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Formulation of Dusty Micropolar Fluid Mathematical Model

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Abstract. Dusty micropolar fluid model is a new generation of fluid model that is embedded with the dust particles which having micro rotation behavioural. The proposed model will incorporate with thermal boundary condition known as Newtonian heating (NH). This paper aims to present the details of formulation on mathematical model of dusty micropolar fluid starting with its basic equations until obtaining a set of solvable form of equations. The derivation of equations involves two set of equations which covering the fluid and dust phases respectively. In order to decrease the complexity, the respective model will be undergoing the suitable similarity transformations to transform the equations into the solvable form. The numerical solution presented by compared with previous study to check the validity of the model. The resulting equation will give advance understanding on dusty micropolar fluid model to the researchers especially mathematician and fluid mechanist.

1. Introduction

Recently, the applications of a flow gas particles mixture in the fluid have a great deal of attention in the industrial field. It is because the fluid is commonly not pure but contain foreign objects or dust particles in its geophysical situations [1]. Micropolar fluids were one of the categories in Non-Newtonian fluids where it has a microstructure that consists of rigid particles. The motions of fluid are illustrated by two velocity vectors that accrue from rotated these fine particles in the fluid known as classical velocity vector and microrotation vector where the particle deformation will be neglected. The theory of micropolar fluid constituted by Eringen [2] was used wisely in the industrial field with various physical conditions. For example, in the engineering process including painting, lubricants, blood, polymers, colloidal fluids and suspension fluids [3]. The research of micropolar fluids be an interesting topic with a variety of effects, for example, Ishak et al. [4] investigated the effect of microrotation on an isothermal continuously moving plane surface and the analysis of steady boundary layer and heat transfer of micropolar fluid were presented. Then continued by Yacob et al. [5] were studied the effect of MHD flow on micropolar fluid towards a vertical permeable plate with prescribed surface heat flux where they found that for assisting and opposing flows there are exist dual solutions. Since that, dual solutions have captured more attention among mathematicians such as [6-8]. In the same way, Sandeep and Sulachana [8] interested in investigating the effect of MHD in micropolar fluid over a stretching/shrinking sheet and Gamal [9] tackled the flow problem of the effect of MHD on thin films on micropolar fluid.

Furthermore, Borrelli et al. [10] also studied MHD oblique stagnation-point flow of a micropolar fluid where they found that oblique stagnation-point flow exists if and only if the external magnetic field is parallel to the dividing streamline. However, Hayat et al. [11] studied the method to solve the MHD flow a micropolar fluid near a stagnation point towards a non-linear stretching surface. The method is known as the homotopy analysis method. According to the increasing industrial applications on aerodynamic extrusion of plastic sheets, hot rolling, metal spinning, artificial fibers, glass-fiber production, paper production and drawing of plastic films Aurangzaib et al. [7] investigated the micropolar fluid flow and transfer due to a stretching/shrinking sheet. Then followed by Hossain et al. [12] also interested in boundary layer flow and heat transfer in a micropolar fluid past a permeable flat plate.

On the other hand, consider the increases of the application involving dusty fluid, the modelling of dusty fluid become attracted to mathematician. For instance, soil salvation by natural winds, lunar surface erosion by the exhaust of a landing vehicle and dust entrainment in a cloud formed during a nuclear explosion are involving dust particles in boundary layers [13]. Siddiqa et al. [13] investigated the effect of dusty fluid on a natural convection flow due to a heated vertical surface with the two-phase boundary layer model of fluid and dust phases. Moreover, Arifin et al. [14] studied the aligned magnetic field of two phases mixed convection flow in dusty Casson fluid over a stretching sheet numerically. Further, Aggarwal [1] dealt with the effect of dust particles by theoretical investigation on micropolar fluid heated and dissolved by electrically conducting from below in the presence of a uniform vertical magnetic field in a porous medium. Then, Izani and Ali [15] studied the convective heat transfer characteristics of an incompressible dusty fluid over an exponentially stretching surface where the behavior of flow with fluid particle suspension have specific exponential function forms.

However, there were studies of micropolar dusty fluid with presence of nanoparticles with the effect of a magnetic field and thermal radiation done by Ghadikolaei et al. [16]. The study considered an incompressible TiO_2-water nanoparticle on micropolar fluid. Ghadikolaei et al. [17-18] investigated the effect of MHD radiation of micropolar dusty fluid with nanoparticles such as graphene oxide (Go) and hybrid nanoparticles ($Cu-Al_2O_3$). Further, Ghadikolaei [19] studied the magnetohyrodynamic CNTs-water as a micropolar dusty fluid with the effect of thermal radiation and joule heating. The models have two phases which consist of fluid and dust phases.

Motivating with all above, this present paper attracted to present the formulation of a dusty micropolar fluid mathematical model. The comparison of the numerical results with the existing results to proves the validity of the present results were also presented. The resulting model will give an advance understanding of the dusty micropolar fluid flow.

2. Mathematical Model

In this present paper, the model of dusty micropolar fluid is considered. This new generation of fluid model is embedded with the dust particle which having micro rotation behavioral and incorporate with thermal boundary condition named Newtonian heating (NH). A general theory and the basic equation of micropolar fluids introduced by [2] and used to solve application of micropolar fluid.

Using the usual boundary layer and Boussinesq approximations, the governing equations for two-phase flow given by [8] and [13] can be written as:

Fluid phase:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\upsilon+\frac{\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial N}{\partial y}+\frac{\rho_p}{\rho\tau_m}\left(u_p-u\right) \tag{2}$$

$$\rho j \left(u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma \frac{\partial^2 N}{\partial y^2} - \kappa \left(2N + \frac{\partial u}{\partial y} \right) \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\rho_p c_s}{\tau_T \rho c_p} \left(T_p - T \right) \tag{4}$$

Dust phase:

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0 \tag{5}$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = -\frac{1}{\tau_m} \left(u_p - u \right) \tag{6}$$

$$u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = -\frac{1}{\tau_T} \left(T_p - T \right) \tag{7}$$

where (u,v) and (u_p, v_p) are velocities components of the fluid and particle phase along x and y axes, respectively. Further, υ is the kinematic viscosity, κ is a vortex viscosity, ρ and ρ_p are the density of fluid and dust phase, N is the component of micro rotation vector normal to the x and y axes. τ_m is called velocity relaxation time of particle phase and τ_T is the thermal relaxation time of particle phase. Next, j is a microinertia density, γ is spin gradient viscosity, T and T_p are the temperature of fluid and dust phase, α is the thermal diffusivity, c_p and c_s are specific heat of fluid and dust phase.

The boundary conditions of NH [20] are in the form

$$u = u_w(x) = ax, \quad v = V_w, \quad N = -n\frac{\partial u}{\partial y}, \quad \frac{\partial T}{\partial y} = -h_s T \qquad \text{at } y = 0, \tag{8}$$

$$u \to 0, \quad N \to 0, \quad u_p \to 0, \quad v_p \to v, \quad T \to T_\infty, \quad T_p \to T_\infty \quad \text{as } y \to \infty.$$

where n is concentration of fluid and h_s is heat transfer parameter for Newtonian heating.

3. Formulation of a mathematical model

This section shows how the derivation of the equations works. By introduced the following similarity transformations,

$$u = axf'(\eta), \quad v = -(av)^{\frac{1}{2}} f(\eta), \quad \eta = \left(\frac{a}{v}\right)^{\frac{1}{2}} y, \quad \psi = (va)^{\frac{1}{2}} xf(\eta), \quad N = ax\left(\frac{a}{v}\right)^{\frac{1}{2}} h(\eta),$$

$$\theta = \frac{T - T_\infty}{T_\infty}, \quad u_p = axF'(\eta), \quad v_p = -(av)^{\frac{1}{2}} F(\eta), \quad \theta_p = \frac{T_p - T_\infty}{T_\infty} \tag{9}$$

where ψ is the stream function defined as $u = \dfrac{\partial \psi}{\partial y}$ and $v = \dfrac{\partial \psi}{\partial x}$. Above similarity transformations then used to reduce the equations (1) – (7) with boundary condition (8).

First, obtained $u = axf'(\eta)$ and $v = -(va)^{\frac{1}{2}} f(\eta)$. After finding the differentiation result of $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = af'(\eta) - af'(\eta) = 0$ then it is satisfying equation (1).

Next, by using Equation (9) yields the following equations,

$$u \frac{\partial u}{\partial x} = axf'(\eta).af'(\eta) = a^2 x \left(f'(\eta) \right)^2 \tag{10}$$

$$v \frac{\partial u}{\partial y} = -(va)^{\frac{1}{2}} f(\eta). \frac{\partial}{\partial y} \left(axf'(\eta) \right) = -(va)^{\frac{1}{2}} f(\eta).axf'''(\eta). \left(\frac{a}{v} \right)^{\frac{1}{2}} = -a^2 xf(\eta) f''(\eta) \tag{11}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y} \left(axf'''(\eta). \left(\frac{a}{v} \right)^{\frac{1}{2}} \right) = ax \left(\frac{a}{v} \right)^{\frac{1}{2}} f'''(\eta). \left(\frac{a}{v} \right)^{\frac{1}{2}} = \frac{a^2 x}{v} f'''(\eta) \tag{12}$$

$$\frac{\partial N}{\partial y} = \frac{\partial N}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = ax \left(\frac{a}{v} \right)^{\frac{1}{2}} h'(\eta). \left(\frac{a}{v} \right)^{\frac{1}{2}} = \frac{a^2 x}{v} h'(\eta) \tag{13}$$

$$u_p - u = axF'(\eta) - axf'(\eta) = ax \left(F'(\eta) - f'(\eta) \right) \tag{14}$$

By substituting equations (10 – 14) into equation (2), obtained

$$f'^2(\eta) - f(\eta) f''(\eta) - \left(v + \frac{\kappa}{\rho} \right) \frac{1}{v} f'''(\eta) + \frac{\kappa}{\rho} \frac{1}{v} h'(\eta) + \frac{\rho_p}{a \rho \tau_m} \left(F'(\eta) - f'(\eta) \right) = 0 \tag{15}$$

Given dynamic viscosity $\mu = v\rho$, material parameter $K = \dfrac{\kappa}{\mu} = \dfrac{\kappa}{v\rho}$, $L = \dfrac{\rho_p}{\rho}$ is the mass concentration of particle phase and $\beta = \dfrac{1}{a\tau_m}$ is the fluid particle interaction parameter. The equation (15) becomes

$$(1 + K) f'''(\eta) + f(\eta) f''(\eta) - f'^2(\eta) + Kh'(\eta) + \beta L \left(F'(\eta) - f'(\eta) \right) = 0 \tag{16}$$

Continuing to find the derivative result as follow:

$$u \frac{\partial N}{\partial x} = axf'(\eta) \left(\frac{\partial}{\partial x} \left(ax \left(\frac{a}{v} \right)^{\frac{1}{2}} h(\eta) \right) \right) = a^2 x \left(\frac{a}{v} \right)^{\frac{1}{2}} f'(\eta) h(\eta) \tag{17}$$

$$v \frac{\partial N}{\partial y} = -(va)^{\frac{1}{2}} f(\eta) \left(\frac{a^2 x}{v} h'(\eta) \right) = -\frac{a^2 x (va)^{\frac{1}{2}}}{v} f(\eta) h'(\eta) \tag{18}$$

$$\frac{\partial^2 N}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{a^2 x}{v} h'(\eta)\right) = \frac{a^2 x}{v}\left(\frac{a}{v}\right)^{\frac{1}{2}} h''(\eta) \tag{19}$$

Equations (17 – 19) then substitute into equation (3) and become

$$\gamma \frac{1}{\rho j v} h''(\eta) - f'(\eta) h(\eta) + f(\eta) h'(\eta) - \frac{2\kappa}{\rho j a} h(\eta) - \frac{\kappa}{\rho j a} f''(\eta) = 0 \tag{20}$$

Used spin gradient viscosity given by $\gamma = \left(\mu + \dfrac{\kappa}{2}\right) j$ and microinertia density $j = \dfrac{v}{a}$ introduced by [1]. Then equation (20) become

$$(2 + K) h''(\eta) - f'(\eta) h(\eta) + f(\eta) h'(\eta) - K(2h(\eta) + f''(\eta)) = 0 \tag{21}$$

By using Equation (9), the following equations performed,

$$u \frac{\partial T}{\partial x} = axf'(\eta)\frac{\partial}{\partial x}\left(T_\infty(\theta(\eta) + 1)\right) = axf'(\eta)\left(T_\infty\left(\frac{\partial \theta}{\partial \eta} \cdot \frac{\partial \eta}{\partial x}\right)\right) = axf'(\eta).T_\infty.\theta'(\eta).0 = 0 \tag{22}$$

$$v \frac{\partial T}{\partial y} = -(va)^{\frac{1}{2}} f(\eta)\left(T_\infty\left(\frac{\partial \theta}{\partial \eta} \cdot \frac{\partial \eta}{\partial y}\right)\right) = -(va)^{\frac{1}{2}} f(\eta).T_\infty.\theta'(\eta).\left(\frac{a}{v}\right)^{\frac{1}{2}} = -aT_\infty f(\eta)\theta'(\eta) \tag{23}$$

$$\alpha \frac{\partial^2 T}{\partial y^2} = \alpha \frac{\partial}{\partial y}\left(T_\infty.\theta'(\eta).\left(\frac{a}{v}\right)^{\frac{1}{2}}\right) = \alpha\left(\frac{a}{v}\right)^{\frac{1}{2}} T_\infty\left(\theta''(\eta).\left(\frac{a}{v}\right)^{\frac{1}{2}}\right) = \alpha \frac{a}{v} T_\infty \theta''(\eta) \tag{24}$$

$$T_p - T = T_\infty\left(\theta_p(\eta) + 1\right) - T_\infty\left(\theta(\eta) + 1\right) = T_\infty\left(\theta_p(\eta) - \theta(\eta)\right) \tag{25}$$

Subsequently, by substitute equations (22 – 25) into an equation (4) and become,

$$\frac{\alpha}{v}\theta''(\eta) + f(\eta)\theta'(\eta) + \frac{\rho_p c_s}{a\tau_T \rho c_p}\left(\theta_p(\eta) - \theta(\eta)\right) = 0 \tag{26}$$

Known $\Pr = \dfrac{v}{\alpha}$ is a Prandtl number, $\tau_T = \dfrac{3}{2}\gamma\tau_m \Pr$ is the relation between thermal relaxation time and velocity relation time, $\beta = \dfrac{1}{a\tau_m}$ is the fluid-particle interaction parameter and $\gamma_T = \dfrac{c_s}{c_p}$ is the specific heat ratio of mixture. Then equation (26) become

$$\frac{1}{\Pr}\theta''(\eta) + f(\eta)\theta'(\eta) + \frac{2}{3}\beta L\left(\theta_p(\eta) - \theta(\eta)\right) = 0 \tag{27}$$

Then obtained equation $\dfrac{\partial u_p}{\partial x} + \dfrac{\partial v_p}{\partial y} = aF'(\eta) - aF'(\eta) = 0$, where indicated that equation (5) was satisfied.

After that, the following equations were obtained

$$u_p \frac{\partial u_p}{\partial x} = axF'(\eta).aF'(\eta) = a^2 xF'^2(\eta) \tag{28}$$

$$v_p \frac{\partial u_p}{\partial y} = -(av)^{\frac{1}{2}} F(\eta).\frac{\partial}{\partial y}\left(axF'(\eta)\right) = -(av)^{\frac{1}{2}} F(\eta).ax\left(\frac{a}{v}\right)^{\frac{1}{2}} F''(\eta) = -a^2 xF(\eta)F''(\eta) \tag{29}$$

$$u_p - u = ax\left(F'(\eta) - f'(\eta)\right) \tag{30}$$

The result of equation (6) after substituting equations (28 – 30) were

$$a^2 xF'^2(\eta) - a^2 xF(\eta)F''(\eta) + \frac{1}{\tau_m} ax\left(F'(\eta) - f'(\eta)\right) = 0 \tag{31}$$

Then, finally become

$$F'^2(\eta) - F(\eta)F''(\eta) + \beta\left(F'(\eta) - f'(\eta)\right) = 0 \tag{32}$$

Next, we have

$$u_p \frac{\partial T_p}{\partial x} = 0 \tag{33}$$

$$v_p \frac{\partial T_p}{\partial y} = -aT_\infty F(\eta)\theta_p{}'(\eta) \tag{34}$$

$$T_p(\eta) - T(\eta) = T_\infty\left(\theta_p(\eta) - \theta(\eta)\right) \tag{35}$$

where will substitute into equation (7) and will obtained

$$\theta_p{}'(\eta)F(\eta) + \frac{1}{a\tau_T}\left(\theta(\eta) - \theta_p(\eta)\right) = 0 \tag{36}$$

$$\theta_p{}'(\eta)F(\eta) + \frac{2}{3}\frac{\beta}{\gamma\,\mathrm{Pr}}\left(\theta(\eta) - \theta_p(\eta)\right) = 0 \tag{37}$$

While the boundary conditions (8) are reduced to

$$f(0) = S,\ f'(0) = 1,\ h(0) = -nf''(0),\ \theta'(0) = -b(1+\theta(0)) \quad \text{at } \eta=0$$
$$f'(\eta) \to 0,\ h(\eta) \to 0,\ F(\eta) \to f(\eta),\ F'(\eta) \to 0,\ \theta(\eta) \to 0,\ \theta_p(\eta) \to 0 \ \text{as } \eta \to \infty \tag{38}$$

Here $b = h_s \left(\dfrac{v}{a}\right)^{\frac{1}{2}}$ is the conjugate parameter for NH.

4. Validation of the formulation

This part presents the validation of the formulation with Turkyilmazoglu [22] and Qasim et al [23].

Table 1 shown the comparison of $-\left(1 + \dfrac{K}{2}\right) f''(0)$ for various values of the material parameter, K

when $n = 1/2$ and $\gamma \rightarrow \infty$ with present of dusty effect neglected where $s = \beta = N = 0$.

Table 1. Comparisons of $-\left(1 + \dfrac{K}{2}\right) f''(0)$ for various values of K when $n = 1/2$, $s = \beta = N = 0$ and $\gamma \rightarrow \infty$.

K	Qasim et al. [23]	Turkyilmazoglu [22]	Present
0	1.000000	1.00000000	1.00000002
1	1.224741	1.22474487	1.22474486
2	1.414218	1.41421356	1.41421347
4	1.733052	1.73205081	1.73205096

This clearly indicates that the available numerical results of [22,23] be consistent with our numerical solutions for the case of ignored dusty effect, and it is noticed that the values increase in increasing values of K.

5. Conclusion

In this study, the derivation of dusty micropolar fluid were considered. The model of micropolar fluid and dusty fluid were taken to reduce until solver equations which is in the form of ordinary differential equation. This resulting equation shown valid by comparison with previous study and will give advance understanding on dusty micropolar fluid model to the researchers especially mathematician and fluid mechanist.

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374

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