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Mathematical modelling of wave overtopping at vertical structures

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Abstract. Considering the threats of wave overtopping that can cause flooding in the coastal area and hence implying various damages to the inland properties, this paper provides a simplified mathematical modelling of wave overtopping calculation at vertical seawall as proposed by Cooker. Taking the assumption that the water jet particles projected against the seawall move as free projectiles, a simple estimation is made from the computation of the water displacement which can pass over top of the wall of finite height by applying the trapezoidal rule to the equations derived from both energy conservation and the pressure impulse, *P* previously proposed by Cooker. The results indicate that the overtopping volume per wave is exponentially decaying with the height of the wall above the still water level and is in good agreement to previous researcher and is used to estimate the overtopping discharge for berm and ditch structures. Both cases are comparable for corresponding freeboard values which concluded that the water jet onto the seawall with berm structure will reach higher in the air compared to that of with ditch.

Keywords: Overtopping; Vertical structure; Mathematical model; Freeboard; Wave impact.

1. Introduction
Rapid development and urbanization in recent years accommodating global population growth together with climate change effects has led to the increasing urge to protect the coastal areas and their resident populations. The observed trend of increased storm intensity and many coastal zones throughout the world that are already exposed to tsunami risks are close examples as a result from climate changes. All these trends combined demonstrate the need for reliable and sustainable flood protection to prevent flooding coastal in which will also reduce the wave overtopping. Overtopping discharge is said to occur when the wave height exceeds the crest of a flood defence and running up the face of the structure.

EurOtop (2007) in [1] states that relatively small vertical walls placed on top of a slope contributes significantly on the reduction of wave overtopping. Thus, while focusing on avoiding or reducing the formation of flooding coastal area, an important criterion for the design of a seawall or any flood defence would be the allowable degree of wave overtopping, so freeboard height must be taken into

consideration as it tends to compensate all the unknown factors that would contributes to run up waves. Owen (1980) in [2] was amongst the earliest to carry out an extensive series of model tests to determine the overtopping discharges for a range of seawall designs subjected to different random wave climates. There are also other experimental approaches based on dimensional analysis and regression of experimental data to estimate the overtopping discharge of seawalls by means of laboratory test by Salauddin and Pearson (2018) in [3] and Franco et.al (1994) in [4], full-scale measurement conducted by Pullen et.al (2004) in [5] and also Pearson et.al (2002) where they performed series of experiments under the VOWS collaborative projects in [6].

Apart from experiments, there was also numerous numbers of theoretical approaches that have been made by researchers to predict the overtopping discharge at coastal structures based on different parameters and coefficients they used in their research. A theoretical study by Jervis and Peregrine (1996) showed that the overtopping volume per wave is roughly exponentially decaying with the height of wall above the still water level in [7]. Allsop et.al (2005) in [8] presented a summary of prediction methods for wave overtopping and highlighted that the two wave conditions need to use different prediction tools. A prediction tool and hazard analysis of wave overtopping also was given by Geeraerts et.al (2007) in [9] showing that seawalls reduce wave overtopping but do not stop it. Other prediction formula was also performed by Etemad et.al (2016) in [10] and Pillai et.al (2017) in [11].

As in [12,13], Cooker and Peregrine (1990, 1995) proposed a mathematical model for pressure impulse theory to study the wave impact at vertical seawall. Thus, in this presented paper, we will continue using Cooker's ideal model with the solution of Laplace's equation in the Fourier series form for the boundary-value problem using hyperbolic terms as in equation (1).

$$P(x, y; \mu) = \sum_{n=0}^{\infty} a_n \sin\left(\frac{\lambda_n y}{H}\right) \frac{\sinh\left(\frac{\lambda_n (x - B)}{H}\right)}{\cosh\left(\frac{\lambda_n B}{H}\right)} \tag{1}$$

with $a_n = \int_{-\mu H}^{0} \frac{-2\rho U}{\lambda_n} \sin\left(\frac{\lambda_n y}{H}\right)$ and $\lambda_n = \left(n + \frac{1}{2}\right)\pi$

2. Simplified model of wave overtopping calculation

In this section, a simplified model of wave overtopping calculation for a vertical seawall is illustrated and we use Cooker's model to calculate the overtopping discharge for the model. This will results in an estimation of the maximum quantity of water that could possibly pass over the crest of the seawall. In this case, to consider the allowable degree of wave overtopping, we refer freeboard which is the vertical distance between still water level and top crest of the vertical seawall as a manipulative variable to the relationship with the run-up wave quantity. The wave overtopping discharge is defined as overtopping volume [m^3] per time [s] and structure width [m]. The sketch of the simplified overtopping model is illustrated as in Figure 1.

In this paper, we consider the case where waves slosh against a vertical structure sending a jet of water up to a height that possibly is as much as three times the incident wave height. Considering that the jet of fluid close to the seawall is thin and that the pressure gradients in it are low, the jet particles are assumed to move as free projectiles. Hence the maximum height achieved by the jet is given in equation (3) which was derived from equation (2) of the energy conservation of the water jet:

$$mgh' = \frac{1}{2} m v_y'^2 \tag{2}$$

$$h' = y_{max} = \frac{v_y'^2}{2g} \tag{3}$$

Non-dimensionalising equation (3) gives us

$$y_{max} = \frac{1}{2} F_r v_y{}^2 \tag{4}$$

where $F_r = U_0^2 / gH$ is the Froude number with is U_0 is the impact speed and H is the total water depth at time of impact. Since the initial upwards velocity (before impact) is zero the velocity afterwards is simply given by equation (5) from equation (1) as follows:

$$v_y = \frac{\partial P}{\partial y} = \sum_{n=1}^{\infty} a_n \lambda_n \cos(\lambda_n y) \frac{\sinh(\lambda_n(x-b))}{\cosh(\lambda_n b)} \tag{5}$$

giving equation (6) as the initial upward velocity at $y = 0$

$$v_y = \lambda_n \frac{\sinh(\lambda_n(x-b))}{\cosh(\lambda_n b)} \tag{6}$$

Hence, the maximum vertical height of jet, y_{max} is given by equation (7)

$$y_{max} = \frac{1}{2} F_r \left(\sum_{n=1}^{\infty} a_n \lambda_n \frac{\sinh(\lambda_n(x-b))}{\cosh(\lambda_n b)} \right)^2 \tag{7}$$

with the Fourier coefficients are those of equation (1). Through this paper, we are interested in knowing when y_{max} exceeds F_b, the freeboard height and hence finding F_b in terms of x_b, the distance at which F_b is achieved as illustrated in Figure 1. Thus, for freeboard height we have

$$F_b = \frac{1}{2} F_r \left(\sum_{n=1}^{\infty} a_n \lambda_n \frac{\sinh(\lambda_n(x_b-b))}{\cosh(\lambda_n b)} \right)^2 \tag{8}$$

The series in (8) diverges at $x = 0$ while converges for $x > 0$ since for large n, the terms in (8) are in the form of $e^{-\lambda_n x} / \lambda_n$
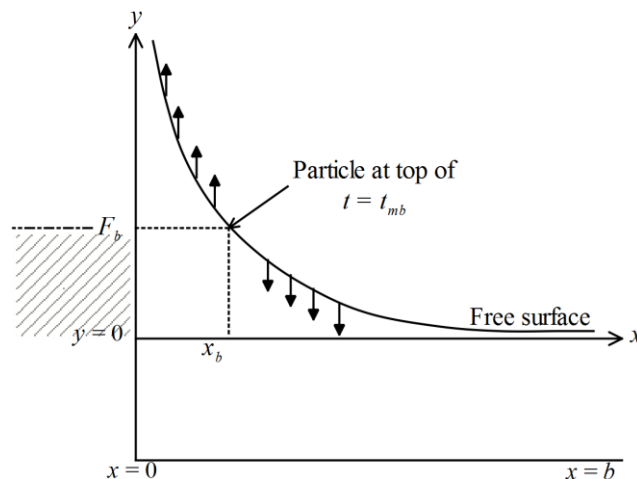

Figure 1. The sketch of the simplified overtopping model

Using both equation (7) and (8), we can then estimate the overtopping discharge, V, as

$$V = \int_0^{x_b} \left(y_{\max} - F_b \right) dx$$

i.e.
$$V = \int_0^{x_b} y_{\max} dx - x_b F_b \tag{9}$$

Taking the shallow water theory as assumption with the impact velocity U_0 to be the wave speed $\sqrt{gH}$ gives $F_r = 1$ and the integration can simply be found numerically using the trapezoidal rule.
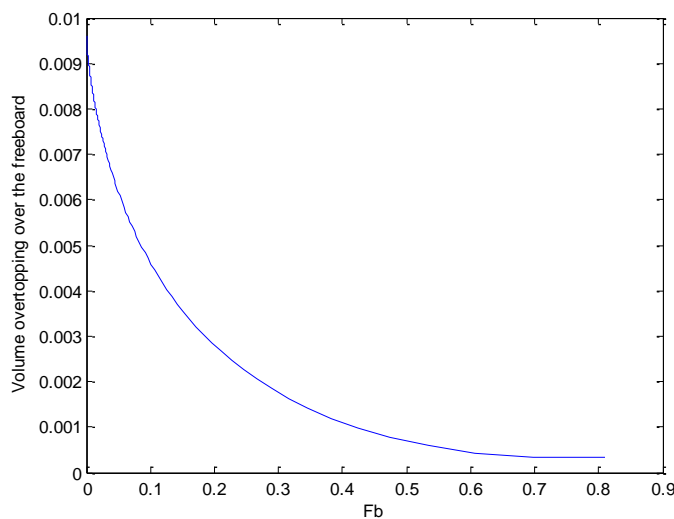

Figure 2. The Overtopping discharge for Cooker's Model for $\mu = 0.1$ Give the units here i.e. F_b is the dimensionless freeboard reference to depth H and V is volume overtopping per unit length of seawall divided by H^2

Figure 2 shows that the overtopping volume decreases as the freeboard height increases, but since we assume earlier that the motion of the particles in jet is free projectiles, then the model will not be valid for small values of F_b. For engineering purposes, it is instructive to translate this result where we need to convert the freeboard and overtopping volume per unit seawall length to dimensional quantities by multiplying by the length scale H and H^2 respectively. With $H = 2m$, $\mu = 0.1$ and $F_b = 0.5$, we have approximately $V = 0.001 H^2$ which is equal to $0.004 m^2$. Through this model, we arbitrarily assume one 100 s wave in every 10 waves impacts onto the wall, resulting in $V = 4 \times 10^{-5} m^3 s^{-1} \approx 0.04 \, l \, s^{-1}$ per metre frontage. This result is comparable with the rules given by Franco et.al (1994) but unfortunately comparison with the VOWS model in [8] is not possible because some of the necessary parameters were not published.

3. Overtopping discharge volume for berm and ditch
Other vertical structures such as berm and ditch have been discussed in detail by Noar (2015) in [14] as Noar investigated the pressure impulse on both structures using Cooker's pressure impulse theory and solve all the eigenfunctions numerically. Thus, in this paper we would also presenting simplified models of wave overtopping for both berm and ditch problems as sketched in Figure 3. All the parameters remain the same as discussed earlier but with different P as found in [14] for berm and ditch problems.
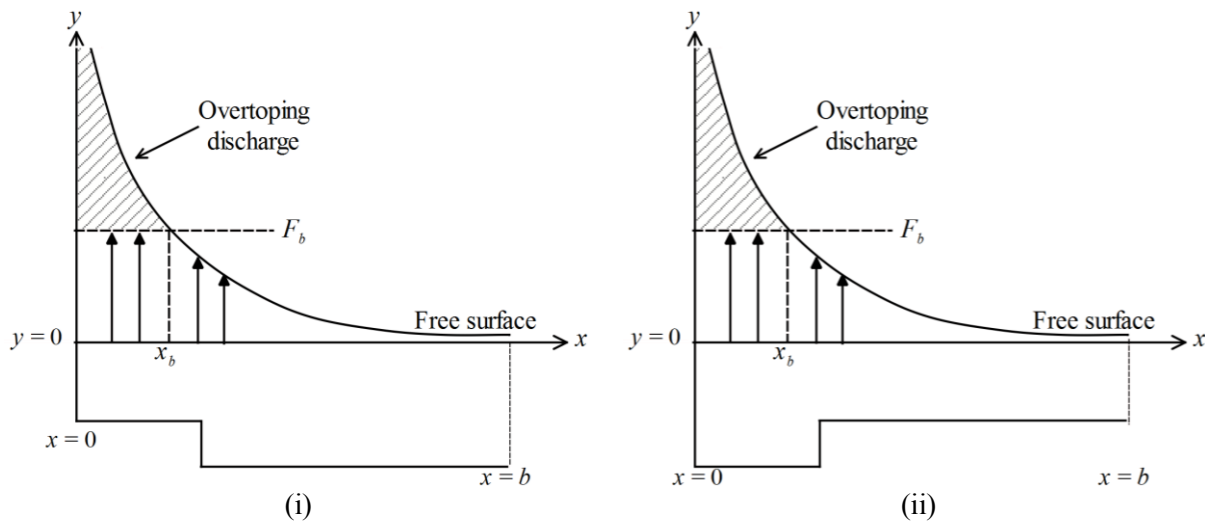
Figure 3. The sketch of the simplified overtopping model for (i) berm and (ii) ditch

Since berm and ditch problems share the same parameters except for the seabed in inner and outer regions, then using the same method used earlier, equation (10) and equation (11) gives the maximum vertical height of jet, y_{max} and the freeboard height F_b for berm and ditch problems respectively.

$$y_{max} = \frac{1}{2} F_r \left(\begin{array}{c} \sum_{n=1}^{\infty} \frac{\lambda_n}{H_b} \frac{1}{\cosh\left(\frac{\lambda_n B_1}{H_b}\right)} \left\{ \alpha_n \sinh\left(\frac{\lambda_n (x - B_1)}{H_b}\right) + \beta_n \cosh\left(\frac{\lambda_n (x - B_1)}{H_b}\right) \right\} \\ + \sum_{n=1}^{\infty} \frac{c_n \lambda_n}{\cosh(\lambda_n B_2)} \sinh(\lambda_n (x - B_2)) \end{array} \right)^2 \tag{10}$$

$$F_b = \frac{1}{2} F_r \left(\begin{array}{c} \sum_{n=1}^{\infty} \frac{\lambda_n}{H_b} \frac{1}{\cosh\left(\frac{\lambda_n B_1}{H_b}\right)} \left\{ \alpha_n \sinh\left(\frac{\lambda_n (x_b - B_1)}{H_b}\right) + \beta_n \cosh\left(\frac{\lambda_n (x_b - B_1)}{H_b}\right) \right\} \\ + \sum_{n=1}^{\infty} \frac{c_n \lambda_n}{\cosh(\lambda_n B_2)} \sinh(\lambda_n (x_b - B_2)) \end{array} \right)^2 \tag{11}$$

The volume of overtopping for vertical seawall with berm and ditch are again estimated as in equation (9) and the results are shown in Figure 4 for the corresponding parameters and are compared against each other.

4. Result and discussion

As we can see from above, the results of our simplified model for wave overtopping discharge for the vertical seawall is tolerable and comparable to that of Franco et.al (1994) but cannot be compared with VOWS model in [8] due to some of the necessary parameters which are not published in this research paper. From Figure 4, we can see that the overtopping discharge for both berm and ditch is comparable for the corresponding values of freeboard height and can be concluded that the freeboard values for berm structure are substantially higher than that for the ditch which means that the water jet onto a vertical seawall with berm structure will reach higher in the air compared to that of the ditch.
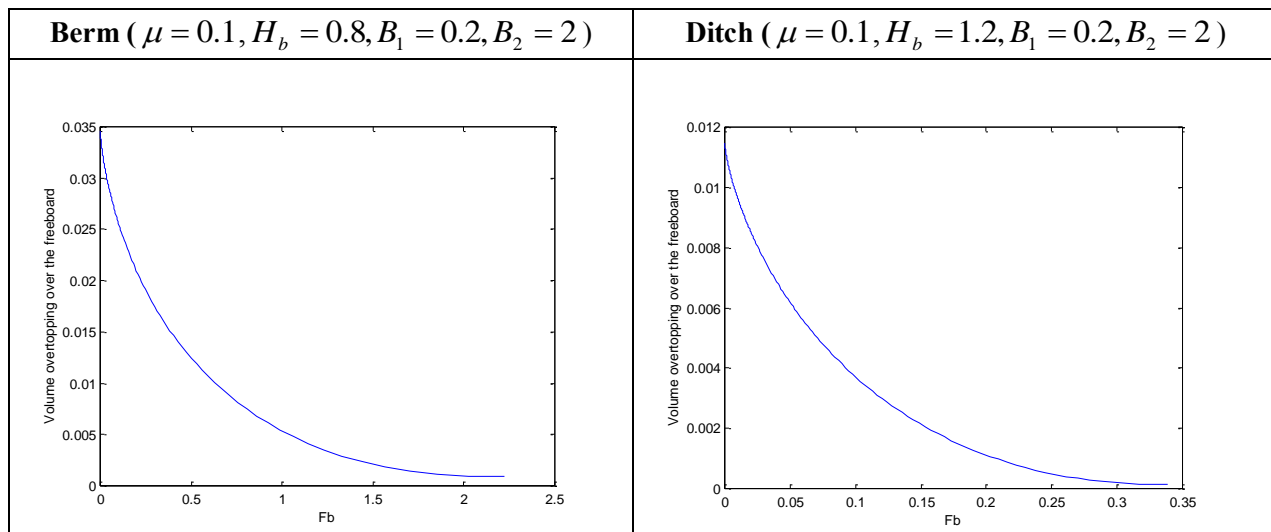
Berm ($\mu = 0.1, H_b = 0.8, B_1 = 0.2, B_2 = 2$)	**Ditch ($\mu = 0.1, H_b = 1.2, B_1 = 0.2, B_2 = 2$)**

Figure 4. Overtopping discharge for seawall with a berm and a ditch with V over F_b

Acknowledgements

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