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MHD Stagnation Point Flow of Micropolar Nanofluid with Soret and Dufour Effects

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Abstract. The effect of Soret and Dufour on magnetohydrodynamic (MHD) stagnation point flow of a micropolar nanofluid over a horizontal stretching sheet is considered. A boundary condition that assumed nanoparticle flux at the plate to be zero is applied in this study. The governing equations are in partial differential equations therefore a similarity transformation variable is employed to reduce the governing non-linear partial differential equations into non-dimensional ordinary differential equations. The ordinary differential equations are computed numerically using Runge-Kutta-Fehlberg (RKF45). Validation of the result is made by comparing the present results with the previous studies. Numerical results are presented in the graphical form for temperature and concentration profile to provide a better insight of this study. It is found that, temperature profile and nanoparticle volume fraction profile increase with an increasing values of Soret and Dufour effects.

1. Introduction

In the last decades, significant interest has been given to the study of fluid flow in micro scale due to the application of this study in micromachining technology and microfluidic system [1]. The fluid flow at micro scale was stated to behave differently than the flow of fluid in micro scale [1]. In an effort to gain understanding on the fluid flow at micro scale, a theory of micropolar fluid was proposed by Eringen [2]. Micropolar fluid theory takes into consideration the microrotation of molecules in the flow which is independent from the velocity of the stream. According to Eringen [2] micropolar fluid is a fluid that has microstructure and belong to a class of fluid that has non-symmetrical stress tensor namely polar fluid. Micropolar fluid theory that was introduced by Eringen [2] neglects the deformation of molecules and support surface and body couples. This theory is very useful in describing fluid that are in narrow channels, liquid containing additives, liquid crystals, polymers, animal blood, suspensions and colloidal solutions. An extensive studies of micropolar fluid has been conducted by previous researchers in different cases of convection heat transfer for instance Waqas et al. [3], Mishra et al. [4] and Anwar et al. [5]. These studies presented that the magnetic parameter coincides to smaller velocity and the angular velocity reduced with an increase value of micromotion. More studies on micropolar fluid are presented in [6-9]. Further, due to many applications of fluid in industry, it has been a concern that the thermal conductivity of a fluid has to be in an optimum level so that the heat transfer process would be efficient.

Recently, new discovery in nanofluid has spark interest in the study of thermal conductivity of nanofluid. Nanofluid is a special type of fluid that was introduced by Choi [10] and can be defined as a fluid containing suspended nanoparticles. This type of fluid has been experimentally proven to enhance



thermal conductivity of the base fluid. In addition to that, nanofluid that contains suspended conducting particles in nanometer scale was found to have less problem of erosion to the components and clogging in small channel than the fluid with suspended micro to millimeter sizes particles. Due to these advantages, numerous studies conducted in order to learn the convective transport of nanofluid. A numerical model of nanofluid was proposed by Buongiorno [11] which include two significant parameters that create relative velocity between nanoparticles and base fluid namely Brownian motion and thermophoresis parameter. This model was investigated numerically in a lot of literatures for instance Khan and Pop [12], Nield and Kuznetsov [13], Kuznetsov and Nield [14] and Hayat et al. [15]. Nield and Kuznetsov [13] and Kuznetsov and Nield [14] investigated nanofluid flow that is immersed in a porous medium and nanofluid flow over a vertical plate with an assumption that nanoparticle volume fraction at the plate is constant. However, due to the difficulty to keep a physically constant value of nanoparticle volume fraction at the plate, they have revised their model in [16-17]. The new boundary condition introduced by Nield and Kuznetsov [16] and Kuznetsov and Nield [17] assumed that nanoparticle flux at the boundary is zero and nanoparticle volume fraction is passively control at the boundary. This boundary condition has been applied in studies conducted by Waqas et al. [18], Tripathi et al. [19], Jusoh et al. [20] and Najib et al. [21] and changes in thermal and concentration boundary layer were reported. Nonetheless, very limited researches on passive control of nanoparticle in micropolar nanofluid can be found. Next, one of the effect considered in this study is thermo diffusion.

Thermo diffusion or Soret effect can be defined as an occurrence of mass flux that generated due to temperature gradient. On the other hand, Dufour effects can be defined as an occurrence of heat flux that caused by a composition gradient. Soret and Dufour effects are often neglected in many studies of heat transfer because these effects are considered to have magnitude that were in smaller order than the other effects that were described by Fourier and Ficks law [22]. However, these effects are significant when the mass and temperature gradient is large. One of the application of this effect is the separation of isotope by thermos diffusion or Soret effects in mixture with very light molecular weight and medium molecular weight [22]. These effects were introduced by Charles Soret in the year 1879 and has been extended in many literatures such as Srinivasacharya et al. [22], Reddy and Chamkha [23], Kasmani et al. [24], Pal and Mondal [25].

Motivated by the above studies, this paper aims to investigate the effect of passively control nanoparticle for MHD micropolar nanofluid over a horizontal stretching sheet with Soret and Dufour effect. This study refers to a model of MHD micropolar nanofluid that was proposed by Anwar et al. [5], the revised model of nanofluid introduced by Kuznetsov and Nield [16] and Soret and Dufour effects studied by Najib et al. [21]. The problem are solved numerically using Runge-Kutta-Fehlberg method. As per author's knowledge, the number of literatures on micropolar nanofluid that passively control volume fractions of nanoparticles are still limited. Therefore, we believe that the results presented here are new and original.

2. Mathematical Formulation

The steady, laminar two-dimensional flow of an electrically conducting micropolar nanofluid over a horizontal stretching sheet is considered. As depicted in figure 1, the stretching surface is confined within the region $y = 0$. The plate is stretched in x -direction with velocity $u_w(x) = ax$ and the ambient fluid flows with velocity $u_\infty = bx$ where a and b are constants with value $a > 0$ and $b > 0$. T_∞ , T_w and C_∞ denoted the ambient temperature, temperature at the wall and ambient concentration of nanofluid. Next, a uniform transverse magnetic field with strength B_0 is applied along the positive y -axis that is perpendicular to the stretching sheet and parallel to the y -axis. Small magnetic field with small magnetic Reynolds number is considered to be negligible. Further, Soret and Dufour effect are considered in this study and the nanoparticle volume fraction at the boundary is assumed to be passively control. Under the boundary layer approximation and physical conditions, the boundary layer governing equations are: (see Anwar et al. [5])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_\infty \frac{du}{dx} + \left(\frac{\mu + k_1^*}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \left(\frac{k_1^*}{\rho} \right) \frac{\partial N^*}{\partial y} + \frac{\sigma B_0^2}{\rho} (u_\infty - u) \tag{2}$$

$$u \frac{\partial N^*}{\partial x} + v \frac{\partial N^*}{\partial y} = \left(\frac{\gamma^*}{j^* \rho} \right) \frac{\partial^2 N^*}{\partial y^2} - \left(\frac{k_1^*}{j^* \rho} \right) \left(2N^* + \frac{\partial u}{\partial y} \right) \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{D_m K_T}{C_s C_\rho} \frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{5}$$

subjected to the boundary condition,

$$u = u_w(x) = ax, v = 0, N^* = -m_0 \frac{\partial u}{\partial y}, T = T_w, D_B \frac{\partial C}{\partial y} + \frac{D_B}{T_\infty} \frac{\partial T}{\partial y} = 0 \text{ at } y = 0 \tag{6}$$

$$u \rightarrow u_\infty = bx, v \rightarrow 0, N^* \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty$$

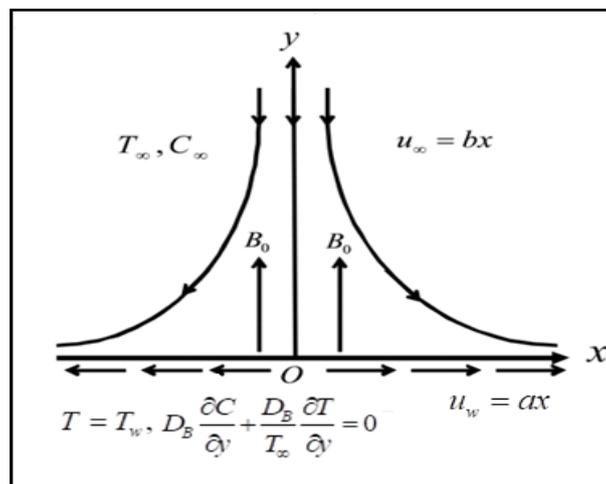


Figure 1: Physical model of the coordinate system

where u and v are the velocity components along the x and y axes. Viscosity, vortex viscosity, and density of the base fluid are represented by μ , k_1^* and ρ whereas σ , γ^* , j^* and N^* symbolized electrical conductivity, spin gradient viscosity, micro inertia per-unit mass and angular velocity or micro rotation.

$\alpha = \frac{k}{(\rho c)_f}$ is signified as thermal diffusivity where k is thermal conductivity and $(\rho c)_f$ is the heat capacity of the base fluid. Then, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ represents heat capacity where $(\rho c)_p$ is heat capacity of nanoparticles. D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient.

In order to reduce the governing nonlinear partial differential equations to nonlinear ordinary differential equations, the following similarity transformation variables are introduced:

$$\psi = (av)^{1/2} xf(\eta), \quad \eta = \left(\frac{a}{v}\right)^{1/2} y, \quad N^* = ax(av)^{1/2} h(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_\infty} \quad (7)$$

where ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. By substituting equation (7) into (1) to (5), we found that equation (1) is identically satisfied and equations (2) to (5) are transformed to the following ordinary differentiation equations:

$$(1 + K)f''' + ff'' - (f')^2 + Kh' + \varepsilon^2 + M(\varepsilon - f') = 0 \quad (8)$$

$$\left(1 + \frac{K}{2}\right)h'' + h'f - hf' - K(2h + f'') = 0 \quad (9)$$

$$\left(\frac{1}{Pr}\right)\theta'' + f\theta' + N_B\phi'\theta' + N_T\theta'^2 + D_f\phi'' = 0 \quad (10)$$

$$\phi'' + Le f\phi' + \frac{N_T}{N_B}\theta'' + SrLe\theta'' = 0 \quad (11)$$

subjected to the following transformed boundary conditions,

$$\begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad h(0) = 0, \quad \theta(0) = 1, \quad N_B\phi'(0) + N_T\theta'(0) = 0 \\ f'(\infty) \rightarrow \varepsilon, \quad h(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0 \end{aligned} \quad (12)$$

where $\varepsilon = \frac{b}{a} (> 0)$, $M = \frac{\sigma B_0^2}{a\rho_f}$, $\nu = \frac{\mu}{\rho}$, $Pr = \frac{\nu}{\alpha}$, $Le = \frac{\nu}{D_B}$, $K = \frac{k_1^*}{\mu}$, $N_B = \frac{\tau D_B C_\infty}{\nu}$, $N_T = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu}$

$D_f = \frac{D_m K_T}{\nu C_s C_\rho (T_w - T_\infty)}$ and $Sr = \frac{D_m K_T (T_w - T_\infty)}{a C_\infty T_m}$ signify velocity ratio parameter, magnetic field

parameter, kinematic viscosity, Prandtl number, Lewis number, Brownian motion parameter, thermophoresis parameter, Dufour parameter and Soret parameter, respectively.

3. Results and Discussion

Ordinary differential equations (8) to (11) are solved numerically using Runge-Kutta-Fehlberg technique. The numerical result and graphical results obtained for parameters Soret Sr , Dufour Df , Brownian motion N_B and thermophoresis N_T for temperature profile and nanoparticle volume fraction are provided in this study. The value for each parameter used are as specified here:

$Pr = 7$, $M = 1$, $N_T = 0.2$, $N_B = 0.1$, $K = 1$, $Le = 5$, $\varepsilon = 0.5$, $Df = 0.01$, $Sr = 0.2$. Prandtl number 7 is adopted from Anwar et al. [5] as this paper is extended from their study. Table 1 presents the comparison of $-f''(0)$ and $g'(0)$ for different value of M and K at $Pr = 0.71$ and $Le = 0.2$. As can be observed from table 1, the present results is compared to the numerical results published by Hsiao [8] that investigated the flow of micropolar nanofluid past a stretching sheet considering the effect of viscous dissipation. The problem was solved using finite difference method whereas present result obtained via Runge-Kutta-Fehlberg method. The results are found to be in a good agreement which confirm the accuracy of the numerical solution obtained in this study.

Table 1: Comparison results $-f''(0)$ and $g'(0)$ for different value of M and K .

M	K	Hsiao [8] $-f''(0)$	Present Results	Hsiao [8] $g'(0)$	Present Results
0.0	0.2	0.9098	0.9098	0.0950	0.0950
0.5	0.2	1.1147	1.1144	0.1051	0.1051
1.0	0.2	1.2871	1.2871	0.1121	0.1121
1.0	0.0	1.4142	1.4142	0.0000	0.0000
1.0	0.5	1.1408	1.1408	0.2112	0.2112
1.0	2.0	0.7697	0.7697	0.3586	0.3586

The effect of Brownian motion on temperature profile and concentration profile are presented in figure 2 and figure 3. Figure 2 illustrates the temperature profiles that are in increasing manner when the value of parameter Brownian motion is increased. In general, Brownian motion described the random movement of particles in a fluid. Thus, an increase value of Brownian motion will intensify the collision between particles in nanofluid, consequently generating more heat. In figure 3, nanoparticle volume fraction is displayed to be decreasing when the value of Brownian motion parameter increase.

The effect of thermophoresis parameter on temperature profile is demonstrated in figure 4. As the thermophoresis parameters increase, the temperature profile is increase as well. The increase in the thermophoresis also leads to the increase of nanoparticle momentum. Increasing in momentum of nanoparticles will cause the nanoparticles to transfer their kinetic energy to the cold region in which increasing the fluid temperature. Further, figure 5 indicates that nanoparticle volume fraction rise when the value of thermophoresis is elevated.

Next, the effect of Dufour parameter on temperature profile and nanoparticle volume fraction are presented in figure 6 and figure 7. Figure 6 shows the increasing value of temperature profile causes by increment in Dufour parameter. The value of Dufour parameter signify the influence of concentration gradient to the thermal energy flux in the flow. Therefore, we have observed the temperature profile increase when the Dufour parameter increase. The following figure 7 shows that nanoparticle volume fraction increases when the values of Dufour parameter increased. Further, figure 8 presents the result of Soret effect on temperature profile. Temperature profile rise when the value of Soret effect is increased. Last but not least, nanoparticle volume fraction is found to be increasing when the value of Soret effect is increased in figure 9.

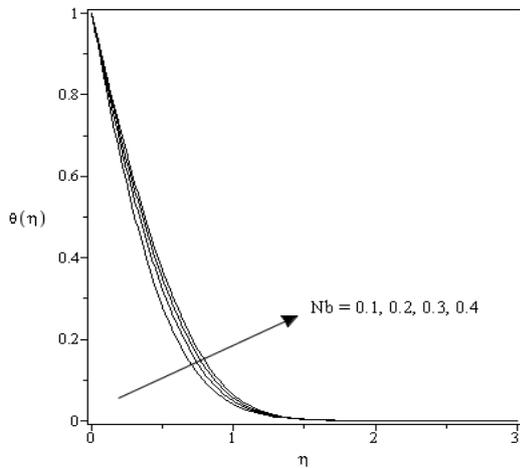


Figure 2: Temperature profile with different values of Nb when $Pr = 7, M = 1, N_T = 0.2, K = 1, Le = 5, \varepsilon = 0.5, Df = 0.01, Sr = 0.2$

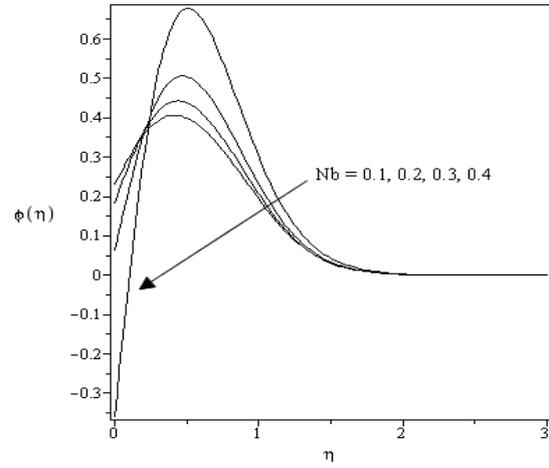


Figure 3: Nanoparticle volume fraction with different values of Nb when $\varepsilon = 0.5, Df = 0.01, Pr = 7, M = 1, K = 1, N_T = 0.2, Le = 5, Sr = 0.2$

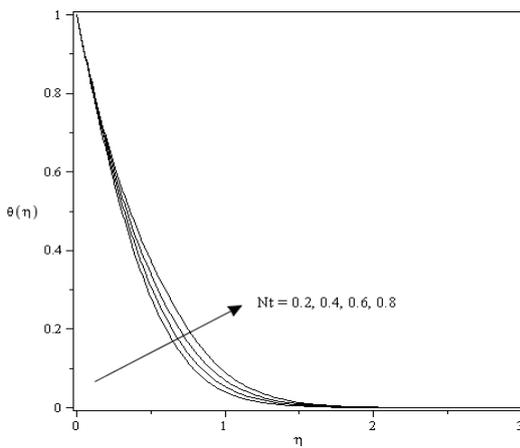


Figure 4: Temperature profile with different values of Nt when $Pr = 7, M = 1, K = 1, N_B = 0.1, Le = 5, \varepsilon = 0.5, Df = 0.01, Sr = 0.2$

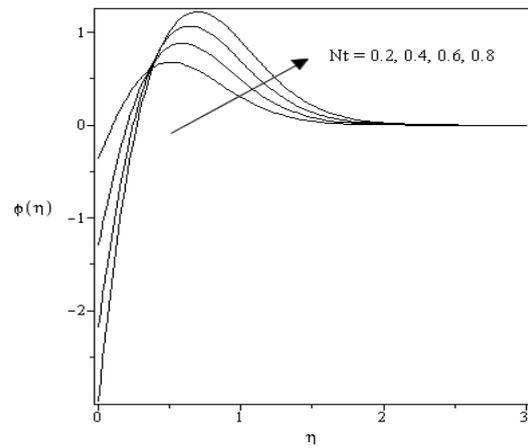


Figure 5: Temperature profile with different values of Nt when $Pr = 7, M = 1, K = 1, N_B = 0.1, Le = 5, \varepsilon = 0.5, Df = 0.01, Sr = 0.2$

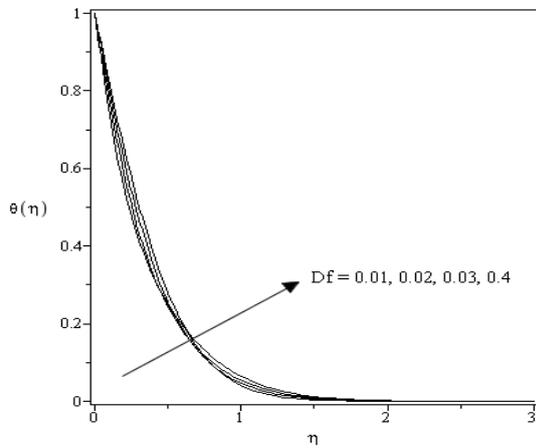


Figure 6: Temperature profile with different values of Df when $M = 1, N_T = 0.2, Sr = 0.2, N_B = 0.1, K = 1, Le = 5, \epsilon = 0.5, Pr = 7$

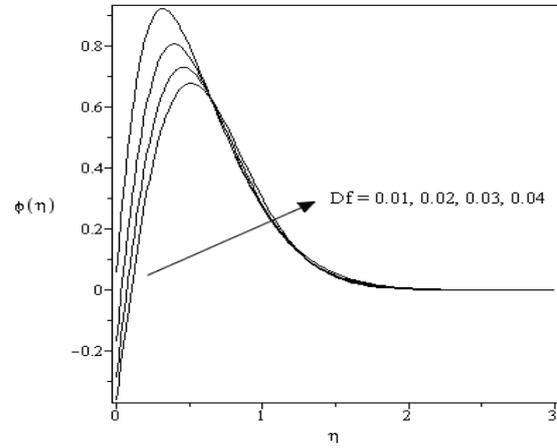


Figure 7: Nanoparticle volume fraction with different values of Df when $M = 1, K = 1, Le = 5, N_T = 0.2, N_B = 0.1, \epsilon = 0.5, Sr = 0.2, Pr = 7$

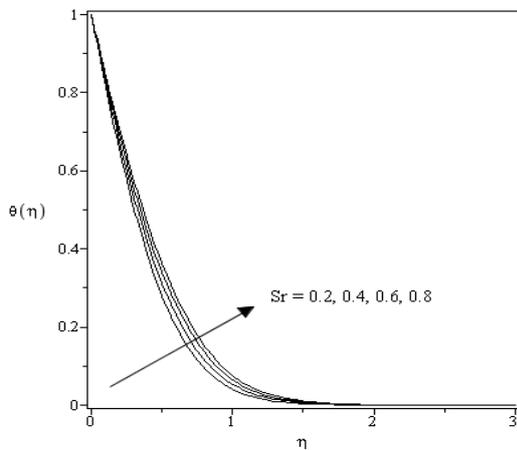


Figure 8: Temperature profile for different values of Sr when $Pr = 7, M = 1, N_T = 0.2, N_B = 0.1, K = 1, Le = 5, \epsilon = 0.5, Df = 0.01$

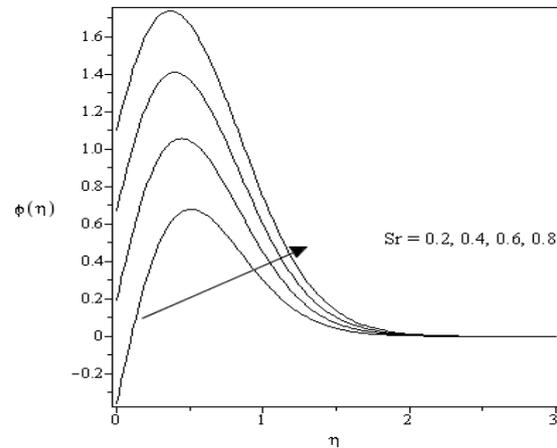


Figure 9: Nanoparticle volume fraction with different values of Sr when $M = 1, K = 1, Le = 5, N_T = 0.2, N_B = 0.1, \epsilon = 0.5, Df = 0.01,$

4. Conclusion

In this study, the problem of MHD micropolar nanofluid fluid over a stretching sheet with the effect of Soret and Dufour is analysed. Nanoparticle fraction at the boundary is assumed to be passively control. Equations (8) to (11) are solved numerically using Runge-Kutta-Fehlberg method. The influence of Brownian motion, thermophoresis and Soret and Dufour effect on temperature and nanoparticle volume fraction profile has been discussed in this paper. The results investigated can be concluded as follows:

- Temperature profile enhanced when the parameter Brownian motion, thermophoresis, Soret and Dufour effect is increased.
- Nanoparticle volume fraction increases with increasing value of thermophoresis, Soret and Dufour effect.
- However, the opposite result is observed for the case Brownian motion parameter.

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References

- [1] Kucaba-Piętal A, 2004 *Bulletin of the Polish Academy of Sciences: Technical Sciences* **52** 209-214
- [2] AC Eringen 1966 *Journal of Mathematics and Mechanics* **16** 1–18
- [3] Waqas M, Farooq M, Khan M I, Alsaedi A, Hayat T and Yasmeen T 2016 *International Journal of Heat and Mass Transfer* **102** 766–72
- [4] Mishra S R, Baag S and Mohapatra D K 2016 *Engineering Science and Technology, an International Journal* **19** 1919–28
- [5] Anwar M I, Shafie S, Hayat T, Shehzad S A and Salleh M Z 2017 *Journal of the Brazilian Society of Mechanical Sciences and Engineering* **39** 89-100
- [6] Pal D and Mandal G 2017 *International Journal of Mechanical Sciences* **126** 308-318
- [7] Hayat T, Khan M I, Waqas M, Alsaedi A and Khan M I 2017 *International Journal of Hydrogen Energy* **42** 16821-33
- [8] Hsiao K L 2017 *International Journal of Heat and Mass Transfer* **112** 983-990
- [9] Kumar T S, Kumar B R, Makinde O D and Vijaya Kumar A G 2018 *Defect and Diffusion Forum* **387** 78-90
- [10] Choi S 1995 *American Society of Mechanical Engineers* **231** 99-105
- [11] Buongiorno J 2006 *Journal of heat transfer* **128** 240-250
- [12] Khan W A and Pop I 2010 *International Journal of Heat and Mass Transfer* **53** 2477-83
- [13] Nield D A and Kuznetsov A V 2011 *International Journal of Heat and Mass Transfer* **52** 5792-95
- [14] Kuznetsov A V and Nield D A 2010 *International Journal of Thermal Sciences* **49** 243-247
- [15] Hayat T, Aziz A, Muhammad T and Alsaedi A 2017 *Chinese Journal of Physics* **55** 1495-1510
- [16] Kuznetsov A V. and Nield D A 2013 *International Journal of Heat and Mass Transfer* **65** 682-685
- [17] Kuznetsov A V. and Nield D A 2014 *International Journal of Thermal Sciences* **77** 126-129
- [18] Waqas M, Khan M I, Hayat T and Alsaedi A 2017 *Computer Methods in Applied Mechanics and Engineering* **324** 640-653
- [19] Tripathi R, Seth G S and Mishra M K 2017 *Advanced Powder Technology* **28** 2630-41
- [20] Jusoh R, Nazar R and Pop I 2018 *Physics of Fluids* **30** 033604
- [21] Najib N, Bachok N, Arifin N and Ali F 2018 *Applied Sciences* **8** 642
- [22] Srinivasacharya D and Shiferaw M 2014 *Arabian Journal for Science and Engineering* **39** 5085-5093
- [23] Reddy P S and Chamkha A J 2016 *Advanced Powder Technology* **27** 1207-18

- [24] Kasmani R M, Sivasankaran S, Bhuvanewari M and Hussein A K 2017 *International Journal of Numerical Methods for Heat & Fluid Flow* **27** 2333-54
- [25] Pal D and Mondal H 2018 *International Journal for Computational Methods in Engineering Science and Mechanics* **19** 49-60