crack develop before attached together with the original vertebrae. The area of the critical region is refined using finer meshes so that the reliable results are necessarily produced especially in the vertebra body.

Consider a mixed mode problem for this case, uncertain mechanical and geometric characteristics for cracked structure that is subject to random loads. Let random variables, X that affected from other parameters are given by

$$X = \{R, E, \upsilon, W, K_c\} \quad (1)$$

where R is crack radius, E is the Young modulus, υ is Poisson ratio, W total body weight and Kc is fracture toughness for this model.

B. Fracture toughness

For simple boundary value problems with idealized crack geometry, the SIF or fracture toughness may be expressed as

$$K = f \cdot \sigma \sqrt{\pi R} \quad (2)$$

where f is the correction factor, σ is applied stress and R is the crack radius. Alternatively, fracture toughness can be derived in terms of the strain energy release rate, G, defined as the change in potential energy per unit increase in crack area. For linear elastic release rate (LEFM), the relationship between strain energy and SIF were close. It can be expressed in terms of the mode I, II, and III as follows:

$$G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu} \quad (3)$$

where μ is the shear modulus, υ is Poisson ratio and E' is defined as follows:

$$E' = E(\text{Young Modulus}) \text{ in plane stress}$$
$$E' = E/(1-\upsilon^2) \text{ in plane strain} \quad (4)$$

The crack was dividing by three type modes of loading condition. Mode I is represented the crack opening, mode II is crack sliding and mode III is tearing. Typically, SIF is given by the subscript such as K_I, K_{II}, or K_{III}. In order to make SIF significant to the real condition, three modes of loading must be considered. However, this complex shape model limits the evaluation. Hence, the assessments only refer to the most significant fracture on the structure which is mode I.

Figure 1 Boundary condition of vertebra

Further modification of this superposition technique was required to obtained the effective SIF method defined in Eq. (5)

$$K_{eff} = \sqrt{K_I^2 + K_{II}^2 + \frac{K_{III}^2}{1-\upsilon}} \qquad (5)$$

where K_{eff} is the combination between three modes of crack loading on the model.

C. *Probabilistic Algorithm*

Uncertainties of parameters were determined which affected to the stress intensity factor. Denote by X and n number of total samples with components $X_1, X_2, \ldots, X_n$ characterizing the load, crack geometry and material properties. By formulating a performance function $g(X)$, which is a function of random variables and typically defined as

$$g(X) = R(X) - S(X) \qquad (6)$$

where $R(X)$ is the limiting strength of material or resistance and $S(X)$ is applied stress. The probability of failure, P_f is given by

$$P_f = P(g(X) \le 0) \qquad (7)$$

It is the likelihood that the stress exceeds the strength, whereas the reliability of the structure is the converse $P_s = 1 - P_f$. Nevertheless, probability of failure may be expressed as

$$P_f = N_f / N_T \qquad (8)$$

where N_f is the number of the sample will be fail divide by the total number of samples, N_T.


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┌─────────────────────────────────────────────────┐
│  Determine the fracture response parameters, X   │
└─────────────────────────────────────────────────┘
                        ↓
┌─────────────────────────────────────────────────┐
│       Generate the parameters randomly, n        │
└─────────────────────────────────────────────────┘
                      i = 1
┌─────────────────────────────────────────────────┐
│      Solve stress analysis and SIF using FEA     │
└─────────────────────────────────────────────────┘
                        ↓
┌─────────────────────────────────────────────────┐
│        Arrange the deterministic information     │
└─────────────────────────────────────────────────┘
                        ↓
┌─────────────────────────────────────────────────┐
│   Probabilistic of failure, Pf = Nf / NT         │
└─────────────────────────────────────────────────┘
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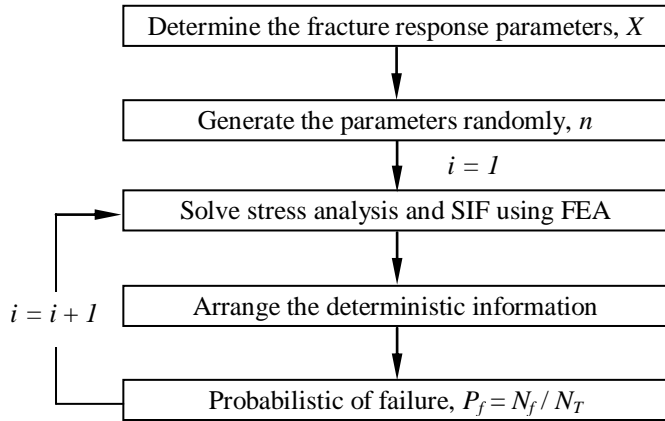i = i + 1

Figure 2 Algorithm of the automatic probability of failure

The probabilistic algorithm in Figure 2 demonstrates the steps have been used to develop programming code using the built-in modules in ANSYS. The most commonly applied probabilistic model is the Monte Carlo method which involves randomly generating values for each variable according to its distribution and then predicting the distribution of performance through repeated trials [9]. This method is computational expensive as the accuracy due to dependent on the number of samples. For this study, number of samples, n equal to 100 after considering the irregular surface, number of elements, and size of crack.

Latin hypercube sampling (LHS) was used to generate the parameters according to its distribution type. It also requires fewer sampling point and more accurate rather than direct Monte Carlo sampling technique (DMCS). Besides it, LHS will decrease time consuming for analyze the data since the number of samples decrease.

D. *Sensitivity Analysis*

In designing the sensitivity parameter, output response parameter was needed to determine first. Requirements of that parameter due to the model failure are based on strength and fracture toughness of the vertebra. So, therefore, the most sensitive parameter was a measure from small changes the input parameters gives huge effect to the output parameter. This determines from gradient correlation of the scatter plot. Commonly, Spearmann and Pearson rank correlation is the most popular used in study the monotonic relationship.

Relative sensitivities are commonly referred to as probabilistic sensitivity factors, α and the change in safety index, β with respect to the standard normal variate, u. The probabilistic sensitivity factor was defined as

$$\alpha_i = \frac{\partial \beta}{\partial u_i} = \frac{\partial p}{\partial u_i} \frac{\partial \beta}{\partial p} \qquad (9)$$

where p equal to a specific probability level.

III. RESULTS & DISCUSSION

The maximum stress in Figure 3 was appeared at the left superior at the pedicle with 4.5 MPa Von mises stress. It is due to the crack exist, and moment effect occurred on the same area. The critical fracture area is commonly on the crack tip due to stress fracture. According to the [16], the maximum stress is located in the inferior part of the pedicle and facet surface. It is convinced that the pedicle is the most critical region in vertebra structure. That value 4.5 MPa is greater than the yield stress which is obtained from [17]. From this observation, the failure envisaged occurred in this stage. However, yield assessment did not account for the plastic deformation in crack tip, and SIF will take place the evaluation of analysis.

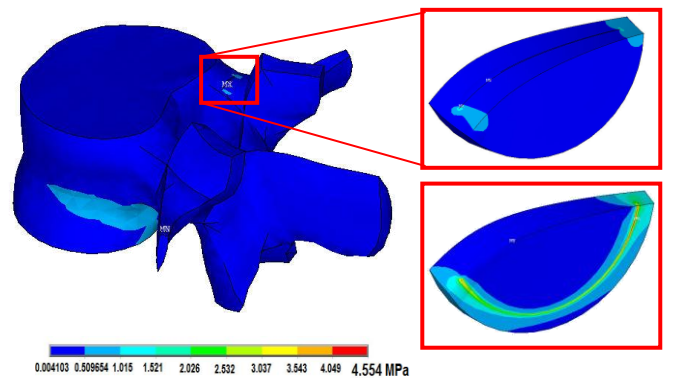0.004103 0.509654 1.015 1.521 2.026 2.532 3.037 3.543 4.049 4.554 MPa

Figure 3: Von Mises stress on crack tip

Furthermore, the deterministic analysis of SIF evaluation indicates that the crack tip occurred stress fracture. Mode I become the greatest crack loading since the shape and loading

conditions are representing to the crack opening. The SIF effective value is 0.532 MPa.m1/2 while the fracture toughness is 1.46 MPa.m1/2. Despite, it shows that the value still not exceeded the critical fracture toughness and assumed the structure is not failed. Nevertheless, the uncertainties are affected the vertebra structure in random variables. The probabilistic analysis was performed to determine the probability of failure based on deterministic analysis. Therefore, Figure 4 illustrated the probability of failure of crack mode I based on maximum SIF.
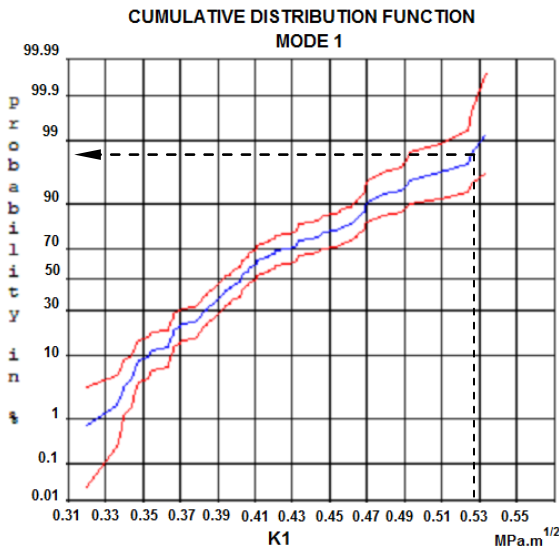
Figure 4 Probability of failure for Mode I

The probability for maximum value of SIF is representing by projected line from 0.525 MPa.m$^{1/2}$ reflects to the 98% of probability in percent. The graph shows that the value greater than the 0.525 MPa.m$^{1/2}$ is referring 1-0.98=0.02. It is meant the probability of failure for this model is 2%. From this result, conclude that the uncertainties inherent the biological structure is to affect the vertebra failure. However, all parameters related to the uncertainty still cannot determine by this figure. Therefore, Figure 5 indicates the sensitivity parameter analysis based on Spearman correlation value.
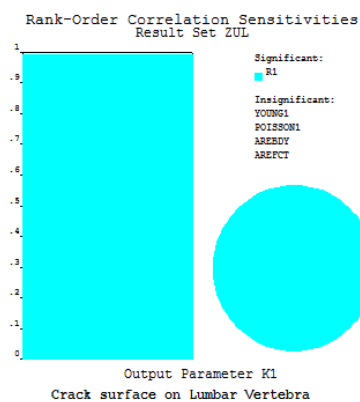

Figure 5 Sensitivity parameter analysis

The sensitivity parameter can be determined by using Spearmann correlation with monotonic relationship. From figure 5, the most significant parameter for the fracture toughness evaluation is crack size. Besides it, that parameter is almost equivalent to one. It symbolizes that only crack size sensitive and significant than the vertebra failure rather than others. Therefore, the crack parameter needs to be emphasized to control the vertebra failure conditions. The insignificant or unimportant random variables have eliminated from the sensitivity chart to improving the computational efficiency.

IV. CONCLUSION

Finite element analysis was performed to measure the SIF for vertebra structure in the unilateral crack. This study objective is achieved to determine the probability of failure of a vertebra under compressive loading. The uncertainties are reflected the structure to be fail depends on the three basic parameters in material properties, loading and geometry. All the uncertainties decided in this study are based on knowledge and experience. In deterministic, the SIF is indicated that the structure is not failed while in probabilistic analysis shows in contrast. Effect of uncertainties in biological structure is significant into representing the real-life phenomena. About 2% probability of failure obtained from probabilistic analysis with respect to the SIF 0.532 MPa.mm$^{1/2}$. Crack size becomes the most significant parameter affect the failure evaluation on sensitivity analysis. Therefore, emphasizing of this parameter is too crucial in order to avoid the system failure. This study is useful to investigate the inherent uncertainties and variations in biological structures.

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