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Cite as: AIP Conference Proceedings **1867**, 020010 (2017); https://doi.org/10.1063/1.4994413
Published Online: 01 August 2017

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The Effect of Uncertainties in Distance-Based Ranking Methods for Multi-Criteria Decision Making

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Abstract. Data in the multi-criteria decision making are often imprecise and changeable. Therefore, it is important to carry out sensitivity analysis test for the multi-criteria decision making problem. The paper aims to present a sensitivity analysis for some ranking techniques based on the distance measures in multi-criteria decision making. Two types of uncertainties are considered for the sensitivity analysis test. The first uncertainty is related to the input data, while the second uncertainty is towards the Decision Maker preferences (weights). The ranking techniques considered in this study are TOPSIS, the relative distance and trade-off ranking methods. TOPSIS and the relative distance method measure a distance from an alternative to the ideal and anti-ideal solutions. In turn, the trade-off ranking calculates a distance of an alternative to the extreme solutions and other alternatives. Several test cases are considered to study the performance of each ranking technique in both types of uncertainties.

INTRODUCTION

There is a considerable existing research on uncertainties in the deterministic multi-criteria decision making (MCDM) methods. Barron and Schmidt [1] proposed two procedures - an entropy based procedure and a least square technique - to test the sensitivity of the attributes (criteria) weights in the multi-attribute value theory (MAVT) method. It is assumed that in the former approach the weights are nearly equal, whilst the latter requires a set of arbitrary weights of the criteria.

Von Winterfeldt and Edwards [2] defined the Flat Maxima Principle to test sensitivity analysis on the multi-attribute utility theory (MAUT) method. Rios Insua [3] described a sensitivity analysis in the traditional MCDM Bayesian model.

In addition, there exist several sensitivity analyses performed on the Analytic Hierarchy Process (AHP) prior to its development by Saaty [4]. Masuda [5] studied the effect of changes in the entire decision matrix vectors on the ranking of the alternatives in the AHP method. Further research was done by Armacost and Hosseini [6], who presented a procedure for determining the most critical criterion for the AHP problem. There is also a software package for the AHP, named Expert Choice, developed in 1990, to carry out the sensitivity analysis of the method where the user can alter the weights of the decision criteria and see how the ranking changes.

Triantaphyllou and Sanchez [7] carried out a sensitivity analysis on the weights of the decision criteria and the performance values of the alternatives to three MCDM methods: Weighted Sum Model, Weighted Product Model and AHP. They determined the smallest changes of the current weights that would affect the existing ranking.

Alinezhad and Amini [8] carried out a sensitivity analysis on the technique for order preference by similarity to ideal solution (TOPSIS) method. They changed the weight of a criterion and observed its effect on the final score of the alternatives. Simanaviciene and Ustinovichius [9] also presented a sensitivity analysis on TOPSIS method. They carried out a comparison with the simple additive weighting (SAW) method. They found out that TOPSIS method is more sensitive to the differ in criteria value than the SAW method.

International Conference on Mathematics: Pure, Applied and Computation
AIP Conf. Proc. 1867, 020010-1–020010-16; doi: 10.1063/1.4994413
Published by AIP Publishing. 978-0-7354-1547-8/$30.00

The existing papers related to uncertainty in the MCDM process address the sensitivity of MCDM models to the change of criteria weights. However, the first task in any decision making process is to identify the set of alternatives for the Decision Maker (DM) to make the choice. Thus, it is also essential to examine the uncertainty in this task of the decision making process, and to determine how it affects the ranking. This paper examines both uncertainties in the MCDM process using other MCDM methods classified as the distance-based ranking techniques.

The rest of the paper is organized as follows. The MCDM methods are reviewed in Section 2; they are presented by the TOPSIS, the relative distance ranking approach, and a new proposed method - the trade-off ranking method. As mentioned above, two types of uncertainty analysis are considered. The first analysis, presented in Section 3, is applied to the changes of alternatives via the fuzzy numbers approach to find a more robust set of alternatives. In Section 4, a second analysis is carried out through the changes of criteria weights. The effects of both analyses on the ranking methods are illustrated by solving some test cases. Finally, the Conclusion is presented in Section 5.

MULTI-CRITERIA DECISION MAKING

The set of alternatives to be considered in a MCDM problem can be produced via a multi-objective optimization technique. Let the design space and the objective space be presented by $X \subset \mathbb{R}^m$ and $Y \subset \mathbb{R}^n$, respectively. The multi-objective optimization problem is then formulated by

$$\begin{aligned} &\text{Minimize} \quad Y = \{Y_1(x), Y_2(x), ..., Y_n(x)\}, \\ &\text{subject to} \quad x \in D, \end{aligned} \tag{1}$$

where $D \subset X$ is the feasible design space and the set $\{Y^*(x) \mid x \in D\}$ is the feasible objective space.

The set of solutions to the multi-objective problem (1) is a compromise solution between the multiple conflicting objectives. Such a solution corresponds to a so-called Pareto solution. A set of Pareto solutions that obtained from the multi-objective optimization problem is then considered as a set of alternatives in the MCDM problem.

A design vector $x \in D$ is Pareto optimal iff there does not exist any $h \in D$ such that $Y_j(h) \leq Y_j(x)$ $(j = 1, ..., n)$ and exists $l \leq n : Y_l(h) < Y_l(x)$.

In general, there are three main steps in the decision making technique: (i) determining the relevant alternatives, (ii) imposing the numerical measures (i.e., weights), and (iii) evaluating a ranking of each alternative.

In the current paper, a sensitivity analysis is applied to steps (i) and (ii). In step (iii), a new ranking method called the trade-off ranking is used.

Distance-Based Ranking Methods

In this subsection, three MCDM methods used in this paper are reviewed. Firstly, a novel trade-off ranking approach is introduced, followed by the TOPSIS and relative distance ranking approaches. Each ranking technique is based on a distance measurement. The difference in the distance measure used in each method is explained further in the appropriate subsection.

A MCDM problem with n criteria and q alternatives can be expressed in a matrix form as

	Criterion				
	Y_1	Y_2	Y_3	...	Y_n
Alternative					
A_1	Y_{11}	Y_{12}	Y_{13}	...	Y_{1n}
A_2	Y_{21}	Y_{22}	Y_{23}	...	Y_{2n}
A_3	Y_{31}	Y_{32}	Y_{33}	...	Y_{3n}
:	:	:	:	:	:
A_q	Y_{q1}	Y_{q2}	Y_{q3}	...	Y_{qn}

,

$$W = [w_1, w_2, ..., w_n],$$

where Y_{ij} is the performance of criterion j in alternative i and w_j is the criteria weights, for $i = 1, ..., q;\ j = 1, ..., n$. Note that the objectives in problem (1) are regarded as the criteria in the MCDM problem. Meanwhile, the $Y_{ij}, i = 1, ..., q;\ j = 1, ..., n$ are obtained as the results of solving the multi-objective problem (1).

Trade-off Ranking

Consider first the trade-off ranking method to determine the ranking of each Pareto solution (alternative). In a conflicting multi-criteria problem, it is not possible to determine an alternative that possess the best value for all the criteria, hence it is wise to seek a compromise solution between those criteria. The formulation of the trade-off ranking method ensures the best trade-off solution. The principle of the method is to favour a solution with the least level of compromise. Generally, each criterion is assumed to be equally important; however, the weighting may be enforced in the trade-off ranking method formulation.

An even and well distributed set of Pareto solutions is a pre-requisite for the method. Such a set efficiently represents the whole Pareto frontier. Thus, a DM is able to make an efficient decision based on the limited set within a time-constraint situation. Such a set can be obtained from optimization methods such as the Normal Boundary Intersection (NBI) method [10], the Normal Constraint (NC) method [11, 12] and the Directed Search Domain (DSD) algorithm [13, 14].

Now, consider q alternatives and n criteria. Suppose that $q > n$. In the trade-off ranking method, the criteria performance Y_{ij} and the criteria weights w_j are normalised by the formulae:

$$f_{ij} = \frac{Y_{ij} - \min\limits_{j} Y_{ij}}{\max\limits_{j} Y_{ij} - \min\limits_{j} Y_{ij}}, \; i = 1, ..., q; \; j = 1, ..., n, \tag{2}$$

$$w'_j = \frac{w_j}{\sum_{j=1}^{n} w_j}, \; j = 1, ..., n. \tag{3}$$

The normalisation step can be ignored if the performance scores vary in the same range and $\sum_{j=1}^{n} w_j = 1$ with $w_j \geq 0, \; j = 1, ..., n$.

Ideally, the set of alternatives includes the anchor points of the multi-objective problem (1). The anchor point is a solution for a single-objective problem. Such a solution is called an extreme solution in the trade-off ranking method, i.e. a solution with the best value in at least one criterion. We presume that the number of extreme solutions in MCDM problem is equal to n. Practically, a k-th extreme solution, $A_k, \; k = 1, ..., n$ is the alternative with the minimal j-th criterion:

$$A_k = \{\min_{1 \leq i \leq q} f_{ij}\}, \; j = 1, ..., n. \tag{4}$$

The ranking measure for the trade-off method is the total distance from an alternative, $A_\alpha = (f_{\alpha 1}, f_{\alpha 2}, ..., f_{\alpha n})^T, \; \alpha = 1, ..., q$, to all the extreme solutions, $A_k = (f_{k1}, f_{k2}, ..., f_{kn})^T, \; k = 1, ..., n$, given by formulae:

$$\begin{aligned} DT1_\alpha &= \sum_{j=1}^{n} w'_j [d_{TOR1}(A_\alpha, A_k)], \quad \alpha = 1, ..., q, \\ &\sum_{j=1}^{n} w'_j = 1, \\ &w'_j \geqslant 0, \; j = 1, ..., n, \end{aligned} \tag{5}$$

where $w'_j, \; j = 1, ..., n$ is the normalized weight of criterion j. The $d_{TOR1}(.,.)$ denotes the distance formula between two points in L_2-metric such as:

$$d_{TOR1}(A_\alpha, A_k) = \left[\sum_{j=1}^{n} \left(f_{\alpha j} - f_{kj} \right)^2 \right]^{1/2}, \; \alpha = 1, ..., q; \; k = 1, ..., n. \tag{6}$$

An alternative with the minimum value of $DT1$, i.e. the degree of first trade-off, is the best alternative with the trade-off ranking method. Such an alternative is the closest option to all the extreme solutions. We choose the extreme solution as the point of reference since it represents the best solution in a single-criterion problem. Hence, by obtaining the best solution in most of the criteria, if not all, is considered as a reasonable compromise solution in the conflicting criteria problem.

In the case of the same minimum value of $DT1$, the trade-off ranking formulation is further applied between the alternatives [15]. The general formula for the weighted distance between an alternative $A_\alpha = (f_{\alpha 1}, f_{\alpha 2}, ..., f_{\alpha n})^T,\ \alpha = 1, ..., q$, and an alternative $A_\beta = (f_{\beta 1}, f_{\beta 2}, ..., f_{\beta n})^T,\ \beta = 1, ..., q$, is:

$$
\begin{aligned}
d_{TOR2}(A_\alpha, A_\beta) = &\left[\sum_{j=1}^{n} w_j'^2 \left(f_{\alpha j} - f_{\beta j}\right)^2\right]^{1/2}, \\
&\sum_{j=1}^{n} w_j' = 1, \\
&w_j' \geqslant 0,\ j = 1, ..., n.
\end{aligned}
\tag{7}
$$

Here, $\alpha, \beta = 1, ..., q$, where q is the number of alternatives. The distance formulae $d_{TOR2}(.,.)$ in the trade-off ranking method implies the distance between two alternatives in terms of weighted criteria values.

The sum of distances from one alternative to all the others, i.e. the degree of second trade-off, is calculated as:

$$DT2_\alpha = \sum_{i=1}^{q} \left[d_{TOR2}(A_\alpha, A_i)\right], \alpha = 1, 2, ..., q. \tag{8}$$

The value of $DT2_\alpha$ implies the total distances of an alternative α to the other alternatives. A smaller values implies a shorter accumulative distances. Hence, an alternative with a smaller value of $DT2$ has a less compromise with the other alternatives, or a less degree of trade-off for an alternative. Here, the trade-off ranking is determined further by the value of $DT2$ where the least value holds the highest ranking.

Thus, the best solution in the trade-off ranking is the solution with the least compromise in regards to (i) the extreme solutions, and (ii) the other alternatives. As an analogy, consider the price and quality as two conflicting criteria. The best quality item is usually offered for a higher price. In turn, the cheapest item is a trade-off of its quality. Given all the feasible options, including those two extreme cases, the trade-off ranking method is able to give the least trade-off option as the best solution. The distance measurement in the trade-off ranking is illustrated in Fig.1.

TOPSIS

TOPSIS is a well-known MCDM method introduced by Hwang and Yoon [16]. TOPSIS is based on having an alternative with the minimum distance to the ideal solution, I_T^+, and the maximum distance to the anti-ideal solution, I_T^-. The first step in TOPSIS process is to normalise the decision variables, f_{ij}, by the formula:

$$r_{ij} = \frac{Y_{ij}}{\sqrt{\sum_{i=1}^{q} Y_{ij}^2}}, i = 1, ..., q, j = 1, ...n. \tag{9}$$

Next, the normalised decision variables are multiplied by their respective weights to be assigned by the Decision Maker:

$$v_{ij} = w_j r_{ij} \text{ where } \sum_{j=1}^{n} w_j = 1, w_j \geqslant 0. \tag{10}$$

The ideal solution, I_T^+, and the anti-ideal solution, I_T^-, are determined by the formulae:

$$I_T^+ = (v_1^+, v_2^+, ..., v_n^+), \tag{11}$$

$$I_T^- = (v_1^-, v_2^-, ..., v_n^-), \tag{12}$$

where

$$
\begin{aligned}
v_j^+ &= \min \{v_{ij}, i = 1, ..., q\}, \\
v_j^- &= \max \{v_{ij}, i = 1, ..., q\},\ (j = 1, ..., n).
\end{aligned}
$$

Then, the distance of an alternative to the ideal and the anti-ideal solutions is calculated, respectively, in the norm of L_2 by formulae:

$$dT_i^+ = \sqrt{\sum_{j=1}^{n} w_j^2 (v_j^+ - v_{ij})^2}, \ i = 1, ..., q, \tag{13}$$

$$dT_i^- = \sqrt{\sum_{j=1}^{n} w_j^2 (v_j^- - v_{ij)})^2}, \ i = 1, ..., q. \tag{14}$$

The ranking in TOPSIS is determined by the formula:

$$D_i = \frac{dT_i^-}{dT_i^+ + dT_i^-}, \ i = 1, ..., q, \tag{15}$$

where the largest value of D_i is regarded as the best solution. One of the main drawbacks of the method is in its ambiguity; an alternative closer to the ideal solution is not necessary farther to the anti-ideal solution.

In comparison to the trade-off ranking, TOPSIS is based on the individual performance evaluation, while the trade-off ranking is based on the overall evaluation. Each alternative is ranked with respect to its differences to the others in the trade-off ranking. Meanwhile, in TOPSIS, the ranking is based on the distance to the ideal and anti-ideal solutions. Such evaluation is the best method for a problem with mutual criteria. However, in a conflicting multi-criteria problem, where it is impossible to have a solution with the best value in all criteria, the trade-off ranking can be a preferable approach since it takes into account the differences of all alternatives, and provides the best compromise option.

Relative Distance Ranking

The relative distance ranking was introduced by Kao [17]. The principle of the method is similar to TOPSIS, it identifies an alternative with the shortest distance to the ideal solution and the longest distance to the anti-ideal solution. The differences between the two methods are related to the weighting calculation and the distance formula. As noted above, TOPSIS uses the priori weights determined by the Decision Makers, while the relative distance method exploits an optimization based on the available data set to identify the weights. For this reason, the quality of the weights strongly depends on the quality of the available data. However, Kao does not exclude the weights determined from the Decision Maker in the relative distance ranking formulation [17]. The key difference of the method is that the distance is measured in the norm of L_1 rather than L_2. As a result, in contrast to TOPSIS, the ranking with respect to the ideal and the anti-ideal solutions are always consistent in the relative distance approach.

The ideal and anti-ideal solutions for the relative distance method are determined without preliminary scaling as follows:

$$I_R^+ = (Y_1^+, Y_2^+, ..., Y_n^+), \tag{16}$$

$$I_R^- = (Y_1^-, Y_2^-, ..., Y_n^-), \tag{17}$$

where

$$Y_j^+ = \min \{Y_{ij}, i = 1, ..., q\},$$
$$Y_j^- = \max \{Y_{ij}, i = 1, ..., q\}, \ , \ j = 1, ..., n.$$

The distance of an alternative to the ideal and anti-ideal solutions is then calculated, respectively, by the formulae:

$$dR_i^+ = \sum_{j=1}^{m} w_j |Y_{ij} - Y_j^+|, \; i = 1, ..., q, \tag{18}$$

$$dR_i^- = \sum_{j=1}^{m} w_j |Y_{ij} - Y_j^-|, \; i = 1, ..., q, \tag{19}$$

$$\sum_{j=1}^{n} w_j |Y_j^- - Y_j^+| = 1,$$

$$w_j |Y_j^- - Y_j^+| \geqslant \epsilon > 0.$$

An alternative with the shortest distance to the ideal solution or the longest distance to the anti-ideal solution is ranked the highest.

The difference in distance measurement between the trade-off ranking, TOPSIS and the relative distance method is illustrated in Fig.1.

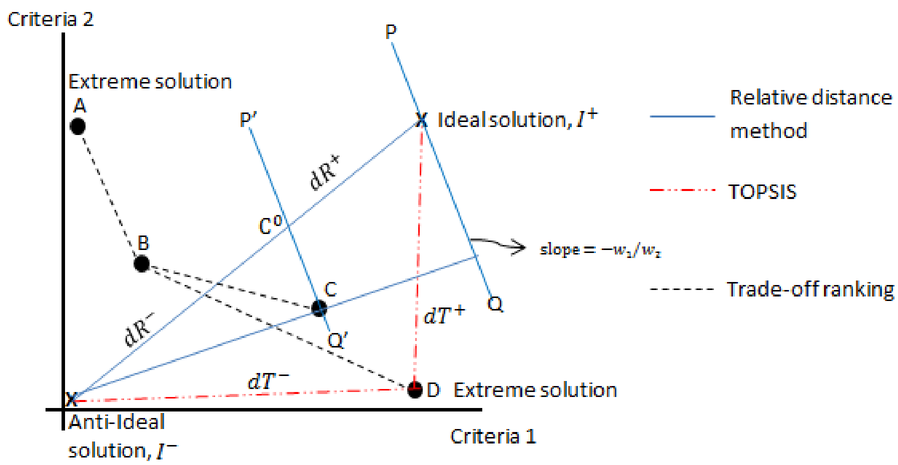

FIGURE 1. Difference in distance measurement between the trade-off ranking, TOPSIS and the relative distance method.

Figure 1 shows the distance measurements of TOPSIS, the relative distance and trade-off ranking methods. Consider four alternatives A, B, C, D, where A and D are the extreme solutions, the ideal and anti-ideal solutions as shown in Fig.1. The intervals between alternatives $[B, A]$ and $[B, D]$ contribute to the distance in the first level of trade-off ranking method where the sum of the distances is $DT1_B$ as formula (5). The second level of trade-off measures the distance of an alternative to all other alternatives, i.e. $[B, A]$, $[B, C]$ and $[B, D]$, using the weighted distance formula in L_2-metric. The sum of them, $DT2_B$, is the basis for the second level trade-off ranking as in formula (8). The intervals $[I^+, D]$ and $[I^-, D]$, represented by dT^+ and dT^- respectively, show the distance in TOPSIS and are also considered in L_2-metric. For the relative distance method, consider line PQ for calculating a relative measurement of each alternative to the ideal/anti-ideal solutions. The line PQ is determined by constructing a straight line with the slope= $-w_1/w_2$ that passing through the ideal solution I^+. Consider line $P'Q'$ with the same slope passing through the alternative C and suppose the interval $[I^-, I^+]$ intersects the line $P'Q'$ at point C^o as shown in Fig.1. Then, C^oI^- is the relative distance of alternative C to the anti-ideal solution, while C^oI^+ is the relative distance of alternative C to the ideal solution. Due to the relative measurement, the distance in the relative distance method is calculated using the L_1 distance metric.

UNCERTAINTY IN THE INPUT DATA

In this section, the first uncertainty in the MCDM process, uncertainty in the input data, is considered. This first type of uncertainty can occur due to a variety of reasons, such as imprecise input parameters, lack of data or inaccurate data during the design process. In multi-objective optimization, the input data are used to generate a set of alternatives for MCDM process. Thus, the uncertainty in the input data may generate a different set of alternatives and give a different ranking solution. In this study, the sensitivity of the MCDM methods to the uncertainty in the input data is considered by using the fuzzy set theory [18], which allows the multi-objective problem to be formulated in a more flexible way for practical applications. The fuzzy theory was used by Erfani and Utyuzhnikov [2010] to handle the uncertainty in the variables and to develop a robust design of the multi-objective optimization problem by finding a less sensitive solution to the uncertainty of the model. There are several other authors who implemented the fuzzy theory in MCDM methods and application problems [20–22].

By using fuzzy numbers, problem (1) can be transformed into a multi-objective fuzzy constrained problem as:

$$\begin{aligned} &\text{Minimize } \widetilde{Y} = \{\widetilde{Y}_1(x), \widetilde{Y}_2(x), ..., \widetilde{Y}_n(x)\}, \\ &\text{subject to } \widetilde{g}_k(x) \leq \widetilde{b}_k, \ , \ k = 1, ..., m, \\ &\qquad x \in D, \end{aligned} \tag{20}$$

where the tilde denotes that the problem is modelled using fuzzy variables.

The fuzzy problem (20) is solved by transforming the fuzzy numbers into a crisp value. To do so, the crisp possibilistic mean value is used [23]. The crisp possibilistic mean value of fuzzy numbers is given by the formulae:

$$Mean(A) = a + \frac{c-b}{6}. \tag{21}$$

Using formula (21) (see Appendix), the fuzzy problem (20) is then converted into a deterministic formulation by substituting the fuzzy variables with their crisp values. Thus, problem (20) is reduced to

$$\begin{aligned} &\text{Minimize } Y^{mp} = Y_1^{mp}(x), Y_2^{mp}(x), ..., Y_n^{mp}(x) \\ &\text{subject to } g_k^{mp}(x) \leq b_k^{mp} \ , \ k = 1, ..., m, \\ &\qquad x \in D, \end{aligned} \tag{22}$$

where the mp notation denotes the possibilistic mean values.

Solution to the deterministic problem (22) gives a set of solutions called the Possibilistic Mean Pareto optimal solution according to the following definition:

Definition: (Possibilistic Mean Pareto Optimality) Vector $x^* \in D$ is called the Possibilistic Mean Pareto optimal of problem (22) iff there does not exist any $h \in D$ such that $Y_j^{mp}(h) \leq Y_j^{mp}(x^*)$ for any $j = 1, ..., n$ and exists $l \leq n : Y_l^{mp}(h) < Y_l^{mp}(x^*)$.

The set of optimal solutions for problem (22) is then treated as Pareto alternatives in MCDM process, where a ranking algorithm is imposed to find the best option.

Robust Set of Alternatives

The DM may prefer a robust (stable) set of solutions, which minimizes the variation of input parameters, rather than an optimal one. In finding for a robust set of solutions, a new function, the measure of robustness, is introduced [19], and is defined as follows:

$$R = \frac{1}{nm} \sum_{j=1}^{n} \sum_{k=1}^{m} \frac{\sigma Y_j}{\sigma x_k}, \tag{23}$$

where m is the number of design variables/parameters which vary, and n is the number of objective functions. The denominator σx_k, which denotes the variance of the parameter, is calculated by the variance of fuzzy number [23] given by (see Appendix):

$$Var(A) = \frac{(b+c)^2}{24}. \tag{24}$$

In turn, the numerator σY_j, which is the variance of the objective function, is calculated using the first order Taylor series [24] as follows:

$$\sigma_{Y_j}^2 = \sum_{j=1}^{n} \left(\frac{\partial Y_j}{\partial p} \right)^2 \sigma_p^2, \tag{25}$$

where p is the uncertain parameters of the model. Here, Eqn.(24) is also used to calculate the variance of the parameters, σ_p.

Eqn.(23) is minimized to find a robust set of solutions, since minimization of R leads to a smaller value of σY_j, despite having greater value of σx_k. Hence, R is added as a new objective function for multi-objective problem (22). Therefore, in searching for a robust set of solutions in multi-objective optimization, problem (22) is then converted to

$$\begin{aligned} &\text{Minimize } Y^{mp} = \{Y_1^{mp}(x), Y_2^{mp}(x), ..., Y_n^{mp}(x)\}, \\ &\text{Minimize } R, \\ &\text{subject to } g_k^{mp}(x) \leq b_k^{mp}\ ,\ k = 1, ..., m, \\ &\qquad x \in D. \end{aligned} \tag{26}$$

The set of robust solutions is then ranked with the MCDM methods. The rankings for both, Possibilistic Mean Pareto and robust solutions, are demonstrated with a test case in the next subsection.

Test Case: Two-bar Truss Structure

In this subsection, a test case is considered to see the difference in the ranking solutions with the alternatives obtained from the Possibilistic Mean Pareto design (22) and the robust design (26). The difference reflects the ranking effect of each MCDM method with respect to the uncertainty in the input data.

The test case is introduced by Messac and Ismail-Yahaya [25] and is shown in Fig.2. The design variables are the diameter of members x_1, and the height of structure x_2. There are two conflicting objective functions to be considered, which are to minimize the total mass of truss members, and to minimize the deflection due to the load $F = (150, 20, 30)$ kN. The parameters of the problem are: member thickness, $t = (2.5, 0.5, 1.5)$ mm, structure width, $w = (750, 100, 50)$ mm, mass density, $\rho = 7.8 \times 10^{-3}$ gr/mm^3 and elastic modulus, $E = 210000$ Nmm^2. The constraints are as follows: the normal stress has to be less than the buckling stress, $1 \leq x_1 \leq 100$ and $10 \leq x_2 \leq 1000$. As can be seen, there are three uncertain variables/parameters in this problem, F, t and w, have been set as the triangular fuzzy numbers.

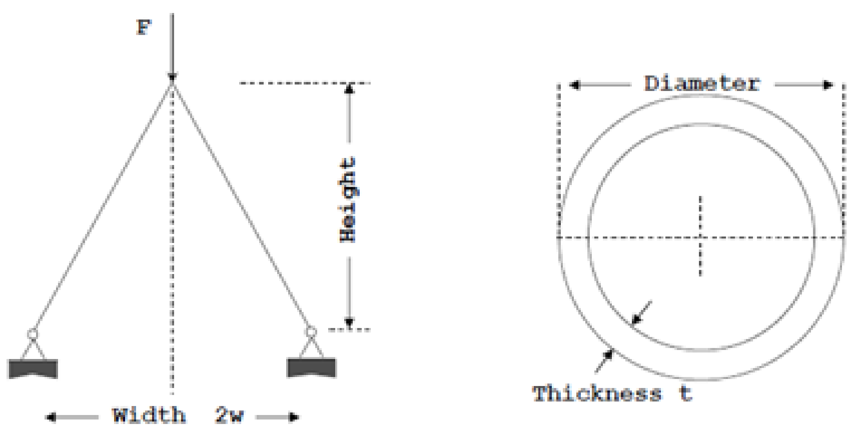

FIGURE 2. Two-bar truss structure.

The full formulation of the problem is as follows:

$$
\begin{aligned}
\text{Minimize Mass, } F_1 &= 2\pi p t x_1 \sqrt{w^2 + x_2^2},\\
\text{Minimize Deflection, } F_2 &= \frac{F(w^2 + x_2^2)^{3/2}}{(2\pi t E x_1 x_2)^2},\\
\text{subject to } s &\leq \frac{1}{8}\pi^2 E \frac{t^2 + x_1^2}{w^2 + x_2^2},\\
1 &\leq x_1 \leq 100,\\
10 &\leq x_2 \leq 1000,
\end{aligned}
$$

where

$$
s = \frac{F}{2\pi t x_1 x_2}\sqrt{w^2 + x_2^2}.
$$

The triangular fuzzy parameters are substituted by their crisp possibilistic mean values of $F = 151.6$ kN, $t = 2.66$ mm and $w = 741.6$ mm (21). Using these deterministic values of F, t and w, problem (22) is then solved to obtain the Possibilistic Mean Pareto solutions.

The robust measure is constructed with the variance of 1020.6, 0.4 and 30.61 for F, t and w, respectively, as follows:

$$
R = \frac{1}{6}\left(\left(\frac{\sigma F_1}{\sigma F} + \frac{\sigma F_1}{\sigma t} + \frac{\sigma F_1}{\sigma w}\right) + \left(\frac{\sigma F_2}{\sigma F} + \frac{\sigma F_2}{\sigma t} + \frac{\sigma F_2}{\sigma w}\right)\right)
$$

.

Both the numerators σF_1 and σF_2 are calculated using Eqn.(25) with respect to the uncertain parameters F, t and w.

Both problems (22) and (26) are solved with the Directed Search Domain (DSD) algorithm [13]. Erfani and Utyuzhnikov [2010] have tackled the robustness problem in their paper. Their work here is now extended to the ranking problem. For each set of alternatives obtained, the MCDM methods are then used to solve the ranking problem with the assumption of equal weights.

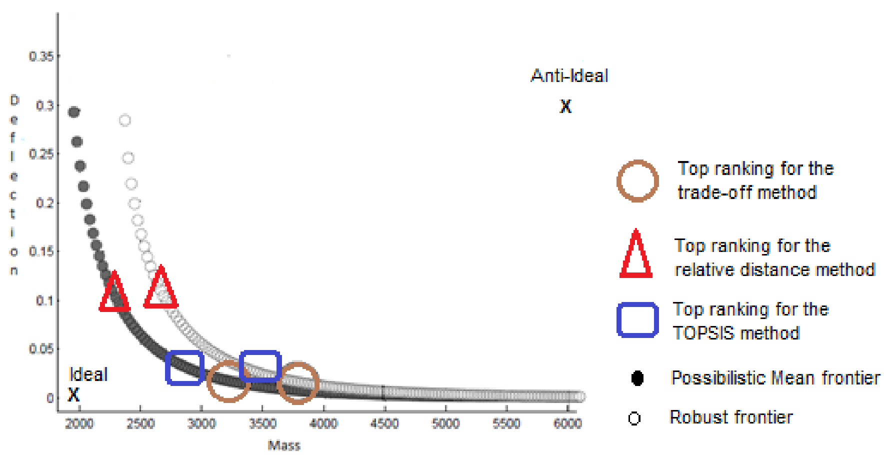

FIGURE 3. Results of the top ranking for Possibilistic Mean and robust Pareto frontier.

Figure 3 shows the results of the top ranking for Possibilistic Mean and robust Pareto frontiers with three different MCDM methods: trade-off ranking, relative distance method and TOPSIS. For these methods, the top ranking areas are marked by the circle, triangular and rectangular, respectively. As can be seen from Fig.3, the top ranking solutions for both the Possibilistic Mean Pareto frontier and the robust one are different for each method. The DM has the option to choose between the Mean Pareto optimal solution and the robust solution. The highest trade-off ranking for the robust frontier is situated in the region, which differs significantly less from the appropriate solutions on the Possibilistic Mean Pareto frontier, than that for any other method. Therefore, it is advisable for the DM to opt for this robust solution, as it is not only robust but also near to the mean Pareto optimal value, with a lower deflection and greater mass. The trade-off ranking method selects the region on the Pareto frontier with the lowest reasonable deflection. Any further decrease of deflection leads to a significant increase of mass.

In the case of the TOPSIS and the relative distance, the top rankings prove to be significantly different for the Possibilistic Mean and robust frontiers. Intuitively, it is preferable to design a truss with a larger mass for obtaining less deflection. This is consistent with the solution provided by the trade-off ranking method in the indicated region of Fig. 3. By testing this first level of sensitivity analysis, the DM is able to determine a set of robust alternatives in MCDM process. Moreover, the DM is able to gain insight into different solutions provided by the MCDM methods and make the best decision out of all solutions.

UNCERTAINTY ON THE PREFERENCE OF EACH CRITERION

In this section, another type of uncertainty in the MCDM process is considered. In the decision making process, the DM may be uncertain about their preferences. The DM may have ambiguity, for instance, in which criterion they prefer the most or how much the preference is, or they do not know exactly how much they prefer a certain alternative. In the decision making process, the weights assigned to the decision criteria represent the importance of the criteria or the preferences of the DM. The criterion with the highest weight is the most important. The DM can make better decision if he/she is able to determine how sensitive the current ranking of the alternatives to the changes of the weights of the decision criteria. The problem can be tackled via the sensitivity analysis of the criteria weights.

Computational Experiment

A computational experiment is undertaken to study how sensitive each MCDM method to the changes of the weight of each criterion. A traditional approach is used, where random weights under constraints are generated for the sensitivity analysis. The random weights reflect the various techniques of weight evaluations as applied to different MCDM processes [17, 26–33]. Regardless of the techniques used, the obtained weights always have deterministic values. The results of the experiment are shown in some extreme cases of weights. The test cases for the experiment are solved with the DSD algorithm [13] for generating a well-distributed set of Pareto solutions. For the ranking analysis, the trade-off ranking method is compared against TOPSIS and the relative distance method.

In all the test cases, shapes are used to show the top choices for each method. The rectangular shape is used for the trade-off ranking method; the triangular, for the relative distance approach; and the circle, for TOPSIS.

ZDT2 problem: the test case is introduced by Shukla and Deb [2007]. The formulation of the problem is given as follows:

$$\begin{aligned}
&\text{Minimize } (F_1(x), F_2(x)) \\
&\text{subject to } 0 \leq x_i \leq 1 \ (i = 1, 2, ..., n) \\
&\quad \text{where} \\
&\quad F_1(x) = x_1 \\
&\quad F_2(x) = g(x)\left(1 - (x_1/g(x))^2\right) \\
&\quad g(x) = 1 + 9/(n-1)\sum_{i=2}^{n} x_i^2
\end{aligned}$$

The optimization problem generates a non-convex Pareto frontier as shown in Fig.4. The results for the best solutions for each ranking approach for the selected weights are also shown in the same figure.

The extreme solutions to this problem is (0,1) and (1,0). As seen in Fig.4, the preferable choices in the trade-off ranking method vary as the weight changes. As F_1 is more preferable than F_2, i.e. $w_1 > w_2$, the top choices in the trade-off ranking are skewed to the F_2 area in the graph. In turn, if the DM opt for criterion F_2 than F_1, the preferable

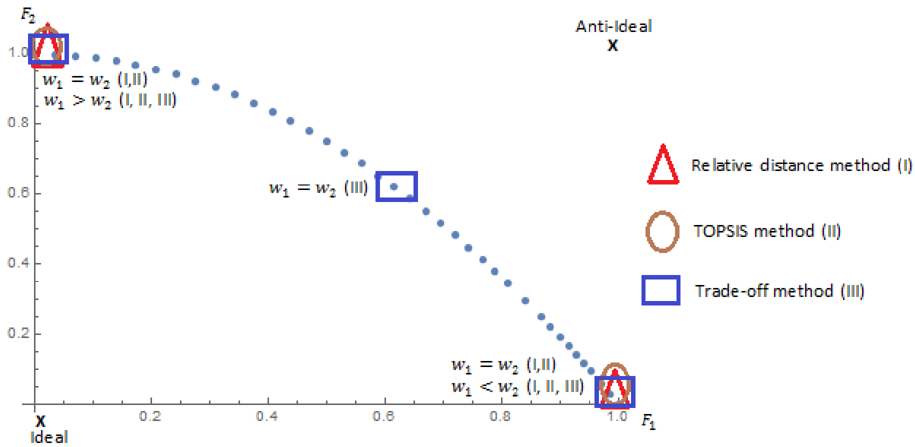

FIGURE 4. Top ranking for each method with each weight case for ZDT2.

choices are on the F_1 area. As the weights are equal ($w_1 = 0.5, w_2 = 0.5$), the top solutions are at the center of the graph, implying a solution with the least compromise among others. In the equal weights case, the second level of trade-off is imposed as the first level calculation revealed the same minimum value of $DT1$. The same ranking situation occurs in the relative distance and TOPSIS methods. They are only differ for the case of equal weights between criteria.

Similar to the trade-off ranking method, as criterion F_1 is more preferable to criterion F_2, the top choices for the relative distance and TOPSIS methods are situated on the left-hand side of the graph. Otherwise, the top ranking alternatives are situated on the right-hand side region.

For the equal weights between criterion F_1 and criterion F_2 ($w_1 = 0.5, w_2 = 0.5$), the top choices for the relative distance method reveal two extreme cases, in which both solutions (0,1) and (1,0) give the same minimum value for the ranking. There is a huge difference in choosing (0,1) as the opposite to (1,0), which implies that it is necessary to choose one criteria while completely ignoring the other. Such ranking solutions occur because the problem has the same extreme values for both criteria, which are $F_1 = 1$ and $F_2 = 1$. Thus, the rankings depend on the weights, i.e. the DM preferences. Therefore, when the weights are equal, the two best solutions occur. In the case of having two or more of the best solutions, TOPSIS and the relative distance methods do not have an extra algorithm to tackle this kind of problem.

DTLZ5 problem: the three dimensional test case is introduced by Deb et.al. [2005].

$$
\begin{aligned}
&\text{Minimize } \ (F_1(x), F_2(x), F_3(x)) \\
&\text{subject to } 0 \le x_i \le 1 \ \ (i = 1, 2, 3) \\
&\quad \text{where} \\
&\qquad F_1(x) = (1 + g(x_3)) \cos(\theta_1) \cos(\theta_2), \\
&\qquad F_2(x) = (1 + g(x_3)) \cos(\theta_1) \sin(\theta_2), \\
&\qquad F_3(x) = 3\,(1 + g(x_3)) \sin(\theta_1), \\
&\qquad g(x) = (x_3 - 0.5)^2 \\
&\qquad \theta_1 = \frac{\pi}{2}\,(x_1) \\
&\qquad \theta_2 = \frac{\pi}{4\,(1 + g(x_2))}\,(1 + 3g(x_3)x_2)
\end{aligned}
$$

In this case, the same alternative, (0,0,1), is chosen twice as the extreme solutions since it has the minimum value in two criteria, F_1 and F_2. The third extreme solution, implies an alternative with the minimum value in criteria F_3, is (0.71,0.71,0). The best alternatives for each ranking method with several chosen weight cases are shown in Fig.5. The chosen weights are given in Table 1.

TABLE 1. Weight cases for DTLZ5

Weight case	*a*	*b*	*c*	*d*	*e*	*f*
Weights	$w_1 = 1$ $w_2 = 0$ $w_3 = 0$	$w_1 = 0$ $w_2 = 1$ $w_3 = 0$	$w_1 = 0$ $w_2 = 0$ $w_3 = 1$	$w_1 = 1/3$ $w_2 = 1/3$ $w_3 = 1/3$	$w_1 = 0.25$ $w_2 = 0.25$ $w_3 = 0.5$	$w_1 = 0.3$ $w_2 = 0.3$ $w_3 = 0.4$

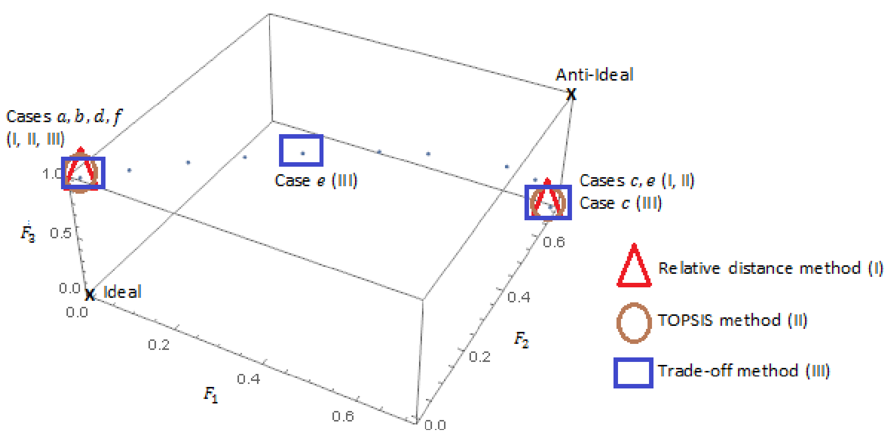

FIGURE 5. Top ranking for each method with each weight case for DTLZ5.

In Table 1, the weight cases *a*, *b* and *c* represent the importance in only one criterion, either F_1, F_2 or F_3, at a time. In turn, the case *d* represents an equally importance criteria.

In Fig.5, the rectangular shapes represent the top choices for the specified weight cases in the trade-off ranking method. The triangular shapes imply the top rankings for the relative distance approach, and the circles denote the top ranking for TOPSIS. From the results shown, it follows that the top ranking in the trade-off ranking method, TOPSIS and the relative distance approach are extreme solutions (0,0,1) and (0.71,0.71,0) as the weight changes. In this problem, the trade-off ranking method gives the same top solution as TOPSIS and the relative distance method. In the cases of *a*, *b* and *c*, the top solution in the trade-off ranking method is an alternative with the best value in each important criterion, respectively. For example, (0,0,1) is the best solution for the cases *a* ($w_1 = 1$) and *b* ($w_2 = 1$) as it has the minimum values at $F_1 = 0$ and $F_2 = 0$.

Apart from other similar ranking results with TOPSIS and the relative distance approach, the trade-off ranking gives the middle point as the best solution in a specific weights case *e*, which are $w_1 = 0.25, w_2 = 0.25$ and $w_3 = 0.5$. In the case *e*, the extreme solutions are said to be equally preferred by the DM as the weights w_1 and w_2 represent the importance of the same extreme solution (0,0,1). In this weight case, the trade-off ranking algorithm is imposed further to the second level, i.e. $DT2$, and hence the middle solution. The result of this weight case is different for the other two methods, in which they give (0.71,0.71,0) as the best solution. Supposedly, in the relative distance method and TOPSIS, it is an alternative that closest to the ideal (0,0,0) and farthest from the anti-ideal (0.71,0.71,1) solutions.

For the equal weights case *d*, the three MCDM methods give the same top solution (0,0,1). The equal weights represent equally important criteria among the three F_1, F_2 and F_3. Again, in the trade-off ranking method, the weights $w_1 = 1/3$ and $w_2 = 1/3$ represent the importance of the same extreme solution (0,0,1) since it is the alternative with the best value in criteria F_1 as well as F_2. Therefore, whenever the preferences towards both criteria F_1 and F_2 exceed the preference towards the third criteria F_3, the extreme solution (0,0,1) is the top solution with the trade-off ranking method. As an example, in the weight case d, the extreme solution (0,0,1) is preferred 2/3 times more than the extreme solution (0.71,0.71,0). The same ranking occurs in the weight case f where the importance of extreme solution (0,0,1) is 0.6 ($w_1 + w_2$) compared to the extreme solution (0.71,0.71.0) which is only 0.4 (w_3).

Objective Weights via the Trade-off Ranking Method

There are two types of weights in a MCDM problem, the subjective and objective weights. The subjective weight is the weight determined by the DM since the human judgements may be affected by their past experience, intuition, biased and etc. On the other hand, the objective weight is calculated from the data. By obtaining the weight via a data calculation, we minimize the uncertainty in the DM preference, giving a set of weights that would optimize the ranking calculation. In this section, we show how to determine the objective weights using the trade-off ranking method.

The weights are determined by minimizing formula (5) such as:

$$\begin{aligned}
&\text{Minimize} \sum_{i=1}^{q}\left[\sum_{j=1}^{n} w_j[d_{TOR1}(A_i, A_k)]\right], i = 1, ..., q; k = 1, ..., n,\\
&\text{subject to} \sum_{j=1}^{n} w_j = 1,\\
&\quad w_j \geqslant 0,\ \ j = 1, ..., n,\\
&\quad \text{where}\\
&\quad d_{TOR1}(A_i, A_k) = \left[\sum_{j=1}^{n}\left(f_{ij} - f_{kj}\right)^2\right]^{1/2},\\
&\quad f_{ij} = \frac{Y_{ij} - \min_j Y_{ij}}{\max_j Y_{ij} - \min_j Y_{ij}}.
\end{aligned} \tag{27}$$

The result of optimization problem (27) is a set of weights that would minimize the total distances of the alternatives and the extreme solutions. Therefore, if the data are skewed to one of the extreme solutions, the result is a set of optimal weights that would give such an extreme solution as the best option. As examples of the optimization results, consider the two trivial data sets as shown in Fig.6.

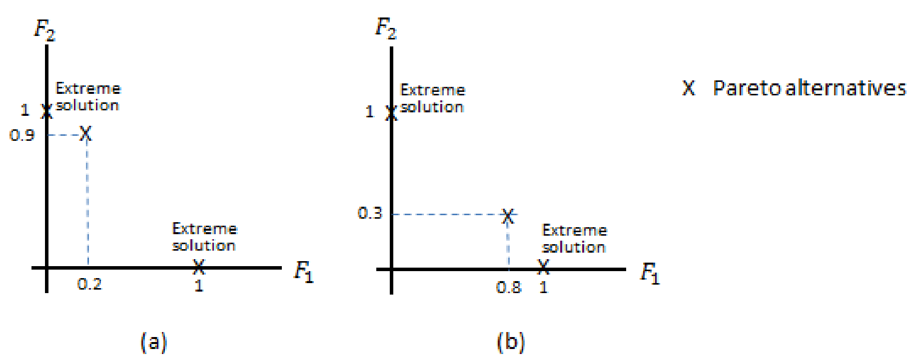

FIGURE 6. Examples for the objective weights calculation.

Figure 6 shows two graphs, 6(a) and 6(b), as examples of the objective weights calculation using the trade-off ranking method for problem (27). In Fig.6(a), the Pareto solutions are skewed to the extreme solution of F_2, while the data in Fig.6(b) show otherwise. By solving optimization problem (27) using the set of alternatives in Fig.6(a), we obtain the optimal weights of $w_1 = 1$ and $w_2 = 0$. The optimal weights give the extreme solution (0,1) as the best option out of three Pareto alternatives in Fig.6(a) with the trade-off ranking method. As can be seen, the extreme solution (0,1) is situated in the F_2 area, which is the area where the data in Fig.6(a) are skewed.

On the other hand, data in Fig.6(b) give $w_1 = 0$ and $w_2 = 1$ as the optimal weights. Such weights give the extreme solution (1,0) as the best option in Fig.6(b). The results are consistent with the data in Fig.6(b) which are scattered towards the F_1 area. Thus, the optimization result for problem (27) depends on the distribution of the data set on the Pareto frontier. If the data are evenly distributed, optimization problem (27) gives finitely many sets of optimal weights as the result. In such a case, Eqn.(7) and Eqn.(8) should be employed for the ranking calculation using the equal weights.

As mentioned before, imposing the objective weights may minimize the uncertainty in the DM preferences. However, when the DM preferences are available, the subjective weights should be considered instead of the objective weights.

CONCLUSION

This paper presents a sensitivity analysis of the distance-based ranking techniques, TOPSIS and the relative distance, including a new MCDM method, the trade-off ranking. Two types of uncertainties in MCDM process: in the data parameters/variables and in the preferences of the Decision Makers towards each criterion are tested. In the data-uncertainty analysis, the fuzzy numbers have been implemented and a new objective function has been introduced in finding a new robust set of alternatives. In the preference-uncertainty analysis, different weights have been used to represent variation in the preferences of the Decision Maker. The effects of the ranking solutions in each analysis have been studied for several test cases.

The first analysis, uncertainty in the data variables, is important for the DM since it provides a robust set of alternatives despite the perturbation in the input data. The DM can also gain insight into different solutions provided by the different MCDM methods. The second analysis, uncertainty in the criteria weights, is valuable because it gives an overall view of the DM preferences. In the second analysis the trade-off ranking method emphasizes on the importance of the criteria. If the DM choose some criteria far greater than the others, the trade-off ranking method is able to give a solution with the best value in the preferred criteria, i.e. one of the extreme solution. Thus, the trade-off ranking method can be acceptable to a different type of DM preferences in the same problem. In comparison to TOPSIS and the relative distance method, the trade-off ranking method has an extra algorithm to tackle a problem with more than one solution. The extra algorithm in the trade-off ranking method is relevant to the conflicting criteria problem. In such a problem, there is no unique solution. One solution may be better than the other, depending on the criteria weights or the DM preferences. Hence, when the criteria are equally preferred, the extra algorithm gives the least-compromise option as the best solution. Finally, the trade-off ranking method gives a solution that may preferred by the DM from the range of the extreme solutions to the least compromise among alternatives.

ACKNOWLEDGMENTS

The first author gratefully acknowledges the research scholarship awarded by the Ministry of Higher Education Malaysia and the University of Malaysia Pahang.

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APPENDIX

Suppose $A = (a, b, c)$ is a triangular fuzzy number where $b > 0$ and $c > 0$ are the left and right-width of the fuzzy number centered at a. Therefore, the α-cut of A is computed by,

$$A_\alpha = [a - (1-\alpha)b, a + (1-\alpha)c] \; \forall \alpha \in [0, 1].$$

The Possibilistic mean value of A is the arithmetic mean of its lower and upper possibilistic mean value [23], i.e.

$$Mean(A) = \frac{(\underline{M}(A) + \overline{M}(A))}{2},$$

where

$$\underline{M}(A) = 2\int_0^1 \alpha \underline{A} d\alpha, \; \overline{M}(A) = 2\int_0^1 \alpha \overline{A} d\alpha.$$

Here, $\overline{A}$ and $\underline{A}$ are the upper and lower bounds of α-cut of fuzzy number A, respectively. It is easy to prove that the possibilistic mean value of the fuzzy number is given by

$$Mean(A) = a + \frac{c - b}{6}.$$

The variance of a fuzzy number is given by the formulae [23],

$$Var(A) = \frac{1}{2}\int_0^1 \alpha(\overline{A} - \underline{A})^2 d\alpha.$$

Again, it is easy to prove that the variance of fuzzy number A can be calculated as

$$Var(A) = \frac{(b + c)^2}{24}.$$