PAPER • OPEN ACCESS

Modified Sine Cosine Algorithm for Identification of Liquid Slosh based on Continuous-time Hammerstein Model

To cite this article: Julakha Jahan Jui *et al* 2020 *J. Phys.: Conf. Ser.* **1529** 042090

Modified Sine Cosine Algorithm for Identification of Liquid Slosh based on Continuous-time Hammerstein Model

Julakha Jahan Jui[1], Mohd Helmi Suid[1], Mohd Riduwan Ghazali[1], Mohd Ashraf Ahmad[1], Mohd Zaidi Mohd Tumari[2]

[1]Faculty of Electrical and Electronics Engineering Technology, University Malaysia Pahang, 26600, Pekan, Pahang, Malaysia.
[2]Faculty of Electrical & Electronic Engineering Technology, Universiti Teknikal Malaysia Melaka, Melaka

Abstract. This paper presents the identification of liquid slosh plant using the Hammerstein model based on modified Sine Cosine Algorithm (mSCA). A remote car that carries a container of liquid is considered as a liquid slosh experimental rig. In contrast to other research works, this paper considers a piece-wise affine function in a nonlinear function of the Hammerstein model, which is more generalized function. Moreover, a continuous-time transfer function is utilized in the Hammerstein model, which is more suitable to represent a real system. The mSCA method is used to tune both coefficients in the nonlinear function and the transfer function of the Hammerstein model such that the error between the identified output and the real experimental output is minimized. The effectiveness of the proposed framework is assessed in terms of the convergence curve response, output response, and the stability of the identified model through the pole-zero map. The results show that the mSCA based method is able to produce a Hammerstein model that yields identified output response closes to the real experimental slosh output with 82.12 % improvement of sum of quadratic error.

1. Introduction

Nowadays, liquid slosh inside a cargo always happens in many situations. For example, ships with liquid container carriers are at high risk of generating sloshing load during operation [1]. In the metal industries, high oscillation can spill molten metal that is dangerous to the operator [2]. Meanwhile, sloshing of fuel and other liquids in moving vehicles may cause instability and undesired dynamics [3]. Hence, it is necessary to completely study the behavior of this residual slosh induced by the container motion. One may study the behavior of liquid slosh through developing the exact mathematical model of liquid slosh. So far, many researchers focus on the first principle approach to model the slosh behavior, while there are few literatures to discuss it from the perspective of nonlinear system identification approach.

On the other hand, block oriented nonlinear system identification has become popular techniques to model a complex plant. The block oriented nonlinear model can be classified into three categories, which are Hammerstein model, Wiener model and Hammerstein Wiener model. In particular, Hammerstein model is a model that consists of a nonlinear function followed by linear dynamic sub-plant, while Wiener model consists of a linear dynamic sub-plant followed by nonlinear function, and finally, Hammerstein-Wiener model contains a linear dynamic sub-plant inserted between two or more nonlinear functions in series. Among these three block oriented models, Hammerstein model is famous due to its simple model structure and it has been widely used for nonlinear system identification. Specifically, the Hammerstein model has been applied to model a real plant such as

Solid Oxide fuel cell [4], bidirectional DC motor [5], oxygen uptake estimation [6], stretch reflex dynamics [7], turn-table servo system [8], pneumatic muscle actuators [9], amplified piezoelectric actuators [10] and multi-axis piezoelectric micro positioning stages [11]. On the other hand, many tools have been utilized to identify the Hammerstein model. There are the iterative method [12]-[14], the subspace method [15]-[17], the least square method [18], the blind approach [19] and the parametric instrumental variables method [20]. Moreover, many also consider the optimization tools for Hammerstein model, such as Bacterial Foraging algorithm [21], Cuckoo search algorithm [22], Particle Swarm optimization [23] and Genetic algorithm [24].

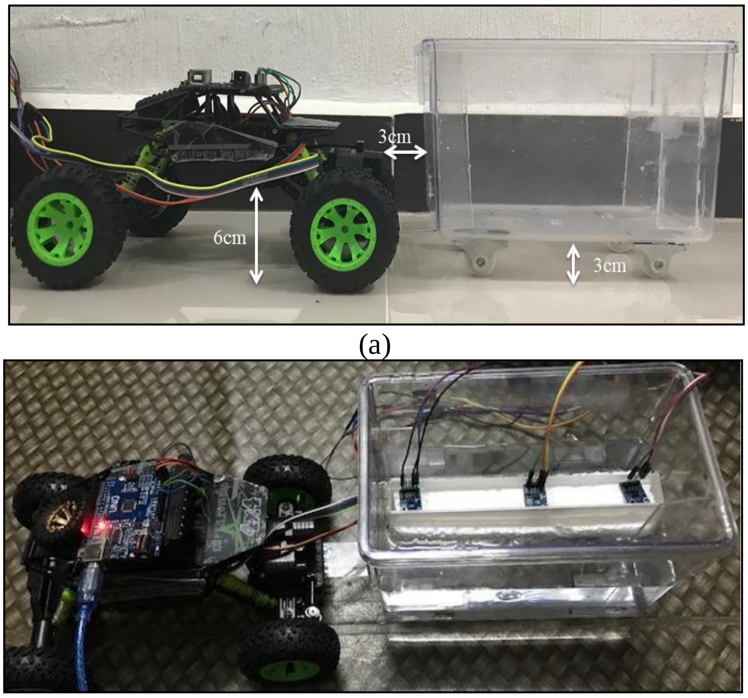
Based on the above literature, several limitations are ineluctable in their works, which are:
(i) Most of the Hammerstein models used in their study are based on discrete-time model, while many real plants can be easily represented in continuous-time model.
(ii) Almost all the methods assume a known structure of nonlinear function, which consists of several basis functions.

Though, our proposed work can solve a more general class of continuous-time Hammerstein model by assuming an unknown structure of nonlinear function. In particular, a piece-wise affine function is adopted with so many basis functions. Due to the introduction of the piece-wise affine function, a high dimensional design parameter tuning is considered in this study, which make the identification problem more complex. On the other hand, Sine Cosine Algorithm (SCA) [25] has become a top notch optimization algorithm which has solved various types of engineering problems [25]-[27]. To the best of our knowledge, there are still few works to discuss on the SCA for identification of Hammerstein model. Moreover, other recent optimization methods are quite complex as compared to SCA which may contribute to high computation time in obtaining the result. Thence, it motivates us to see the effectiveness of the SCA in modelling the liquid slosh plant from the real experimental data. Moreover, based on our preliminary works on this problem, the standard SCA is still not able to provide high accuracy of liquid slosh model. Therefore, it motivates us to modify the standard SCA algorithm such that a better accuracy of liquid slosh plant can be obtained.

This paper presents the identification of liquid slosh plant using the Hammerstein model based on modified SCA (mSCA) method in [26]. A remote car that carrying a container of liquid is considered as the liquid slosh experimental rig. The mSCA method is used to tune both coefficients in the nonlinear function and transfer function of the Hammerstein model such that the error between the identified output and the real experimental output is minimized. The effectiveness of the proposed framework is assessed in terms of the convergence curve response, output response, and the stability of the identified model through the pole zero map.

2. Liquid Slosh Experimental Rig

In this study, a mobile liquid slosh plant is considered to replicate real situation of a moving container carrying liquid, as shown in figure 1. In particular, a remote control car is used to carry a small tank filled with liquid. The tank is also equipped with four plastic wheels so that it can move smoothly as shown in figure 1(a). Moreover, three accelerometer sensors (ADXL335) that are floated on the surface of liquid are used to measure liquid oscillation as shown in figure 1(b). For simplicity of our study, the liquid slosh data from only one of the sensor is recorded and only z-axis output data is considered. Figure 2 shows a general schematic diagram of liquid slosh experimental rig. In particular, an Arduino UNO is used as a data acquisition platform to process the input and output data. Here, we generate a voltage from the Arduino UNO to the remote car and concurrently the Arduino UNO also will acquire the slosh data from the accelerometer. Both the input and output data can be monitored and analysed from the personal computer using the LabVIEW software. In order to identify the model of liquid slosh, the remote car is required to move to a certain distance and suddenly stop to generate a liquid oscillation or slosh inside the tank. Thence, we apply the input voltage as shown in figure 3 to move the remote car. Concurrently, the liquid slosh data is recorded as shown in figure 4. These two data are then used to develop the Hammerstein model based SCA, which is discussed in the next section.



(a)

(b)

Figure 1. Liquid slosh experimental rig

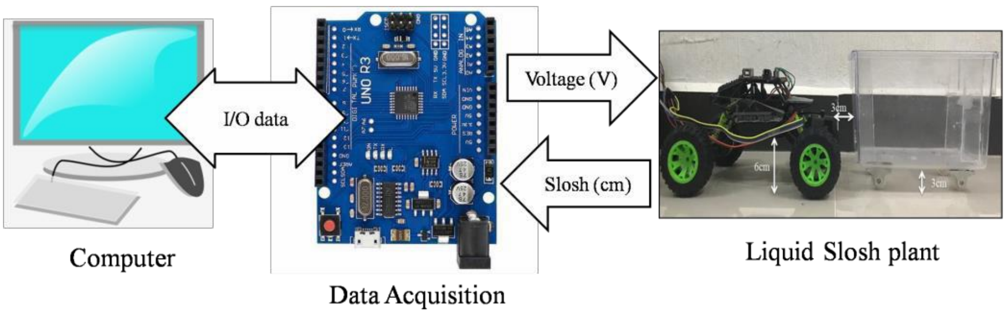


Figure 2. Schematic diagram of liquid slosh experimental rig

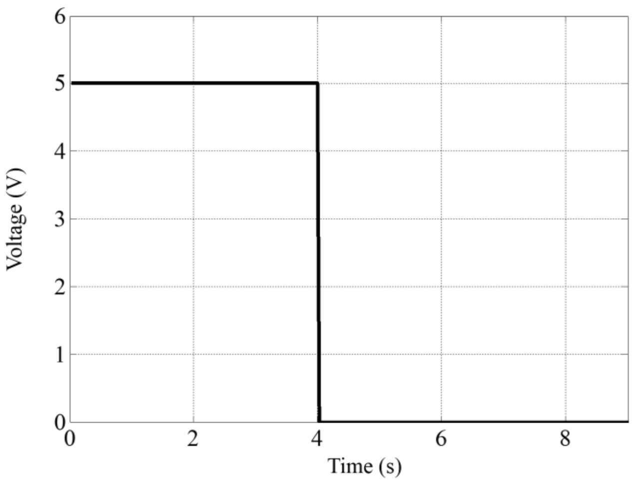


Figure 3. Input voltage applied to the remote car

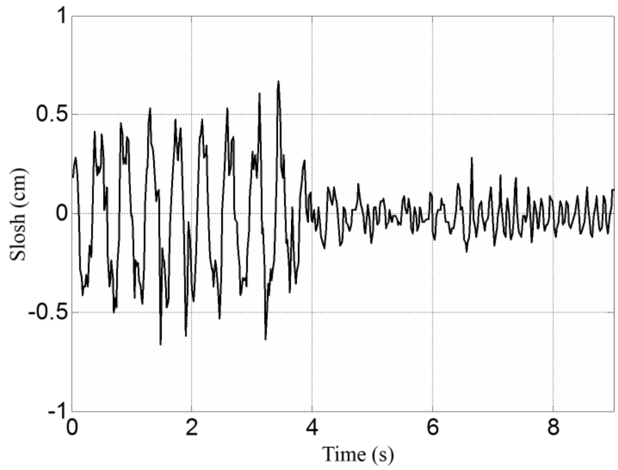


Figure 4. Output slosh from the accelerometer

3. Identification of Liquid Slosh using Hammerstein based modified SCA

In this section, the proposed modified Sine Cosine Algorithm (mSCA) for identification of liquid slosh plant in Section 2 based on Hammerstein model is presented. Firstly, a problem formulation to identify the liquid slosh plant is explained. Then, it is shown on how to apply the mSCA method to identify the liquid slosh based on Hammerstein model.

Figure 5 shows a complete block diagram to identify the liquid slosh model in Section 2. The proposed Hammerstein model consists of nonlinear function $h(u)$ followed by the transfer function $G(s)$. The nonlinear function is a piece-wise affine function given by

$$h(u) = \begin{cases} c_0 + m_1(u - d_0) & \text{if} \quad d_0 \le u < d_1, \\ c_1 + m_2(u - d_1) & \text{if} \quad d_1 \le u < d_2, \\ \vdots & \\ c_{\sigma-1} + m_\sigma(u - d_{\sigma-1}) & \text{if} \quad d_{\sigma-1} \le u < d_\sigma, \end{cases} \tag{1}$$

and the transfer function $G(s)$ is given by

$$G(s) = \frac{B(s)}{A(s)} = \frac{s^m + b_{m-1}s^{m-1} + \cdots + b_0}{a_m s^m + a_{m-1}s^{m-1} + \cdots + a_0} \tag{2}$$

In (1), the symbol $m_i = (c_i - c_{i-1})/(d_i - d_{i-1})(i = 1, 2, ..., \sigma)$ are the segment slope with connecting input and output points as $d_i\,(i = 0, 1, ..., \sigma)$ and $c_i\,(i = 0, 1, ..., \sigma)$, respectively. For simplicity of notation, let $\boldsymbol{d} = [d_0, d_1, \ldots, d_\sigma]^{\mathrm{T}}$ and $\boldsymbol{c} = [c_0, c_1, \ldots, c_\sigma]^{\mathrm{T}}$. Note that the total number of input or output points are $\sigma + 1$. The input of the real liquid slosh plant and the identified model is defined by $u(t)$, while the output of the real liquid slosh plant and the identified model are denoted by $y(t)$ and $\tilde{y}(t)$, respectively. Thence, the expression of the identified output can be written as

$$\tilde{y}(t) = G(s)h(u(t)) \tag{3}$$

Moreover, several assumptions are adopted in this work, which are:

(i) The order of the polynomial $A(s)$ and $B(s)$ are assumed to be known.

(ii) The nonlinear function $h(u(t))$ is one-to-one map to the input $u(t)$ and the values of $d_i\ (i = 1,2,...,\sigma)$ are pre-determined according to the response of input $u(t)$.

Next, let t_s be a sampling time for the real experimental input and output data ($u(t)$, $y(t)$) (t = 0, t_s, $2t_s$, …, Nt_s). Then, in order to accurately identify the liquid slosh model, the following objective function in (4) is adopted in this study:

$$E(G,h) = \sum_{\eta=0}^{N} \left(y(\eta t_s) - \tilde{y}(\eta t_s)\right)^2 \tag{4}$$

Note that the objective function in (4) is based on the sum of quadratic error, which has been widely used in many literature [28]-[29]. Finally, our problem formulation can be described as follows.

Problem 1. Based on the given real experimental data ($u(t)$, $y(t)$) in figure 1, find the nonlinear function $h(u)$ and the transfer function $G(s)$ such that the objective function in (4) is minimized.

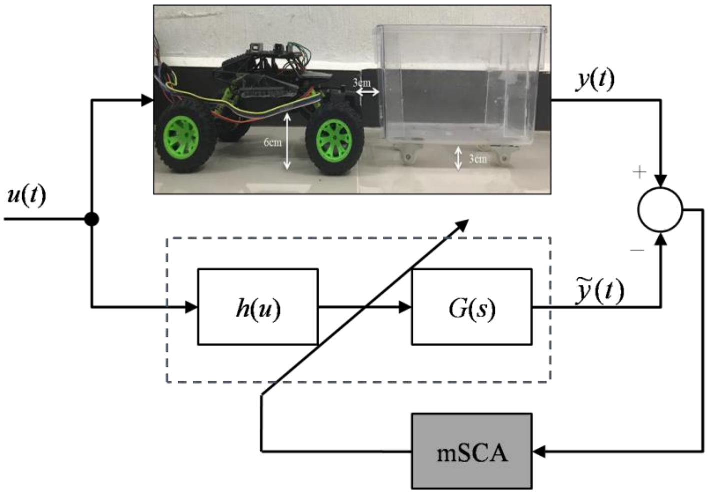


Figure 5. Block diagram of Hammerstein model based mSCA

Furthermore, it is shown on how to apply the SCA in solving **Problem 1**. For simplicity, let the design parameter of **Problem 1** is defined as $\boldsymbol{x} = [b_0\ \ b_1 \cdots\ \ b_{m-1}\ a_0\ \ a_1 \cdots a_m\ \ c_0 \cdots\ \ c_\sigma]^{\mathrm{T}}$, where the elements of the design parameter are the coefficients of both the nonlinear function and the transfer function of the continuous-time Hammerstein model. In SCA framework, let $\boldsymbol{x}_i (i = 1,2,...,M)$ be the design parameter of each agent i for M total number of agents. Then, consider x_{ij} (j = 1, 2,..., D) be the j-th element of the vector $\boldsymbol{xi} (i = 1,2,...,M)$, where D is the size of the design parameter. Thence, by adopting objective function in (4), a minimization problem is expressed as

$$arg \min_{\boldsymbol{x}_i(1),\, \boldsymbol{x}_i(2),...} E(\boldsymbol{x}_i(k)) \tag{5}$$

for iterations k = 1, 2, …, until maximum iteration $k_{\max}$. Finally, the procedure of the mSCA in solving **Problem 1** is shown as follows:

Step 1: Determine the total number of agents M and the maximum iteration $k_{\max}$. Set k = 0 and initialize the design parameter $\boldsymbol{x}_i(0) (i = 1,2,..., M)$ according to the upper bound $\boldsymbol{x}_{\mathrm{up}}$ and lower bound $\boldsymbol{x}_{\mathrm{low}}$ values of the design parameter.

Step 2: Calculate the objective function in (4) for each search agent i.

Step 3: Update the values of the best design parameter ***P*** based on the generated objective function in **Step 2**.

Step 4: For each agent, update the design parameter using the following equation:

$$x_{ij}(k+1) = \begin{cases} \dfrac{x_{ij}(k) + P_j}{2} + r_1 \sin(r_2) \times \left\| r_3 P_j - x_{ij}(k) \right\| & \text{if} \quad r_4 < 0.5, \\ \dfrac{x_{ij}(k) + P_j}{2} + r_1 \cos(r_2) \times \left\| r_3 P_j - x_{ij}(k) \right\| & \text{if} \quad r_4 \geq 0.5, \end{cases} \tag{6}$$

where

$$r_1 = 2\left(1 - \left(\frac{k}{k_{\max}}\right)^{\alpha}\right)^{\gamma} \tag{7}$$

for maximum iteration $k_{\max}$. In (7), the symbols α and γ are the positive constant values that are introduced to regulate the portion of exploration and exploitation during the tuning process. Note that r_2, r_3 and r_4 are random values that are generated independently and uniformly in the ranges $[0, 2\pi]$, $[0, 2]$ and $[0, 1]$, respectively. The detailed justification on the selection of the coefficients r_1, r_2, r_3 and r_4 are clearly explained in [26]-[27]. In (6), the symbol $P_j (j = 1, 2, ..., n)$ is denoted as the best current design parameter in j-th element of ***P*** that is kept during tuning process.

Step 5: After the maximum iteration is achieved, record the best design parameter ***P*** and obtained the continuous-time Hammerstein model in figure 1. Otherwise, repeat **Step 2**.

4. Results and Analysis

In this section, the effectiveness of the modified SCA based method for identifying the liquid slosh system using continuous-time Hammerstein model is demonstrated. In particular, the convergence curve response of the objective function in (4), the pole-zero mapping of linear function and the plot of nonlinear function, will be presented and analyzed in this study.

Based on the experimental setup in Section 2, the input response $u(t)$ as shown in figure 3 is applied to the liquid slosh plant, and the output response $y(t)$ is recorded as shown in figure 4. Here, the input and output data are sampled at $t_s = 0.02$ s for $N = 450$. In this study, the structure of $G(s)$ is selected as follows:

$$G(s) = \frac{B(s)}{A(s)} = \frac{s^3 + b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}. \tag{8}$$

after performing several preliminary testing on the given data ($u(t)$, $y(t)$). The fourth order system is used by considering a cascade of 2nd order system for both dc motor of remote car and the slosh dynamic. Meanwhile, the input points for piece-wise affine function of $h(u(t))$ are given by $\boldsymbol{d} = [0, 0.2, 0.4, 0.6, 0.8, 1, 2, 3, 4, 5]^T$. The selection of vector $\boldsymbol{d}$ is obtained after several preliminary experiments. The design parameter $\boldsymbol{x} \in \boldsymbol{R}^{18}$ with its corresponding transfer function and nonlinear function is shown in Table 1. Next, the mSCA algorithm is applied to tune the design parameter with initial values of design parameter are randomly selected between the upper bound $\boldsymbol{x}_{\mathrm{up}}$ and lower bound $\boldsymbol{x}_{\mathrm{low}}$ as shown in Table 1. Note that the values $\boldsymbol{x}_{\mathrm{up}}$ and $\boldsymbol{x}_{\mathrm{low}}$ are obtained after performing several preliminary experiments. Here, we choose the number of agents $M = 40$, maximum iterations $k_{\max} = 5000$, the values $\alpha = 0.03$ and $\gamma = 0.9$.

Figure 6 shows the response of the objective function convergence with the value of $E(G,h) = 0.1477$ at $k_{\max} = 5000$ with 82.12 % of objective function improvement to produce the best design parameter ***P*** as shown in the final column of Table 1. It shows that the mSCA based method is able to minimize the objective function in (4) and produce a quite close output response $y(t)$ as compared to the real output $\tilde{y}(t)$, which can be clearly seen in figure 7. Note that the identified output response tends to yield high oscillation when input is injected to the system and it start to attenuate when the input is zero, which is quite similar to the response of real experimental output.

In the real experimental setup, we can say that the liquid slosh system is stable since the liquid slosh output is reduced gradually as $t \to \infty$. In order to validate our model regarding the stability, we use the pole-zero map of the identified transfer function $G(s)$ as shown in figure 8. From the pole-zero map, all the poles are located at the left hand side of y-axis. In particular, the obtained values of poles $-0.0515 \pm j14.2755$, -0.9167 and -3.1230, while the obtained values of zeros are -34.4282 and $-0.2859 \pm j0.4780$. On the other hand, we also can observe the feature of nonlinear function by plotting the obtained piece-wise function as depicted in figure 9. Note that our nonlinear function is not restricted to any form of nonlinear function (i.e., quadratic), which is more generalized and provide more flexibility of searching a justifiable function.

Table 1. Design parameter of liquid slosh plant

x	Coefficients	x_{low}	x_{up}	P
x_1	b_2	–5	35	35.0000
x_2	b_1	–5	35	19.9949
x_3	b_0	–5	35	10.6786
x_4	a_4	–5	35	–2.2643
x_5	a_3	–2200	–1	–9.3801
x_5	a_2	–2200	–1	–468.8715
x_7	a_1	–2200	–1	–1864.7569
x_8	a_0	–2200	–1	–1321.0211
x_9	c_0	–5	5	1.2708
x_{10}	c_1	–5	5	–0.5736
x_{11}	c_2	–5	5	1.2831
x_{12}	c_3	–5	5	–3.3464
x_{13}	c_4	–5	5	0.9364
x_{14}	c_5	–5	5	–4.2235
x_{15}	c_6	–5	5	–1.3499
x_{16}	c_7	–5	5	–2.6970
x_{17}	c_8	–5	5	–3.5508
x_{18}	c_9	–5	5	4.5195

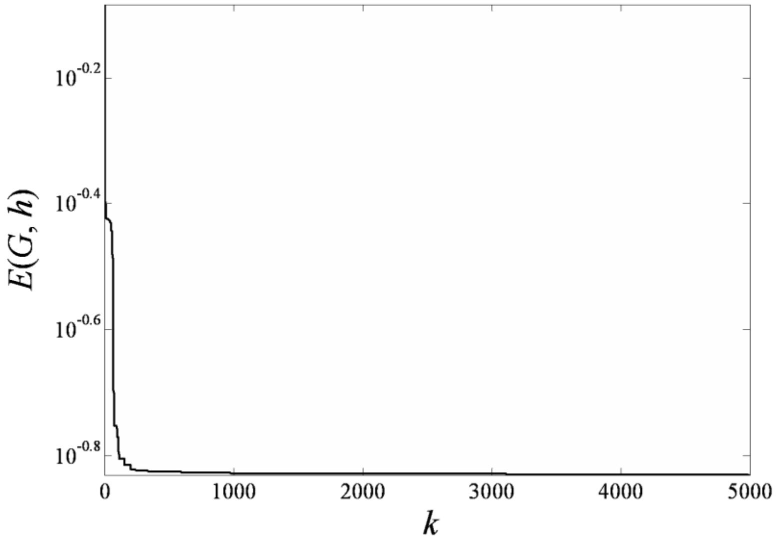


Figure 6. Convergence curve response

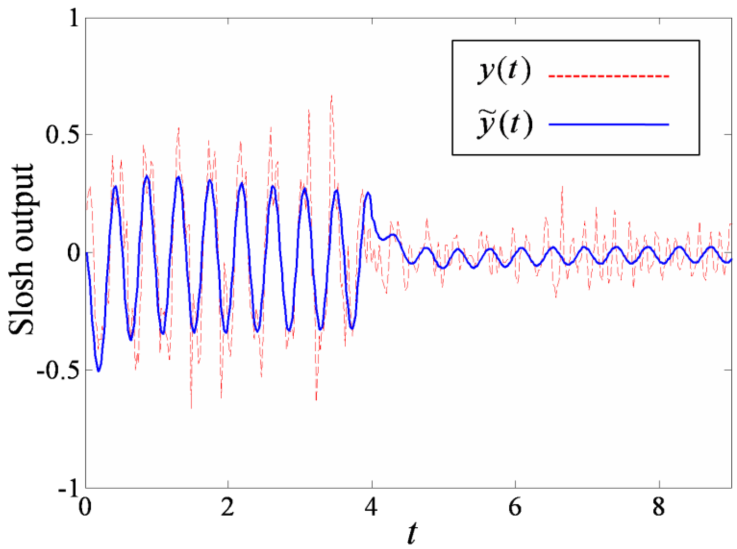


Figure 7. Response of the identified output $\tilde{y}(t)$ and real output $y(t)$

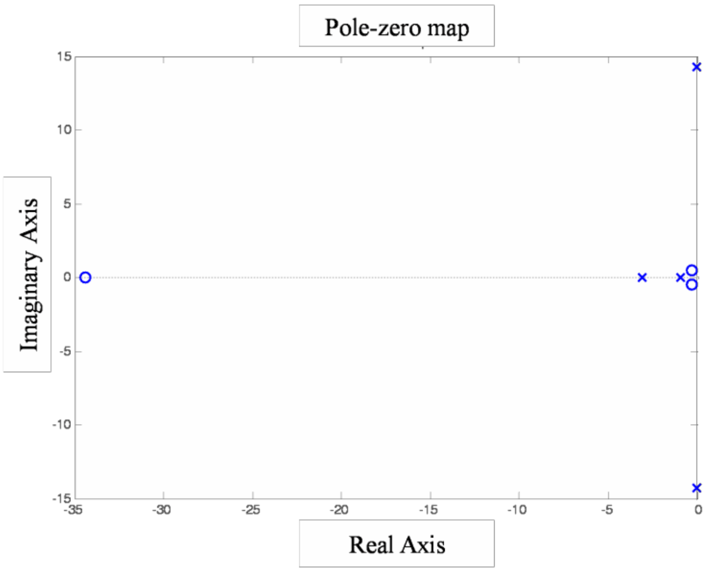


Figure 8. Pole-zero map of transfer function $G(s)$

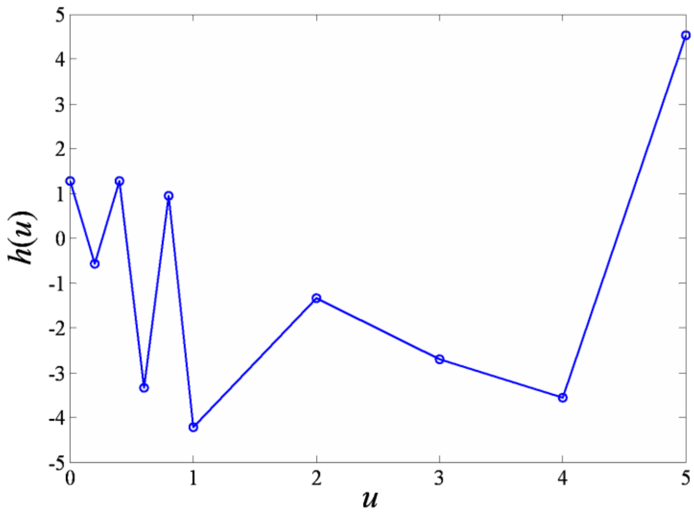


Figure 9. Resultant of piece-wise affine function $h(u)$

5. Conclusion

In this paper, an identification of liquid slosh plant using continuous-time Hammer-stein model based on modified Sine Cosine Algorithm (mSCA) has been presented. The results demonstrated that the proposed generic Hammerstein model based on mSCA has a good potential in identifying the real liquid slosh behavior. In particular, it is shown that the proposed method is able to produce a quite close identified output with real liquid slosh output. Moreover, the resultant linear model has been proved to be stable based on the pole-zero map. It is also shown that the used of piecewise-affine function gives more flexibility for the mSCA to search more generic nonlinear function. In the future, our work can be extended to various types of nonlinear function such as continuous-time Wiener and Hammerstein-Wiener.

Acknowledgments

The authors gratefully acknowledged Research and Innovation Department of Universiti Malaysia Pahang under grant RDU1703153 for the financial support.

References

[1] Rizzuto E and Tedeschi R 1997 Surveys of actual sloshing loads on board of ships at sea *Proc. of Int. Conf. on Ship and Marine Research* p 7.29-7.37.

[2] Terashima K and Schmidt G 1994 Sloshing analysis and suppression control of tilting-type automatic pouring machine *Proc. of IEEE Int. Symp. on Industrial Electronics* p 275-280.

[3] Acarman T and Ozguner U 2006 Rollover prevention for heavy trucks using frequency shaped sliding mode control *Vehicle System Dynamics* vol 44(10) p 737-762.

[4] Li C, Zhu X, Cao G, Sui S and Hu M 2008 Identification of the Hammerstein model of a PEMFC stack based on least squares support vector machines *J. of Power Sources*vol 175 p 303-316.

[5] Kara T and Eker I 2004 Nonlinear modeling and identification of a DC motor for bidirectional operation with real time experiments *Energy Conversion and Management* vol 45(7-8) p 1087-1106.

[6] Su SW, Wang L, Celler BG and Savkin AV 2007 Oxygen uptake estimation in humans during exercise using a Hammerstein model *Annals of biomedical engineering*vol35(11) p 1898-1906.

[7] Westwick DT and Kearney RE 2001 Separable least squares identification of nonlinear Hammerstein models: Application to stretch reflex dynamics. *Annals of Biomedical Engineering* vol 29(8) p 707-718.

[8] Zhang Q, Wang Q and Li G 2016 Nonlinear modeling and predictive functional control of Hammerstein system with application to the turntable servo system *Mechanical Systems and Signal Processing* vol 72 p 383-394.

[9] Ai Q, Peng Y, Zuo J, Meng W and Liu Q 2019 Hammerstein model for hysteresis characteristics of pneumatic muscle actuators. *Int. J. of Intelligent Robotics and Applications* vol 3(1) p 33-44.

[10] Saleem A, Mesbah M and Al-Ratout S 2017 Nonlinear Hammerstein model identification of amplified piezoelectric actuators (APAs): Experimental considerations *4th Int. Conf. on Control, Decision and Information Technologies* p 0633-0638.

[11] Zhang HT, Hu B, Li L, Chen Z, Wu D, Xu B, Huang X, Gu G and Yuan Y 2018 Distributed Hammerstein Modeling for Cross-Coupling Effect of Multiaxis Piezoelectric Micropositioning Stages*IEEE/ASME Transactions on Mechatronics* vol 23(6) p 2794-2804.

[12] Bai EW and Li D 2004 Convergence of the iterative Hammerstein system identification algorithm *IEEE Transactions on automatic control* vol 49(11) p 1929-1940.

[13] Hou J, Chen F, Li P and Zhu Z 2019 Fixed point iteration-based subspace identification of Hammerstein state-space models *IET Control Theory & Applications* vol 13(8) p 1173-1181.

[14] Ge Z, Ding F, Xu L, Alsaedi A and Hayat T 2019 Gradient-based iterative identification method for multivariate equation-error autoregressive moving average systems using the decomposition technique *J. of the Franklin Institute* vol 356(3) p 1658-1676.

[15] HouJ, Liu T, Wahlberg B and Jansson M 2018 Subspace Hammerstein model identification under periodic disturbance *IFAC-PapersOnLine* vol 51(15) p 335-340.

[16] Hou J, Liu T and Wang QG 2019 Subspace identification of Hammerstein-type nonlinear systems subject to unknown periodic disturbance. *Int. J. of Control, (just-accepted)* p 1-29.

[17] Jamaludin IW and Wahab NA 2017 Recursive Subspace Identification Algorithm using the Propagator Based Method *Indonesian J. of Electrical Engineering and Computer Science* vol 6(1) p 172-179.

[18] Wang D and Zhang W2015 Improved least squares identification algorithm for multivariable Hammerstein systems *J. of the Franklin Institute* vol 352(11) p 5292-5307.

[19] Bai EW 2002 A blind approach to the Hammerstein–Wiener model identification *Automatica* vol 38(6) p 967-979.

[20] Ma L and Liu X 2015 A nonlinear recursive instrumental variables identification method of Hammerstein ARMAX system. *Nonlinear Dynamics* vol 79(2) p 1601-1613.

[21] Lin W and Liu PX 2006 Hammerstein model identification based on bacterial foraging *Electronics Letters* vol 42(23) p 1332-1333.

[22] Gotmare A, Patidar R and George NV 2015 Nonlinear system identification using a cuckoo search optimized adaptive Hammerstein model *Expert systems with applications* vol 42(5) p 2538-2546.

[23] Al-Duwaish HN 2011 Identification of Hammerstein models with known nonlinearity structure using particle swarm optimization *Arabian J. for Science and Engineering* vol 36(7) p 1269-1276.

[24] Zhang H and Zhang H 2013 Identification of Hammerstein Model Based on Quantum Genetic Algorithm *Telkomnika* vol 11(12) p 7206-7212.

[25] Mirjalili S 2016 SCA: a sine cosine algorithm for solving optimization problems. *Knowledge-Based Systems* vol 96 p 120-133.

[26] Suid MH, Tumari MZ and Ahmad MA 2019 A modified sine cosine algorithm for improving wind plant energy production. *Indonesian J. of Electrical Engineering and Computer Science* vol 16(1) p 101-106.

[27] Suid MH, Ahmad MA, Ismail MRTR, Ghazali MR, Irawan A and Tumari MZ 2018 An Improved Sine Cosine Algorithm for Solving Optimization Problems *IEEE Conf. on Systems, Process and Control* p 209-213.

[28] Mjahed M. and Ayad H 2019 Quadrotor Identification through the Cooperative Particle Swarm Optimization-Cuckoo Search Approach. *Computational intelligence and neuroscience*p 1-10.

[29] Gupta S, Gupta R and Padhee S 2018 Parametric system identification and robust controller design for liquid–liquid heat exchanger system *IET Control Theory & Applications* vol 12(10) p 1474-1482.