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Estimating Rotational FRFs of a Complex Structure Using FE Model Updating, Mode Expansion and FRF Synthesis Method

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Abstract. Estimating rotational frequency response functions (FRFs) from experimental data is very challenging, and the estimation often leads to poor FRFs due to the presence of spurious resonance peaks. In practice, special equipment is essentially required in the measurement process, unless the rotational FRFs are synthesised using measured modal data. This paper presents an alternative approach for estimating the rotational FRFs of a geometrically complex structure by using finite element modal updating, mode expansion and the FRF synthesis method. The applicability and accuracy of the proposed approach are demonstrated on a car front-end module A simplified finite element (SFE) model of the test structure is introduced and the SFE model is then updated based on the measured modal model obtained from experimental modal analysis. The mode shapes of the updated SFE model are expanded to the test model to obtain the entire translational and rotational modal vector. The rotational FRFs of the expended experimental model is derived via the FRF synthesis method. The derived translational FRF is compared with the measured counterparts for validation purposes. It was found that the derived translational and measured FRF are in a perfect match, and the derived rotational FRF has successfully estimated all the resonance peaks within the frequency of interest. The findings from this work may be useful in improving the accuracy of the experimental rotational FRFs, which are crucial for particular structural dynamics analysis.

1. Introduction

Experimental rotational degrees of freedom (DOFs) are crucial in many areas of dynamic investigations, such as experimental dynamic substructuring [1-2], model reduction/updating [3], structural modification [4], and vibration control [5]. For instance, in the automotive domain, a crash imparts tremendous energy to the occupants, typically in the form of significant rotational inertia. Therefore, the acceleration experienced by the rotating part of the structure is often an essential parameter during system design. It is noteworthy that measuring rotational DOFs data, which usually requires special measurement sensors, is very difficult to be executed and is not a straightforward assignment. There has been a variety of techniques reported by [6-7] employing two spatially separated and sensitivity-matched accelerometers to estimate rotational acceleration. For instance, an X-block attachment was used to allow the attachment of several translational accelerometers to estimate the rotational frequency response functions (FRFs)of a component in the attempt to perform a structural modification of a helicopter tail [8]. Another striking point is a T-block mass was attached to the structure and was de-coupled to expand and obtain the rotational FRFs. This approach is also known as transmission simulator [9]. However, the reported methods only work best particularly when the rotational responses are high.

Recently, rotational accelerometers have been introduced to efficiently allow the measurement of the rotational FRFs of a test structure[10]. However, it was reported in [11] the measurement only limited to force excitation responses which means moment excitation of experimental substructures are required to be expanded based on the modal model to obtain a full DOF of the structure. Although there are several techniques [7,10-11] to directly measure the rotational data, the techniques usually require special equipment such laser vibrometeror rotational accelerometers, which may not be readily available in most laboratories [12].

This paper puts forward an approach of estimating the rotational FRF data of a geometrically complex structure using the updated FE model based mode expansion method. This approach is developed based on model updating, mode expansion and the FRF synthesis method. The process flow of the proposed approach is presented in Figure 1. In this work, a simplified finite element (SFE) model is introduced in order to overcome the difficulties of modelling a complex geometrical structure [13-14]. Later, the updated SFE model is then expanded to the experimental model to obtain all the translational and rotational FRFs.

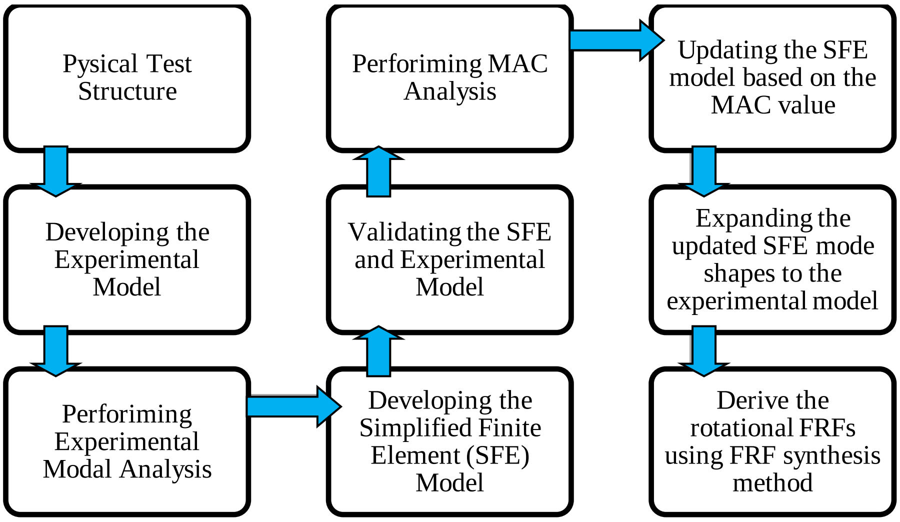

Figure 1. Process flow of the proposed approach

The proposed approach is performed on a car front end module structure as shown in Figure 2. The structure is used as a case study because its configuration has played a crucial contribution to the vibration level of a car. The one end of the test structure as shown in Figure 2 is selected as the reference point because the point covers all modes of interest of the structure.

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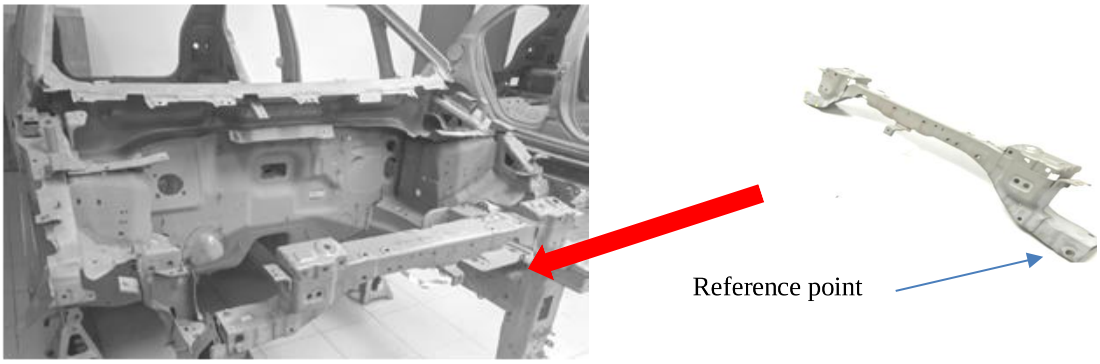


Figure 2. The front-end module of a car body in white

2. Development of the SFE model

Previous works [7,10] reported that rotational accelerometers can be used to directly measure the rotational responses of the test structure. However, the measured responses are only for the force-excitation. Normally, the moment-excitation responses are expanded by using modal expansion methods based on the numerical modal model [11] but the modal expansion method relies on the accuracy of the numerical model. For particular cases involving complex structures, developing accurate finite element models are time consuming and difficult. This work employs an approximation modelling approach, where the geometry of the FE model of the structure is simplified. By employing the approach, the FE modelling time is reduced.

Figure 3 depicts the construction process of the SFE model of structure. 2D-Shell elements with an approximate and simplified outline of the experimental model were used in constructing the SFE model. The procedure adopted gives the necessary details of embodying the physical test structure. The use of 2D shell element allows the rotational data to be extracted from the SFE model [13] and its parameters to be correlated with the experimental model [15,16]. To increase the effectiveness of the SFE model, the structure was discretized into seven different parts, allowing more parameters to be used for updating. The updating has led the mode shapes of the updated SFE model to be well correlated with the experimental ones. Subsequently, the updated SFE model was employed in the expanding process of unmeasured rotational data to the experimental model.

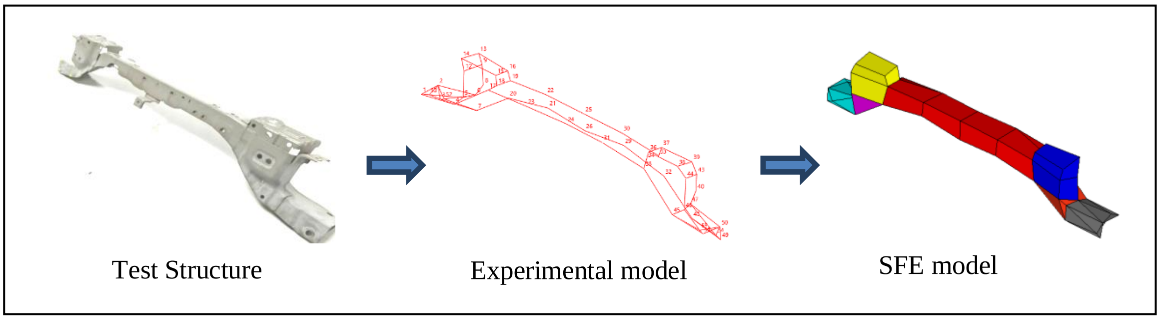


Figure 3. The test structure, experimental model and SFE model

3. Experimental Modal Analysis

The updating of the SFE model relies heavily on the experimentally measured data. In this work, the test structure was tested by using experimental modal analysis to measure the natural frequencies and mode shapes. The test was performed under free boundary conditions as demonstrated in [17]. The experimental configuration was designed as to the finite element model. Figure 4 illustrates the test configuration of the test structure in which three 10mV/g uniaxial accelerometer and a 21.65mV/N impact hammer were used in the measurement. Leuven Measurement System was used to acquire the data. The frequency bandwidth was set between 0 Hz to 512 Hz.

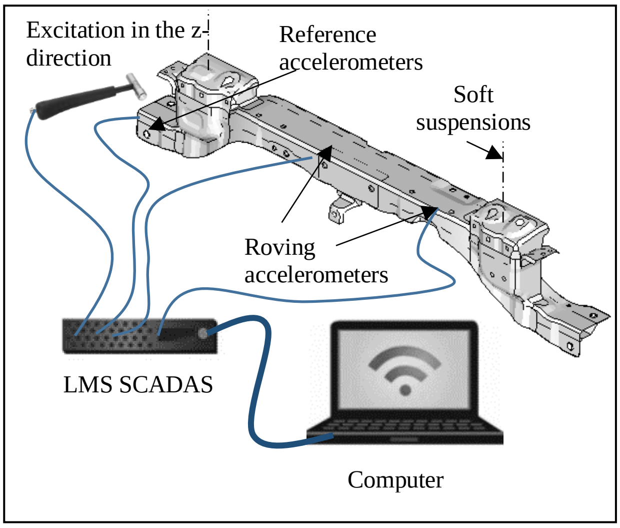


Figure 4. The experimental modal analysis configuration of the test structure

There are eight modes identified within the frequency of interest. The damping values of the modes are also taken into account for FRF synthesis purposes. The natural frequencies and damping values of the structure are tabulated in Table 1. The structure can be considered as a lightly damped structure because the average damping value of each mode is only 0.49 per cent, which the first mode is the highest with 0.84 per cent.

Table 1. The natural frequencies and damping values of the test structure

Mode	Frequency (Hz)	Damping (%)
1	98.78	0.8490
2	162.12	0.5129
3	201.37	0.1553
4	217.89	0.5039
5	302.15	0.5025
6	328.94	0.6566
7	382.65	0.3831
8	483.46	0.4383

4. Model Updating Method

The purpose of updating the SFE model was to improve the accuracy of the model by reconciling certain parameters in the light of the experimental data. The sensitivity analysis can be performed as follows

$$\mathbf{S} = \Phi_i^T \left[\frac{\delta \mathbf{K}}{\delta \theta_j} - \omega_i \frac{\delta \mathbf{M}}{\delta \theta_j} \right] \Phi_i \tag{1}$$

where matrix **S** is the sensitivity matrix, Φ, ω and θ are the eigenvector, eigenvalue and parameter respectively. The j is the j-th parameter and i represents the i-th eigenvalue. The sensitivity analysis is used to quantify the impact of parameters used in modelling on the predicted responses which are the natural frequencies and mode shapes.

Before performing the modal updating method, the accuracy of the predicted mode shapes of the SFE model is quantified using Modal assurance criterion (MAC) analysis. The detailed information on the MAC analysis can be found in [21]. In this work, the parameters of the SFE model were reconciled by referring to the MAC values. This is because only the accurate mode shapes are required for mode expansion purposes. Therefore, the objective function containing only the MAC number is defined in order to reduce the errors between the FE model and EMA mode shapes.

MSC NASTRAN Solution 200 adopted by [22] was used for the updating process and a total of eight mode shapes was employed. The comparison between the EMA and updated SFE mode shapes of the SFE model is presented in Table 2. The MAC values are also included in the table.

Table 2. The comparisons between EMA and updated SFE mode shapes

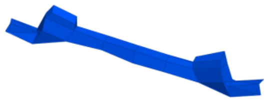
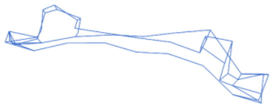
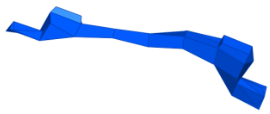
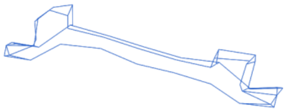
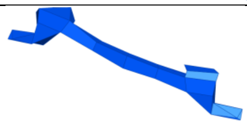
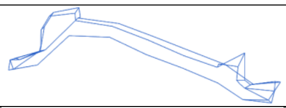
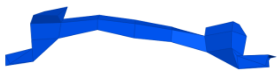
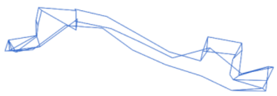
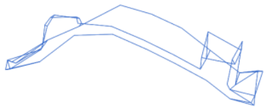
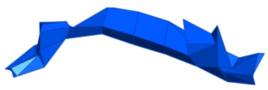
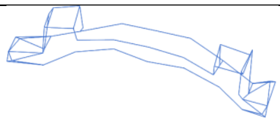
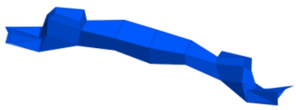
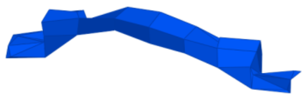
Mode	EMA Mode Shapes	Updated SFE Mode Shapes	MAC Percentage
1			80.00 %
2			83.90%
3			80.90%
4			70.60%
5			89.60%
6			72.70%
7			60.10%
8			62.40%

Table 2 indicates that the average MAC value between the EMA and updated SFE model mode shapes is 75.02 per cent. The value is within an acceptable range, considering the complexity of the structure [21]. Meanwhile, the 7th mode records the lowest MAC value of 60.10 per cent. The lowest record might be because of the incapability of the developed SFE model of accurately simulating the local deformations of the structure. However, based on engineering judgement and observation, the correlation of the mode shapes between both models is relatively strong.

5. Mode Expansion of the Experimental Model

The purpose of the mode expansion of the EMA mode shapes is to obtain the unmeasured rotational data in the light of the updated SFE mode shapes[23]. In this work, the unmeasured rotational modal vectors in the experimental model were expanded by using the system equivalent reduction expansion process (SEREP) method[24]. The expansion process was carried out based on the modal vectors of the updated SFE model. For this method, the expended experimental modal vector with a full set of DOFs, $\mathbf{E}_n$ is expressed in the form of:

$$\mathbf{E}_n = \begin{bmatrix}\mathbf{E}_a\\ \mathbf{E}_d\end{bmatrix} = \mathbf{T}_u\mathbf{E}_a = [\mathbf{U}_n\mathbf{U}_a]^g\mathbf{E}_a = \begin{bmatrix}\mathbf{U}_a\\ \mathbf{U}_d\end{bmatrix}\mathbf{U}_a{}^g\mathbf{E}_a \qquad (2)$$

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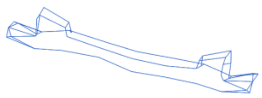
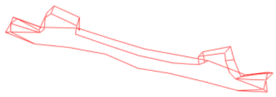
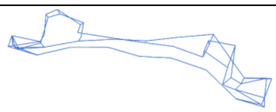
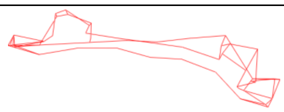
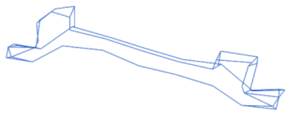
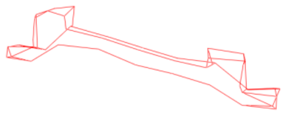
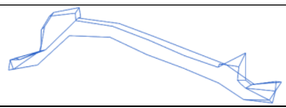
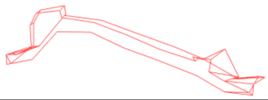
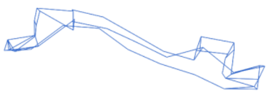
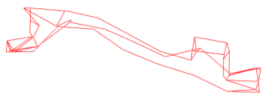
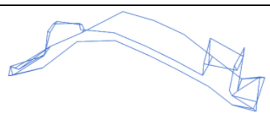
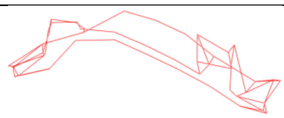
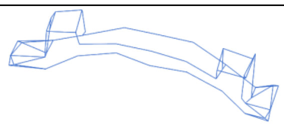
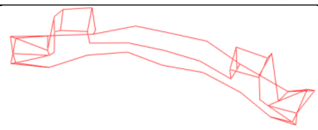
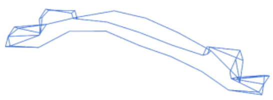
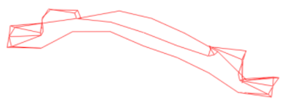
where **E**,**U** and **T** are experimental modal vectors, analytical modal and transformation matrices. Subscripts *a*,*d* and *u* indicate tested set of experimental DOFs, deleted set of DOFs and modal vectors respectively. Table 3 presents the mode shapes comparisons between the EMA and expanded experimental model. The accuracy of the expanded mode shapes is quantified using MAC analysis.

Table 3. The comparison between EMA and expanded mode shapes

Mode	EMA Mode Shapes	Expanded Mode Shapes	MAC Percentage
1			82.70 %
2			88.70%
3			80.90%
4			79.60%
5			89.60%
6			83.90%
7			77.50%
8			90.80%

From Table 3, it was found that there is a high correlation between the EMA and expanded mode shapes. The average MAC value of the whole mode is 84.18 per cent. The 8th mode of the expanded model records the highest correlation with EMA with MAC value of 90.80 per cent. Therefore, the use of the expanded model in predicting the dynamic behaviour of the physical front-end module is reliable. The 3D view of MAC comparison between EMA and expanded mode shapes presented in Figure 5 also indicates that there are no swapped modes between the two models.

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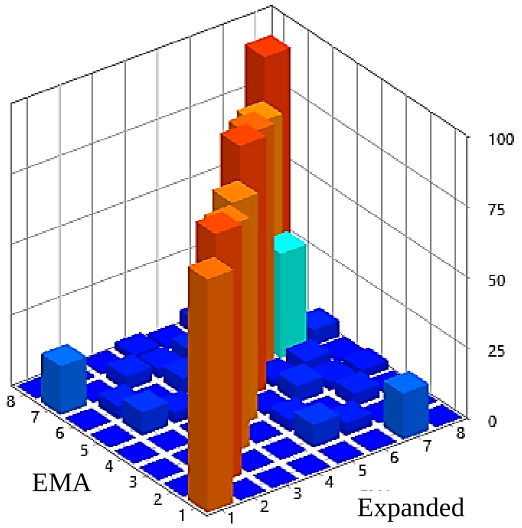

Figure 5. 3D view of MAC comparisons between EMA and expanded mode shapes

6. FRF Synthesis of the Expanded Experimental Model

The objective of the FRF synthesis of the expanded model was to obtain the FRF based on the modal model. The process was carried out for validation purposes. The FRF synthesis method does not require a matrix inversion of a potentially large dynamic stiffness matrix. Only one eigenvalue analysis is needed to determine the natural frequencies and mode shapes[12]. Therefore only one has to be synthesised the FRFs of the DOFs and frequencies of interest. In this work, the x and z-axis translational FRFs at the reference point of the expanded experimental model were derived. The y-axis FRF was not included in the validation process because the sensor cannot be mounted at the reference point of the structure. For this method, the synthesised FRF matrix $\mathrm{H_{syn}}$ $(\boldsymbol{\omega_k})$ and mode shapes are expressed in the form of:

$$\mathrm{H_{syn}}(\omega_\mathrm{k}) = \sum_{i=1}^{N} \frac{\{\emptyset\}_i\{\emptyset\}_i^T}{\left(\omega_{n_i}^2 - \omega_k^2\right) + j2\xi_i\omega_k\omega_{n_i}} \tag{3}$$

where N represents the number of calculated modes, $\{\emptyset\}_i$ represents the ith mass normalised mode shapes, ω_{n_i} represents ith natural frequency and ξ_i represents the ith modal damping ratio. Figures 6 and 7 show the comparisons between the x and z-axis expanded model FRF (orange) and that of EMA at the reference point.

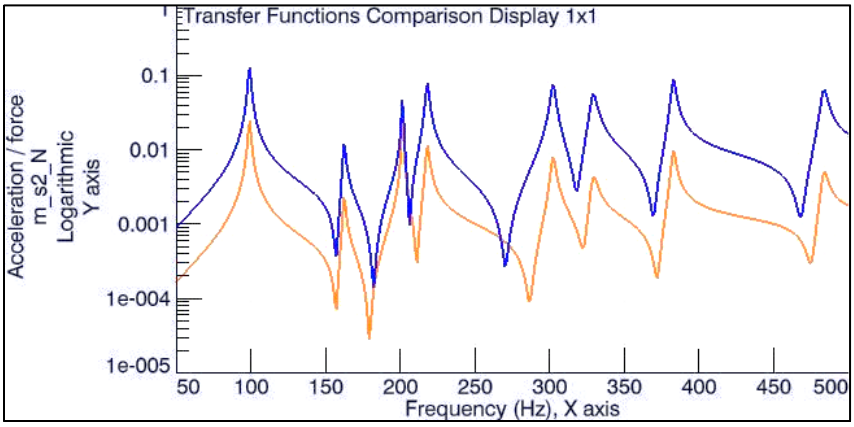

Figure 6. Comparisons between x-axis expanded model (orange) and EMA (blue) FRF at the reference point

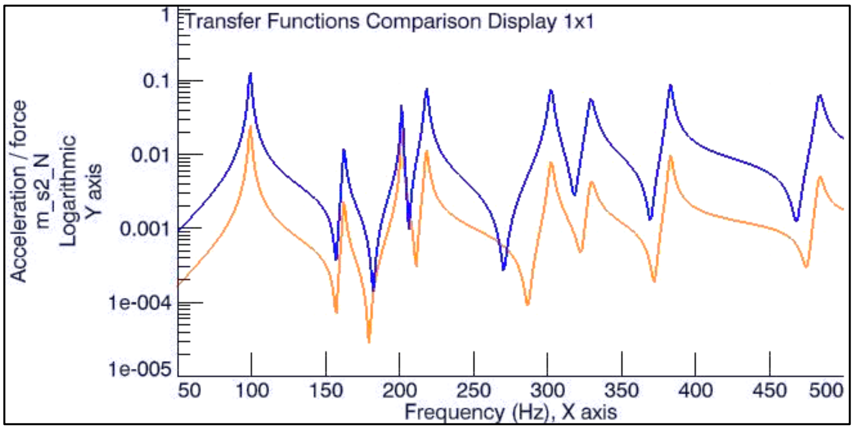

Figure 7. Comparisons between z-axis expanded model (orange) and EMA (blue) FRF at the reference point

Figures 6 and 7 clearly indicate that the resonance frequencies are well matched with the EMA FRF data. In addition, the anti-resonance frequencies have recorded minor discrepancies in the frequency range of 250-300 Hz. The discrepancies observed may be due to the effect of residual modes in the FRF synthesis of the expanded model [25] . On the other hand, the amplitudes of the synthesised FRF are lower than that of the EMA FRF. The low amplitudes may be due to the normalization between the measured and SFE model modal vector during the expansion process. However, based on the pattern of FRFs calculated for the resonances as well as anti-resonances, the SFE model is found to be in good agreement with the experimental model. Since there is a strong correlation between the synthesised and EMA FRFs, the rotational FRFs are then derived and presented in Figures 8 and 9.

Figure 8 presents the FRFs of the force excitation and rotational responses in x,y and z axis , while Figure 9 presents the FRFs of the moment excitation and rotational response in x,y and z axis. In this work, the validation of the rotational FRFs was not performed because the FRFs were not measured during the activities of EMA due to rotational accelerometers were unavailable. From the figures, it can be clearly seen that there are 8 visible resonance peaks appeared in the FRFs. This indicates that each mode experiences rotational acceleration during resonance. It is worth noting that the rotational resonance might introduce damage and unwanted vibration to the structure due to the external forces or moment excitations. Therefore, the rotational responses must be definitely considered in the system design.

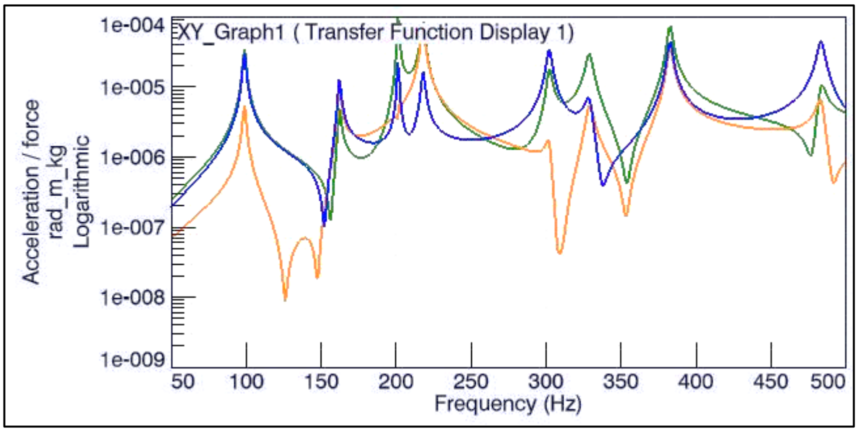

Figure 8. Force x-rotational x (blue), force y-rotational y (orange) and force z-rotational z (green) FRFs.

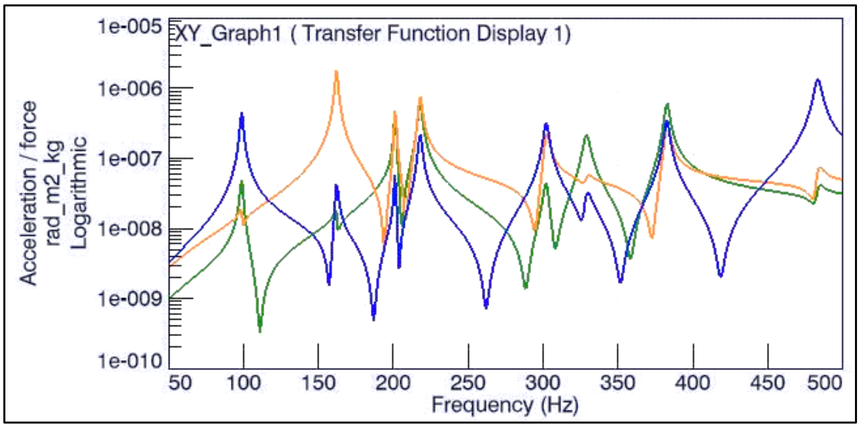

Figure 9. Moment x-rotational x (blue), moment y-rotational y (orange) and moment z-rotational z (green) FRFs.

7. Conclusions

The rotational FRFs estimation approach based on the simplified finite element (SFE) model, model updating, mode expansion and FRF synthesis method has been proposed and presented. The proposed approach has been demonstrated on a car front-end module structure. It was found that the rotational FRFs comprising force and moment excitations have been successfully derived from the expanded experimental model. This finding reveals that the approach used is capable of significantly minimising the heavy reliance on the rotational FRFs measurement, which is very difficult to be performed. The approach used in this research could be employed for other dynamic investigations without a dramatic decline in efficiency and accuracy. Although the rotational responses are less popular in most structural dynamics cases, indeed they are very important especially in the field of experimental dynamic substructuring (modal and frequency based) and structural modification methods.

Acknowledgments

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