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# A Progress on the Development of Mathematical Model on Two-Phase Flow over a Vertical Stretching Sheet

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# A Progress on the Development of Mathematical Model on Two-Phase Flow over a Vertical Stretching Sheet

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**Abstract**. The investigation on flow and heat transfer are important in the engineering applications, for example, extrusion of metal and polymer, cooling of electronic devices, heat exchanger and chemical processing equipment. Since the investigations on the real applications are costly and sometime hazardous, the study on mathematical model representing the fluid flow and heat transfer's problem are considered to overcome this limitation. This paper will present the progress on the development of mathematical model on non-Newtonian two-phase model where the fluid flow is studied together with the dust particles. The review will include the discussions on existing problems, its methodology as well as the significance outcomes. The main contributions on this paper is to present the research gap and the possible development for future investigations on the problem under the mathematical model of fluid flow and heat transfer.

*Keywords*: n*on-Newtonian fluid, two-phase model, flow and heat transfer*

## 1. Introduction

Two-phase flow problem has attracted considerable attention of many researchers in past few decades due to its potential in investigating the flow characteristics of binary mixture of fluid and solid particles, gas and liquid along with liquid and solids. Meanwhile, the advancement in fluid mechanics has led to the development of the innovative way in investigating the suspension of those particles in a fluid flow known as a two-phase flow model, which describes the behavior of fluid flow together with the dust particles. Two-phase flow industrial applications like transportation of petroleum, treatment of waste-water, emission of smoke from vehicles, piping of power plants and corrosive particles in mining generally involve the activities of the fluid-solid movement, [1]. Where the interaction of these phases is significant, in conjunction with these implementations, a number of research activities in the respective flow can be found in the literature with various circumstances, for instance in numerous geometries, boundary conditions, and type of fluid based. Non-Newtonian fluids are necessary for some industries (e.g., food and pharmaceutical); it operates in many activities such as circulation system, polymer processing, and electronic packing. These fluids are also useful in several fields (e.g., heat exchangers, ink-jet pointing, coating, lubricants, enhanced oil recovery, hot rolling, solar collectors, wet cement aggregates, flow in a shock absorber and gaseous diffusion, drilling muds, and the flow in journal bearings),[2-4] . Most clay particles such as emulsions, biological fluids, synthetic lubricants, polymers, paints, charcoal in water and food such as jam a few examples of the behavior of non-Newtonian fluids.

These types of shear -thickening and shear thinning methods, which are considered non-Newtonian fluids, react differently in heat transfer operations. Thus, highlighting the heat transfer characteristics of non-Newtonian fluids under different operating conditions provides the opportunity to promote heat transfer efficiency. In addition, the nature of non-Newtonian heat transfer can be more complex because of the presence of an additional particle phase. Among all means of promoting heat transfer can be applied, consequently, the heat transfer improvement methods may be applied to the end. For fluids, the aggregate of techniques to enhance the convective heat transfer and those to boost up the process of heat transfer, still an important topic of a considerable amount of research. Convective heat transfer has been applied in lots of various operations (e.g., Warmth exchangers, electromagnetic pump, solar creditors, fridges stills, and electronic tool cooling). Siddiqa et al.[5],investigated natural convection go with the flow of a two-phase segment dusty non-Newtonian fluid alongside a vertical surface, an important element arising from the fluid flow issues of the boundary layer is the thermal transfer of non-Newtonian fluid. Moreover, the exploration of other kinds of glide models of dusty non-Newtonian liquids has received considerable attention among researchers under different conditions. In literature, the fluid flow problem has been solved for various type of fluid without considering the dusty part (e.g.: viscous model [6], Erying Powel models, viscoelastic model[7], micropolar model [8] and there have been further studies for the simultaneous flow problems.

## 2. The type of fluid phase

Type of fluid flow can be classified into several different classifications, including those based on fluid flow regularity or flow surface such as liquid water or liquid water with water vapor single-phase and multiple phase fluid are considered two main categories for the fluid flow itself . Previous studies on fluid dynamics force at the flow of a single homogeneous phase (water, air, steam), all of the fluid equations and its relationships are usually categorized with the flow of a phase of fluid whether or not liquid or vapor.

A single-phase fluid is a fluid whose substance is in the liquid or gaseous state. The data collected from studies covered the microscale single-phase fluid go with the flow and issues related to investigations at microscale flow were summarized. Kumar et al. [9], have presented the experimental and numerical results on single-phase fluid flow distribution and thermal transfer in microstructure reactors. Where this particular case has been commonly used in the last 10 years in manufacturing and scientific activities,[10]. In most cases, research concentrated on a single flow (pure fluid). The mixed convection arises in that flow when the free convection and forced convection act together. Thus, the flow occurs because of temperature variance in the surface and surrounding fluids, which provide buoyancy strength, [11]. There are several techniques; ware established for modeling the nanofluids, and they are categorized underneath main models, specifically: single-phase and two-phase models,[12, 13]. The flow pattern and heat transfer of $AL_2O_3$-nanofluid water in a micro-plane abrupt expansion with continuous heat flow conditions are examined numerically. $AL_2O_3$ has believed to be a non-neutron mono-liquid fluid with variable physical properties, [14].

In fluid mechanics, a multi-stage phase is the interactive phase of two separate phases, every one act for a size issue or mass, with shared interfaces in a channel. A single-component or multi component may contain Two-phase flow at the same time, [15]. Together with the material flow with two or more levels of thermodynamics. These phases may also consist of a single chemical agent (water flow and water vapor), or many exceptional chemical additives (oil and water flotation). Two-Phase flow can be described while gas/vapor and liquid capacities need to simultaneously be vacated via the equal valve, or if the liquid at an upstream temperature is higher than its saturation temperature beneath the hole pressure, [16].

A phase has classified as continuous if it occupies a continually related space whereas a kind of disperse is completed whilst the segment occupies disconnected areas, the non-stop phase maybe both gaseous or a liquid and the disperse phase can include either a solid, fluid or gas,[17]. Two famous topologies can be diagnosed, disperse flows and separated flows, the previous being the ones including of finite particles, drops or bubbles allotted within a continuous section. The latter is defined as including or extra continuous streams of fluids separated with a resource of interfaces, [18]. Myre et al,[19] discusses the integration of the performance evaluation of multiphase, and single-phase, multicomponent Lattice-Boltzmann fluid float simulations on GPU clusters. Two-phase flows are the maximum acknowledged of multiphase flows that have contained the liquid-liquid flow, gas- fluid flow, gas-solid flow, and fluid-solid flow. These varieties of flows are maximum large in terms of industry, and well-reviewed by the scholars.

## 3. Development of the problem of two-phase fluid

The simultaneous flow of liquid and gas arise at certain important locations in fluid flow systems. The issue should be resolved as two-phase flow; the simple relationship techniques performed to analyze the single-phase flow are unrelated for analyzing two-phase flow. To this end, the numerical simulation of transient immiscible and incompressible two-phase flows is proposed, demonstrating how to deal with multi-phase flow problems, [20]. This classification is very important because most fluid flow equations generated based on the fact that the complication of the two-phase fluid equations, and the study of change flow from mono to binary and vice versa, because the characteristics of the material changed when the condition changed, the density of the fluid varies and the speediness of sound in the fluid varies.

In addition to the change of pressure and other characteristics, other than that, the study of two-phase flow in which the solid spherical particles are scattered in a fluid is fundamental in technical problems such as packed beds, sediments, environment pollution, centrifugal separation of particle and blood rheology, may also be considered as three-phase flow. Other areas of interest include the study of two-phase flow in climatic systems such as clouds, dust from cement plants and metal smelting in industrial processes is one of the sources of air pollution. Thus, people had exposure to health risks and may be fatal. Much observe has been performed on the character of the two-stage flow in such cases, so engineers can layout in opposition to design against potential pipe failures, loss of pressure, etc,[20].

For the gas flow systems particles, particulates flow field and the mutual interaction of gas with the particle cloud are governed by four similarity parameters which are the velocity equilibration, thermal equilibration, momentum interaction, and thermal interaction. The binary mixture of viscous fluid and dust particles is called as dusty fluid and has attracted much attention because of its importance in engineering applications such as powder technology, shipping of liquid slurries in nuclear processing, chemical processing, and indifferent geophysical situation. From features make two-phase flow an exciting and hard department of fluid mechanics that may supply upward push to different counter-intuitive, negative resistance-kind instabilities, like leading instability, geyser, chugging, and flow misdistribution instabilities as examples of static instabilities, and different dynamic instabilities, [21]. Ghiaasiaan. [22], have investigated the important aspect of nuclear energy, which is safety alongside one of the most serious accidents in nuclear power generation, the loss of coolant.

Two-phase flows happen in several environmental and technological cases. For instance, the soil or problems with dust, air, and sediment in the water, corrosion, and silt, and problems with household equipment or engines (such as helicopters) and power plants in such an environment. Also, many methods of converting energy and chemical methods involve flows of two phases. Boil liquid along with water or sodium and draw heat by condensing in an exceptional environment provides realistic energy flow processing. A set of criteria has been developed to allow the identification of flow systems and their transitions during the test the function of probability density to break the vacuum. Based on these criteria, the flow models of the flow system were evaluated in the literature on liquid gas flows of two stages in two phases,[23]. Numerical study of two-phase modeling of nanofluids in a rotary system with permeability sheet and heat transfer in a rotary system, which is investigated numerically using the fourth-order,[24] . On the other hand, the heat and mass transfer of the two-phase flow was studied with the effects of the electrical double layer induced by the peristaltic propulsion in the presence of the magnetic field of the transverse and heat transfer in two phases,[25]. Akbari. [26], and others conducted a numerical study of two-phase fluids, the study was based on a model for the flow of nanofluid in a curved three-dimensional tube. In one study, the hypothesis that fluid mechanics and thermal vectors involved in sterile treatment could be formulated, the researchers wrote the FORTRAN limited variation program (using the four-step Runge Kutta method, which is appropriate in four phases) to calculate nanoparticles and particle velocity during the full-motion flow of three-dimensional in traditional movement and the Pie Holding tubing , [27]. Li et al. [28], have investigated a two-phase a out model of debris flows on corrosive layers. A theoretical study has been carried out, to analyze the evaporative heat transfer of excessive- speed airflow in two- phases of air in a small vertical tube under heating situations for regular constant wall temperature and continuous heat flow and simplified two-phase circulation flow, the layer model became used to evaluate the evaporation heat transfer traits of the two-phase cyclic flow, [29]. The design of the two-phase glide rings applied to the thermal manage structures of the destiny spacecraft requires understanding of the 2-phase glide and temperature switch phenomena in microgravity situations. The basic approach of evaporators or cold plate has been studied in two-phase flow rings,[30]. Forces such as buoyancy, inertia, viscosity and surface tension, have determined the flow of liquefied natural gas (LPG) into the tube, leading to the formation of different flow patterns, the interaction of two-phase flow patterns with the surface of the inner tube results in an irregular heat transfer coefficient,[31].

Additionally, in the petroleum and chemical industries, the flow of two-phase corrugated oil pipes is one of the main exported worldwide. A decrease in an oil temperature below a certain point may cause the oil phase to behave as a non-Newtonian fluid, which causes the flow activity to be more complex, [32]. Have shed a light on numerical simulation of the flow of the two-phase class corrugated tubular tube on non-Newtonian water as well as heat transfer in their experiment, the thermal transfer of the flow can also be affected, by the non-Newtonian properties of the two-phase flow. However, the flow of oil and water in two phases in the assembly pipeline becomes complicated [33, 34].As a result of a decrease in temperature, in the transport process, non-Newtonian traits emerge with more complex pipeline flow forms. On the other hand, the non-Newtonian traits of two-phase flow may influence heat transfer to tube flow. Gamal al-Abbadi.[35], studied the two-phase electromagnetic vibrations caused by the flow in the hot vertical upward flow and reveiled the importance of two-phase flow in heat exchangers and several applications, like , pipe engineering techniques, communication between construction movement and fluid fluctuations in heat exchangers, small channels, semi-refrigerant cooling flow, and continuous flow of heat.

## 4. Summary

**Table 1.** Type of phase flow.

| | Definition and state |
|---|---|
| Single | Fluid flow of a single homogeneous phase.<br>E.g.: liquid phase & gas phase. |
| Multiple (two phase) | The interactive flow of two distinct phases—each phase representing a mass or volume of matter—with common interfaces in a channel.<br>E.g.: gas and liquid, gas and solid particles, liquid and solids and gas liquid and solids. |

**Table 2.** Development of the problem of two-phase fluid.

| Authors | Year | Contribution |
|---|---|---|
| Wallace [36] | 1978 | Highlight the critical two-phase models, the HEM model and describe their features through the uniform distribution of liquid and vapor, with constant velocity movement, the thermal balance of phase. |
| Gurtinand all [37] | 1996 | Developed a standard framework for Navier-Stokes / Cahn-Hilliard equations using, as a general rule, microforce balancing law in conjunction with basic equations that correspond to a mechanical version of the second law. As a numerical application of theory, |
| Attia et al. [38] | 2012 | Study of experimental data reports and sizing analysis of forced load of foams and microfluids in laminar go with the flow in circular and rectangular tubes as well as in tube bundles. Foams and microchips are two-phase liquids (shear thinning) along with bubbles tightly packed with diameters starting from tens of microns to a few millimeters. |
| Esmaeilnejad et al. [39] | 2014 | Thermal transfer and flux of nanoparticles with a non-neutron base fluid in a microchannel were examined numerically the usage of a two-phase mixture model, numerical thermal transfer of non-Newtonian nanotubes, the thermal resistance of nanotubes the using of nanoparticles, and reporting of large improvement in heat switch the using nanoparticles. |
| Behroyan, et al. [40] | 2015 | The investigation has also provided an appropriate analysis of important elements such as Brownian motion, fluid behavior factors and effective conductivity of nanoparticles that should be considered or changed by each model. The study also compared the predictions of five types of computational fluid dynamics (CFD) models, (I.e. neutron and non-neutone) and three two-phase models, and the study of the forced load of nanotubes in a tube with the constant thermal flow on the pipe wall. |
| Wang et al. [41] | 2017 | Analyzed heat and mass transfer with the transverse magnetic area on the peristaltic motion of two-phase flow (particle-fluid suspension) via a planar channel were tested. The flow observes below have an effective impact on the electrical subject and chemical response, heat and mass transfer with a transverse magnetic area is studied, the actual solution is provided for fluid segment and particle phase and the non-Newtonian Jeffrey fluid is modeled with two-phase phenomena. |
| Wang et al. [41] | 2017 | The power-law model was used to describe the behavior of non-Newtonian liquids and to create an analytical model for predicting pressure gradient, vacuum brake and wave behavior in pump flow, and analysis of how liquid |

| | | |
|---|---|---|
| | | viscosity affects the non-Newtonian pressure and fracture gradient. |
| Siddiqa et al. [5] | 2017 | Presented a boundary-layer evaluation of two-phase dusty non-Newtonian fluid flow alongside a vertical surface by way of using a modified energy-law viscosity model. This investigation mainly reports the flow conduct of spherical debris suspended within the non-Newtonian fluid, velocity, temperature, streamlines, and isotherms are presented for critical parameters, It discovered that energy-law fluids are much more likely to promote the thermal transfer rate. |
| Ali, et al. [42] | 2018 | This aimed to take a look at the effect of the magnetic field and its effects inside the human frame, especially inside the blood. Blood is an instance of a non-Newtonian Cason. The blood flow in a vertical cylinder with heat transfer is because of the blended conviction due to buoyancy and outside pressure، the effect of the magnetic area on blood velocities and magnetic molecules. |
| Mahanthesh & Gireesha [43] | 2018 | The effects of Marangoni on convection in the flow of magnetic fluid Cason by suspending dust particles, the problem is governed by the laws of conservation of mass and momentum and energy for the phase of particles of liquids and dust and use the expansion modulation technique to form ordinary differential equations of partial differential equations. |
| Bhatti et al. [44] | 2018 | The combined effects of heat and mass transfer on MHD peristaltic propulsion of the two-phase liquid flow are studied through a Darcy-Brinkman-Forchheimer porous medium, effects of mass and heat transfer on particle-fluid have been examined, and perturbation solutions have been presented using HPM. |
| Li et al. [32] | 2018 | A mathematical model for the float of non-Newtonian oil-flow stratified tubular tubes under unequal conditions changed into incorporated into the bipolar coordinate system and evaluation of the effect of non-Newtonian performance on oil buoy and heat transfer. |
| Bozorg & Siavashi [45] | 2019 | The assorted convection was analyzed in a square cavity filled with Cu-water nanofluid simulated using the Eulerian two-phase mixture model. The primary fluid is assumed a non-Newtonian fluid followed by energy law, heat transfer in two phases and analysis of the generation of entropy of non-Newtonian nanofluid inside a cavity with an internal rotating heater and cooler. |

| | | |
|---|---|---|
| Lamraoui et al. [46] | 2019 | A numerical study of the hydrodynamic behavior and heat transfer of $Al_2O_3$-nanofluidate in the fusion of the cleft-cleft spot was developed. A set of criteria was developed to allow the determination of flow systems and their transitions by examining the function of the vacuum-density density density. Flow flow models were evaluated in flow literature Two phase liquid gases in two stages. |

During the last decades, a considerable amount of effort have been made in the study of mathematical Model on non-Newtonian fluid on Two-Phase flow in industrial application such studies have practical application. Based on the reported studies the literature review, the forced convection flow of temperature-dependent viscosity of non-Newtonian Erying Powell fluid over a vertical stretching sheet and Free/mixed convection flow of temperature-dependent viscosity of non-Newtonian Erying Powell fluid over a vertical stretching sheet is considered in the proposed problems that are difficult to handle and no report has been found so far discussed about that boundary condition. On top of that, the dusty fluid model is the two-phase flows which consist of fluid and dust phases.

## 5. Conclusion

In available open literature, few studies have been carried out on non-Newtonian two-phase mathematical model representing the fluid flow and heat transfer's problem where several studied different classifications of fluid flow including those based on fluid components and have been widely used in industrial and scientific applications. The two-phase flows are the most known class of multiphase flows and are of the most significant in terms of industry. This paper presented the progress on the development of mathematical model on non-Newtonian two-phase model where the fluid flow is studied together with the dust phase

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