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Theory and derivation of unsymmetrical rotor equation of the foil-air bearing rotor system and its validation with experiment data

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Abstract. This manuscript presents the theory and derivation of rotor equation using rigid body modelling used to validate the modal transformation approach for solving foil-air bearing rotor system. Numerical coefficient of the rotor equation for both methods only different less than 3%. The reasonable correlation in term of orbit's orientation with experimental data, has proved that the rotor was correctly modelled and adapted with simultaneous solution technique and modal model of foil structure.

1. Introduction

The recently developed simultaneous solution for solving nonlinear FAB-rotor system [1] has paved the way for other methods to be integrated into. This is proven when the author managed to develop a modal-based method for modelling the bump foil structure of FAB in the simulations of FAB-rotor problems [2] and neural network identification to replace the full numerical FAB model [3]. The latest method [2] requires the bump foil structure of the FAB to be modelled in FE software to obtain its modal properties wherein the bump interaction and inertia are considered. In this study, the capability of the method is adapted into a basic symmetrical FAB-rotor system which widely used in most of FAB's studies.

However, for validation purpose with experimental data, the rotor must be modelled according to the configuration of the test rig. The rotor equation has been modelled using modal transformation in which needs the *eigenvalues* and *eigenvector* of the rotor, obtained by finite element method and verified by impact testing. Alternatively, basic rigid body modelling approach is also applicable if the facilities to perform FE works are unavailable. This method needs accurate information of the rotor's dimensions, mechanical properties as well as the reaction and external forces act on the system before the center of gravity and the mass moment of inertia of the rotor are calculated. Since in the point of interest is at the FAB location, the actual mass of the rotor has to be shifted to equivalent mass at the FAB, denoted as $m_{eq,J}$.

2. Theory and derivation of the rotor equation using basic rigid body modelling

As can be seen in figure 1, the unsymmetrical test rotor is supported by a precision ball bearing at the left end and a FAB at the other end with the rotor disc overhanging. The self-aligning bearing at O (figure 1 (top)) is considered as a pivot and the bending stiffness of the coupling (between the left end of the shaft and the air motor) is neglected. Considering the operating speed range (up to around 30



krpm), the polar moment of inertia of the rotor and the gyroscopic effect is assumed to be negligible. The validation of the rotor’s model was done by impact tests [4]. The rotor equations derived from both methods are combined with the air film and foil equations of the FAB using the new foil modal model approach as in [2].

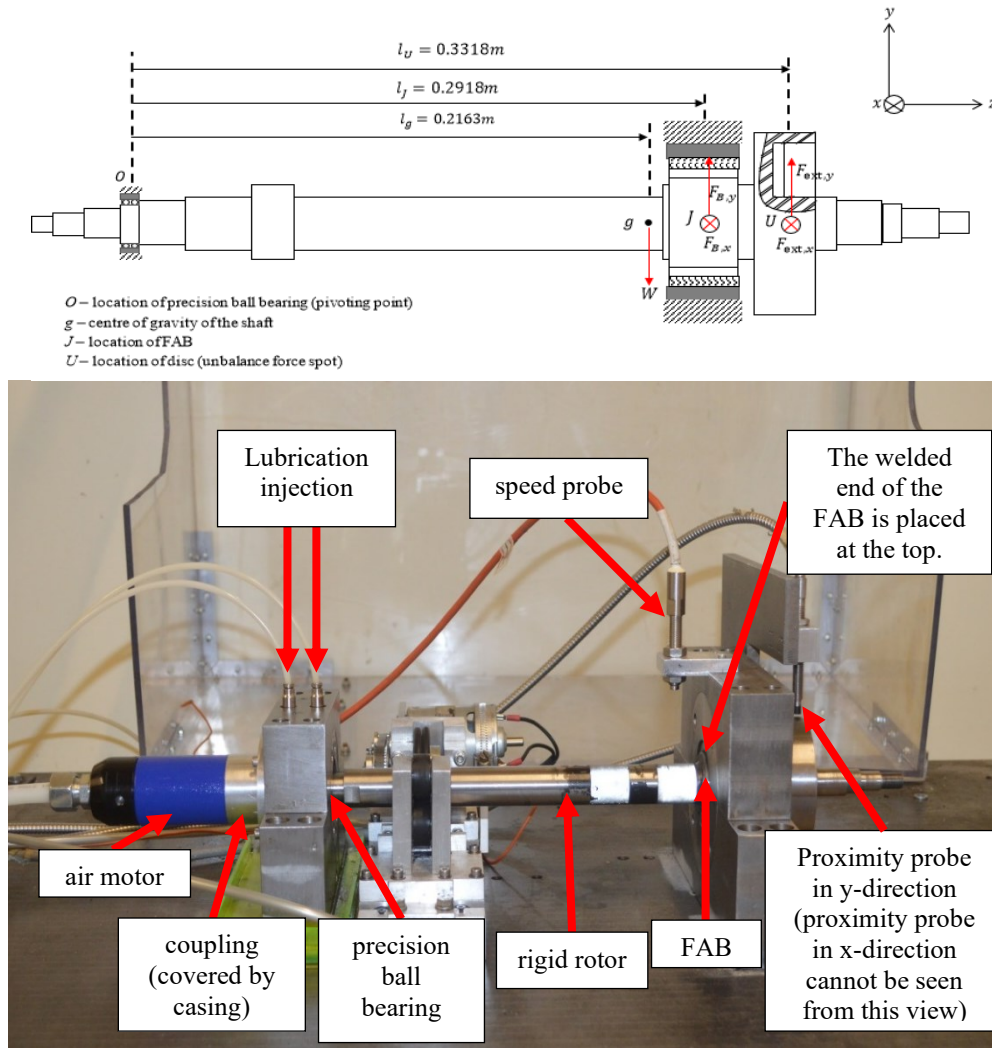


Figure 1. Test rig axial cut through (top) and FAB-rotor test rig configuration (bottom).

The equation of the test rotor as shown in figure 1 in x - and y - direction is given by:

$$F_{J,x}l_J + F_{U,x}l_u = I_O\ddot{\theta}_{x,z} \tag{1a}$$

$$F_{J,y}l_J + F_{U,y}l_u - Wl_g = I_O\ddot{\theta}_{y,z} \tag{1b}$$

where $F_{J,y}$, $F_{J,x}$ are hydrodynamic forces of the FAB, $F_{U,y}$, $F_{U,x}$ are the rotor unbalance forces at the disc, W is static load of the shaft, l_J , l_U are the distances of the FAB and disc from the pivot O , I_O is the mass moment of inertia of the shaft about a vertical or horizontal axis through the pivot point O , and $\ddot{\theta}_{x,z}$, $\ddot{\theta}_{y,z}$ are the angular accelerations in planes xz and yz respectively.

Dividing the eqs. (1a,b) by l_J results in the rotor equations as:

$$F_{J,x} + F_{U,x} \frac{l_u}{l_j} = \frac{I_O}{l_j} \ddot{\theta}_{x,z} \tag{2a}$$

$$F_{J,y} + F_{U,y} \frac{l_u}{l_j} - W \frac{l_g}{l_j} = \frac{I_O}{l_j} \ddot{\theta}_{y,z} \tag{2b}$$

It is known that $\dot{y}_J = l_J \sin \ddot{\theta}_{y,z} \approx l_J \ddot{\theta}_{y,z}$ and $\dot{x}_J = l_J \sin \ddot{\theta}_{x,z} \approx l_J \ddot{\theta}_{x,z}$ and note that $\frac{I_O}{l_j^2}$ term is defined as equivalent mass at point J , $m_{eq,J}$. Thus, the rotor motion can be expressed as:

$$m_{eq,J} \ddot{x}_J = F_{J,x} + F_{U,x} \frac{l_u}{l_j} \tag{3a}$$

$$m_{eq,J} \ddot{y}_J = F_{J,y} + F_{U,y} \frac{l_u}{l_j} - W \frac{l_g}{l_j} \tag{3b}$$

Converting eqs. (3a,b) to non-dimensional time and displacement with $\tau = \Omega t/2$, ()' denoting derivative of () with respect to τ , and $\tilde{y}_J = \frac{y_J}{c}$, $\tilde{x}_J = \frac{x_J}{c}$ yields:

$$\tilde{x}_J''(\tau) = 4 \left[F_{J,x} + F_{U,x} \frac{l_u}{l_j} \right] / m_{eq,J} c \Omega^2 \tag{4a}$$

$$\tilde{y}_J''(\tau) = 4 \left[F_{J,y} + F_{U,y} \frac{l_u}{l_j} - W \frac{l_g}{l_j} \right] / m_{eq,J} c \Omega^2 \tag{4b}$$

where Ω is the rotational speed of the rotor and c is the radial clearance between the shaft and the top foil. The mass moment of inertia I_O of the rotor about the vertical or horizontal axis through the pivot O (figure 1) is calculated based on two different shapes of the shaft: (i) uniform solid cylinder of radius R , length L and centre of mass distant d_O from point O ($\frac{1}{4}MR^2 + \frac{1}{12}ML^2 + Md_O^2$); (ii) thin circular hoop of radius $R_{average}$ with centre distant d_O from point O ($\frac{1}{2}MR_{average}^2 + Md_O^2$). With reference to figure 1, the total mass of the shaft is 1.94405 kg, l_g is 0.2163 m, l_j is 0.2918 m, l_U is 0.3318 m. I_O was calculated to be 0.1131 kgm² using the procedure above. Hence, the equivalent mass at J, $m_{eq,J} = \frac{I_O}{l_j^2} = 1.3282$ kg. With reference to eq. (4b), the term $W \frac{l_g}{l_j}$ is equal to $W_{eq,J}$ where $W_{eq,J}$ is the equivalent static load at J, 14.1367 N. Hence, the final rotor equation derived from basic rigid body modelling can be written as:

$$\tilde{x}_J''(\tau) = 4[F_{J,x} + 1.1367F_{U,x}]/1.3282c\Omega^2 \tag{5a}$$

$$\tilde{y}_J''(\tau) = 4[F_{J,y} + 1.1367F_{U,y} - 14.1367]/1.3282c\Omega^2 \tag{5b}$$

It has therefore been proven that the rigid body rotor equations derived from rigid body modelling agrees with the rotor equation obtained using modal transformation approach [5], with the numerical coefficients differing by less than 3%. For validation with experimental data, since the displacement probes are located at the disc (figure 1), the computed journal displacements (eqs. (5a,b) were converted to the dimensional displacements at the disc using the following formulae

$$x_U(\tau) = c \frac{l_U}{l_j} \tilde{x}_J(\tau) \text{ and } y_U(\tau) = c \frac{l_U}{l_j} \tilde{y}_J(\tau) \tag{6a,b}$$

3. Coupling of rotor equation with air film and foil

The rigid rotor eqs. (5a,b) presented in section 2.1 are then coupled with the FAB modal model and air film equation as follows:

$$\Psi' = \mathbf{g}_{RE}(\Psi, \mathbf{q}_f, \epsilon) \tag{7a}$$

$$\begin{bmatrix} \mathbf{q}_f' \\ \mathbf{q}_f' \end{bmatrix} = \begin{bmatrix} \mathbf{q}_f' \\ -\frac{4}{\Omega^2} [(\Omega/2)\mathbf{D}\mathbf{q}_f' + \Delta\mathbf{q}_f + \mathbf{H}_{y_f}^T \mathbf{f}_p(\Psi, \mathbf{q}_f, \epsilon)] \end{bmatrix} \tag{7b}$$

$$\begin{bmatrix} \epsilon' \\ \epsilon' \end{bmatrix} = \begin{bmatrix} \epsilon' \\ \frac{4}{m_{eq,J}C\Omega^2} \begin{bmatrix} F_{J,x}(\Psi, \mathbf{q}_f, \epsilon) + \epsilon m_{eq,J}u\Omega^2 \cos 2\tau \\ F_{J,y}(\Psi, \mathbf{q}_f, \epsilon) + \epsilon m_{eq,J}u\Omega^2 \sin 2\tau - 14.1367 \end{bmatrix} \end{bmatrix} \tag{7c}$$

where $m_{eq,J}$ refers to the coefficient and equivalent mass at the FAB location. The time history of the response to a rotational unbalance $m_r u$ was obtained by solving state-space from equations of eqs. (7a,b,c) in Matlab using the implicit time domain integrator *ode23s*. This gave the full nonlinear response. Further understanding for solving this simultaneous equation of the rotor equation with air film and foil modal model can be referred to [5].

4. Results and discussion

The orbit response (y_U and x_U) at the disc of the FAB-rotor system computed from simulation using both rotor’s modelling approaches and its comparison with experimental data are shown in figure 2. The predicted orbit responses were steady-state results obtained after integrating over 30 shaft revolutions at given shaft speed and the experimental orbit response is limited to one and two revolutions since it was taken from some specific speeds from *coasting down signal*. The orbit responses from both modeling approaches (rigid body modeling – *first column* & modal transformation – *second column*) are almost identical with little discrepancies. This remarkable similarity is strong enough to conclude that both modelling approaches are valid to be used.

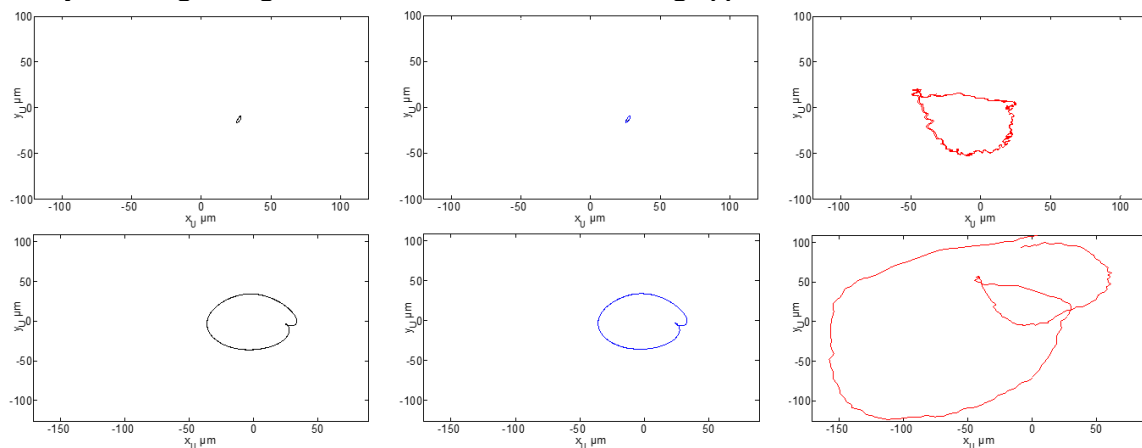


Figure 2. Steady-state orbit response of the FAB-rotor system obtained from simulation with rigid body modelling (first column), modal transformation modeling (second column) and experiment (third column) at low (4,015 rpm) and intermediate (15,031 rpm) speeds under 10 g mm unbalance.

In details, it is clearly seen that the predicted orbit response is an elliptical periodic orbit and operates within small amplitude and linear range. At 15,031 rpm the predicted orbit is not elliptical

and a frequency analysis [5] shows that it exhibits period-two motion. The experimental results show that at those three speeds, the pattern of the orbit response evolves in a manner similar to the theoretical results, with a significant difference in amplitude, approximately 3 to 4 times bigger. The remarkable similarity in the pattern of the predicted and theoretical orbits at 15,031 rpm should be noted. It is suspected that the difference in amplitude between the theoretical and experimental results is due to the run-out of the rotor at the measurement position (the disc U). It is noted that, even though the rigid body modelling does not require FE platform as in modal transformation approach, it is not valid to be used if the rotor speed of the FAB-rotor system exceeds the first bending moment, as the rotor starts to behave as a flexible structure (no longer rigid body). This issue can be handled by using modal transformation approach e.g. using modal coordinate instead of physical coordinate for the rotor equation, as it has been successfully applied by the author in his publication [4].

5. Conclusion

This manuscript has revealed and shown that the rotor system running on foil-air bearing can be modelled by rigid body and modal transformation approaches. Both approaches lead to the same rotor equation with less than 3% different in their numerical coefficient. The validation with experimental results that shows a reasonable correlation in term of orbit's orientation, has proved that the rotor equation was accurately modelled and adapted with simultaneous solution technique and modal model of foil structure. However, the advantage of the modal transformation approach over rigid body approach is it can consider a number of modes when the rotor no longer behaves as a rigid body at high speed operation. On the contrary, rigid body modelling is easy to be developed since it does not require FE platform but only requires knowledge on structural dynamic modelling for rotating structure.

Acknowledgments

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