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Mathematical Representation of Joint Angle Measurement using Step Index Optic Fibre and Linear Array Photodiode Sensor

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Abstract. Human joint angle measurement involves various applications, for example monitoring of spine movement in sagittal and frontal planes, upper limb motion such as shoulder, neck and elbow, as well as lower limb motion such as hip, knee, and ankle. As the range of motions of different human joints are not similar, the output estimation of the designed sensor for these applications need to be carried out prior to sensor development. In this paper, the conceptual design of low-cost plastic optical fibre based on single step index fibre is illustrated, the mathematical modelling is explained, and the sensor's hardware assemble is designed using CATIA software, before the estimated sensor output is presented. These results will be used as a reference values for actual optical fibre sensor design in the near future.

1. Introduction

Optical fibre sensor technology is now becoming more popular in various human health applications including spine movement, upper limb motion, lower limb motion, respiration and heart rate detections. Among different types of fibres, the use of plastic fibre has received great attentions from researchers due to the cost reduction in optical fibre production as well as its associated components such as high intensity light source and high sensitive photodiodes [1]. This paper presents a suitable optical-based technique to measure the movement of human joints, particularly the knee. Several possible sensor designs and their respective working principals are discussed in brief before one of the methods is proposed for the actual sensor development. The mathematical representation, the output estimation as well as hardware design development using CATIA software for the proposed technique are also presented.

2. Different knee monitoring sensor design using optical fibre

Generally, the angular movement of human joint can be measured using various devices, namely accelerometer, ultrasonic sensor, strain gauge, piezo-resistive sensor as well as optic fibre sensor (OFS). The OFS can be applied using different modulation approaches such as intensity [2], phase [3] and wavelength [4] modulation techniques. The proposed optical fibre sensor here is slightly different from the currently available methods. In this case, the angular movement of the joint is represented by the light intensity detection at different points on a photodiode sensor array. Hypothetically, the proposed optical fibre sensor should give better accuracy results due to the fast signal transmission and response at the very minimal range of angular movement as compared to other non-optical devices. This is supported by increasing demand of optical fibre based devices use in the medical field as compared to the conventional devices [5, 6, 7].

Based on previous works, the implementation of optic fibre technology resulted in an improved result and better sensor characteristics [1]. But, it has been a great challenge to implement OFS

technology for standard application due to the range of detection angle limitation. The reasons behind this are because some techniques may cause the cable to break when bent to much or it may cause the light signal to scatter in wide angle range [8]. Hence, the proposed design concept offers an alternative solution to implement fibre optic technology for human joint monitoring especially for applications with large range of angle movement such as knee and shoulder.

Three different possible approaches to implement the OFS technique for human join angle monitoring is presented in this section.

2.1 Moveable output fibre arrangement

The first sensor arrangement shown in Figure 1, is based on the use of an LED, multiple photodiodes and a combination of gear and shaft to allow the fibre to move when the human joint (i.e. knee) is rotating at different angles.
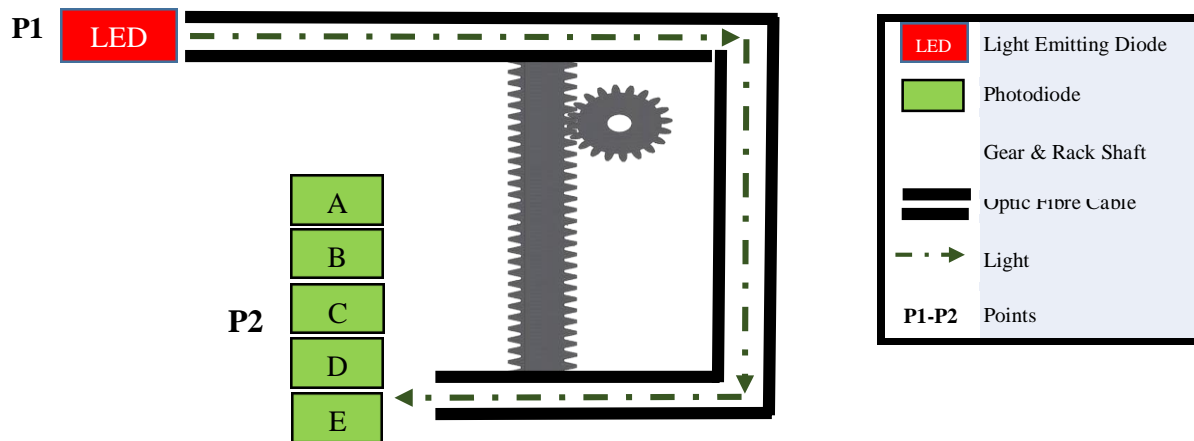

Figure 1. Sensor arrangement using moveable output fibres

The sensor mechanism using this arrangement is as follows;

- Light travel from P1 to P2.
- When knee joint moves, the gear will revolve.
- Motion of the gear causes the rack shaft to move vertically up and down.
- End of the optic fibre cable is attached to the base of the rack shaft.
- Vertical movement of the shaft causes the light to be detected by different detector (A, B, C, D, and E) at P2, where each detector represents different range of angle.

2.2 Light reflection to multiple photodiodes

The second sensor arrangement proposed for this application is based on the use of a reflective mirror in between of the input and output fibres. The LED and photodiodes are fixed pre-determined locations. As the human joint moves, the mirror is allowed to rotate, rendering the light to be channelled into different photodiodes.
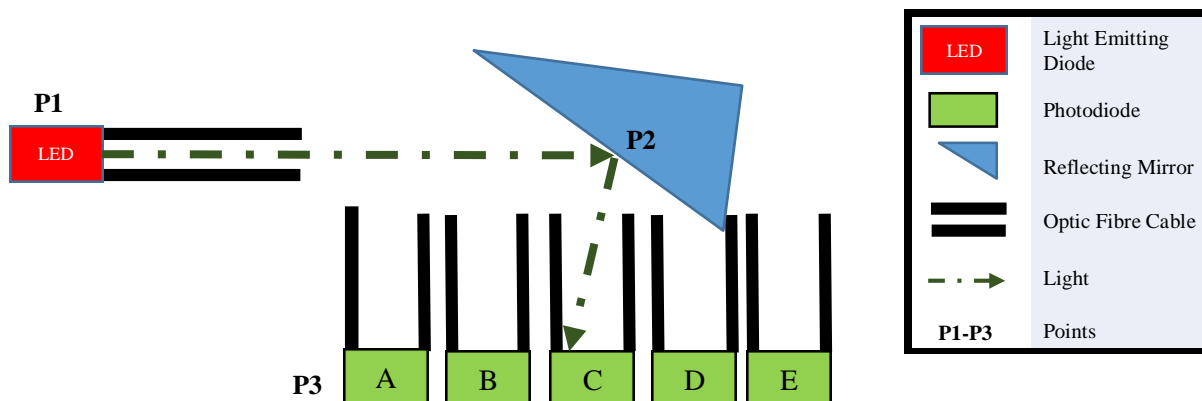

Figure 2. Sensor arrangement using multiple detectors and a reflector

The sensor mechanism using this arrangement is as follows;
- Light travel from P1 to P2 and then to P3.
- Light reflect with an angle on surface of P2.
- Light travel through optic fibre cable to reach a detector at P3.
- Each detector represents different range of angle.

2.3 Light supply from multiple light sources

The last sensor arrangement proposed for this application is based on the use of several LEDs (light sources) and a photodiode. When knee joint moves, the gear will revolve. Then, the motion/ revolving in the gear causes the rack shaft to move vertically up/ down. Movement of the shaft causes the optic fibre cables to move vertically at P1. Movement of cables at P1 varies the amount of light intensity entering into the cables.

The sensor mechanism using this arrangement is as follows;
- Light travel from P1 into the optic fibre cable to P2.
- P2 gathers the light coming from all light sources, and light travels to P3.
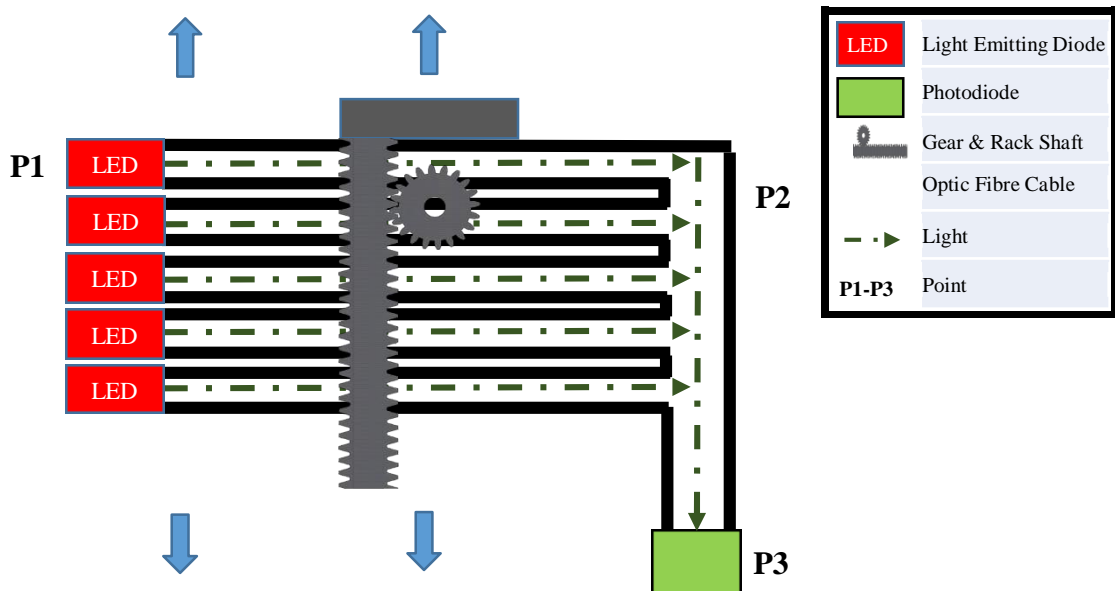- P3 detects the light intensity and converts the value into angle in degree.

Figure 3. Sensor arrangement using multiple LEDs

3. Proposed Hardware Design Assembly

Based on preliminary study, the actual sensor development is assembled based on the detection mechanism using the first sensor arrangement (As in Figure 1). It consists of plastic components and are assembled in a way such that it can translate a rotary movement (human joint movement) into linear movement (fibre movement along the photodiode array). The sketch of the sensor part and its physical components are shown in Figure 4 and Figure 5, respectively.
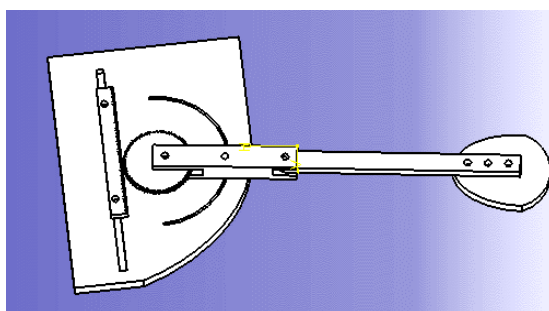

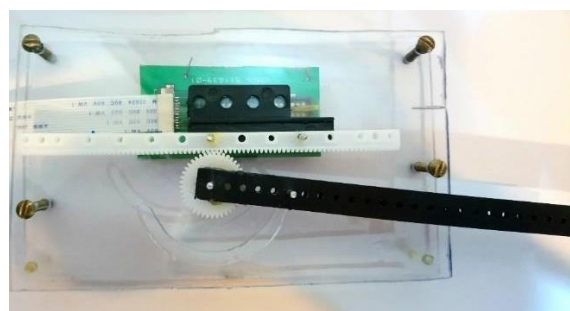**Figure 4**. Hardware Design Assembly Illustration

Figure 5. Actual Hardware Design Assembly.

As can be observed in Figure 5, all components are assembled to perform the sensing mechanism starting from the crank holder (the one that should be attached on the side of the leg) which moves simultaneously with the knee movement. As the crank shaft attached to the shaft holder, the motion of the knee is channelled directly to the gear holder and then to the gear. As a result, rotary motion of the knee gives an equivalent motion to the gear. Hence, for every degree of the knee joint rotation, the movement will drive the gear and is then it is translated into a linear displacement on the rack shaft as it is connected directly with the gear. The rack shaft slides forward and backward together with the gear rotary direction.

3.1 Mechanism of Sensing using CMOS Linear Array Photodiode
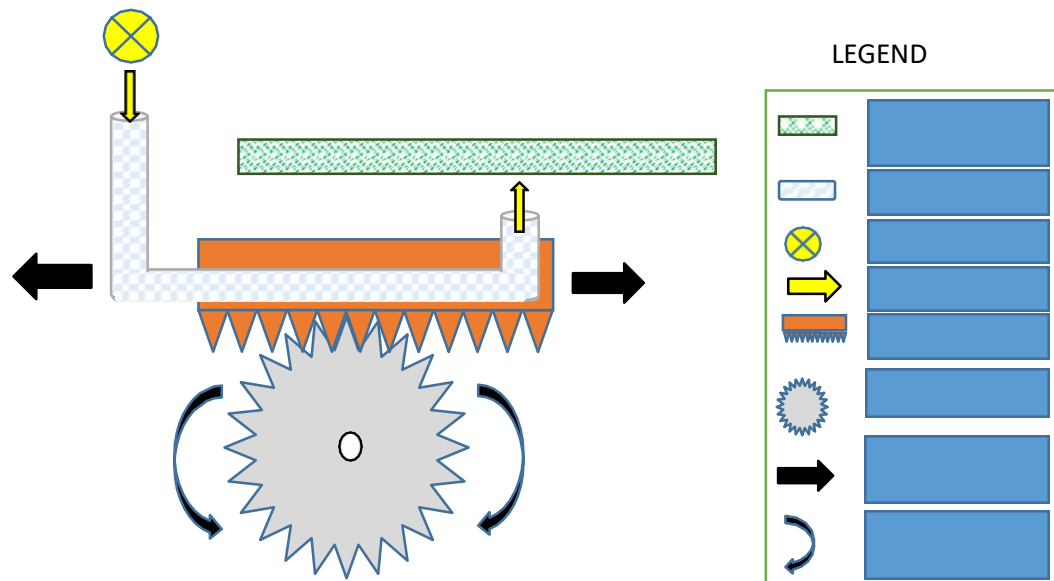
Figure 6. Illustration of the proposed sensor mechanism

Figure 6 illustrates the working mechanism of proposed system and its functionality is as described in the following steps:
1. LED is connected to optic fibre cable.
2. Optic fibre cable is attached and fixed on the rack shaft, and the cable carries the light and transmits it to the other end of the cable facing the linear array photodiode sensor.
3. Gear rotates simultaneously with the knee angular movement.
4. Upon gear rotation, rack shaft moves linearly left or right depending on gear's angular direction.
5. Linear movement of the rack shaft varies the light beam pointing position on the sensor, allowing different pixel positions in the sensor to detect the light beam.
6. Different pixel position refers to different angle.
7. Hence, angular motion variation is translated to linear motion variation and a mathematical formulation is generated to translate the pixel positions into angular reading.

4. Mathematical Modelling

Knee flexion in normal condition goes from zero degree to 155º, in some conditions the knee angle may flex 10º higher or lower than the neutral position (zero degree). Hence, maximum knee flexion can be considered as 165º [9], as shown in Figure 7. A gear complete cycle is 360º. Hence, when detection range needed is 165º, it means the gear in the hardware design should rotate a maximum of 165º and translate that range of rotation into sensor's detection range.

In this design, whenever the gear rotates a range of degree, the rotation is to be translated into linear displacement on the photodiode sensor array at a similar range of angle degree. Therefore, detection range needed is 165º. Mathematical representation of the relationship between gear rotation (represents knee movement) versus sensor detection range is presented in the next paragraph.
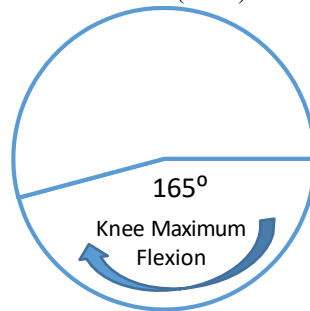
Figure 7. Knee Maximum Flexion Chart.

Before the detection range of the proposed OFS for knee monitoring application is calculated, the actual dimension of the sensitive area on the photodiode array is identified. Based on the sensor's specification sheet by the distributor, the maximum detection distance is 2.867 cm as shown in Figure 8. The sensor name is CMOS linear image sensor S11369-01 by Hamamatsu as shown in Figure 9 [10].
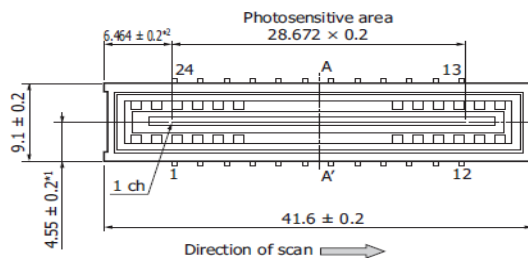
Figure 8. Sensor schematic showing photosensitivity area (Unit: mm) [9].
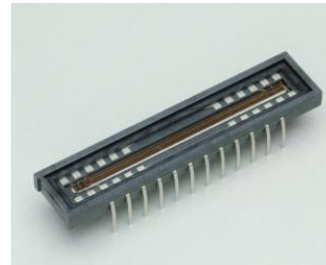
Figure 9. The CMOS linear image sensor S11369-01.

From this dimension, when knee is at maximum flexion of 165º, the distance to be travelled on gear perimeter (in a circle) should be similar to the length of sensor photosensitive length which is 2.867 cm. Hence, in order to obtain the gear diameter which is required to be implemented in the hardware design, the following steps need to be followed. Starting with obtaining the gear internal perimeter (circular perimeter by ignoring threads of the gear).

$$\frac{\text{Sensor photosensitive area}}{\text{Maximum angle to be measured}} = \frac{\text{Gear internal perimeter}}{\text{Circle complete angle}}$$

$$=> \frac{2.867cm}{165^o} = \frac{X}{360^o}$$

$$=> \textbf{X = 6.255 cm} \ \{\text{Gear's internal Perimeter}\}$$

So, the radius of the gear can be obtained using the following formula,

1. Circumference = $2\pi r$ => 6.255 cm = 2(22/7) r

2. Radius of the Gear, => Radius, r = $\dfrac{6.255}{2\pi}$ = **0.996 cm.**

3. Diameter, D is 2R, so D = 2 x 0.996 **= 1.992 cm** {Diameter of the Gear required in the Design}

Hence, the following terms and values are used for the mathematical representation;
1. Sensor detection distance => L_S = 2.867 cm
2. Gear diameter => D_G = 1.992 cm
3. Gear perimeter => L_G = 6.255 cm

4. Gear rotation range desired = 165º {based on the pre-determined / desired angle)
5. *Gear rotation angle => $\angle S$*
6. Gear full rotation angle => $\angle G$ = 360º

Upon identifying required variables for the design, a relationship formula between knee range of motion (angular movement) and sensor detection distance (linear motion) is created as shown in the following steps.

The origin formula to develop the sensor is as follow:

Variable Inputs	$\begin{cases}\end{cases}$	**Sensor Detection Distance** = **Gear Internal Perimeter**	$\begin{cases}\end{cases}$	Fixed Inputs
		Gear Rotation/ Knee Angle **Gear Full Rotation Angle/360º**		

From equation above, the relationship between sensor detection distance (L_S) and knee angle ($\angle S$) is,

$$\frac{L_S}{\angle S} = \frac{L_G}{\angle G} \Rightarrow \frac{L_S}{\angle S} = \frac{6,255}{360^o} = 0.017375$$

Similarly, to find knee angle, $\angle S$

$$\Rightarrow \frac{\angle S}{L_S} = 57.554 \text{ . So, the result gives } \angle S = 57.554 \times L_S$$

Relationship between knee angle and sensor detection distance is obtained, the number 57.554 is a fixed value in the relation and when the value of Ls is inserted, $\angle S$ will be obtained and vice versa.

5. Proving Formula's Reliability

Maximum distance can be travelled on sensor detection area is, L_S = 2.867 cm. By implying the formula of the relationship between Ls and $\angle S$ **,** we should be getting 165º as the maximum desired angle need to be measured when Ls is 2.867 cm. Hence,

$$\angle S = 57.554 \times L_S = 57.554 \times 2.867 = 165.007^o$$

165º is maximum desired angle to be measured when sensor detection area is at its maximum Ls = 2.867 cm. Due to that, formula of relationship between Ls and $\angle S$ is proven correct.

6. Expected Outcome

Ten data of Ls and $\angle S$ in three decimal places is taken to draw the expected result outcome graph for the knee to flex from neutral position till maximum flexion when shank touches the hamstring. Table 1 below shows the data for angle measurement versus distance travelled for knee flexion from neutral to maximum flexing point.

Table 1. Data for angle measurement versus gear distance travelled

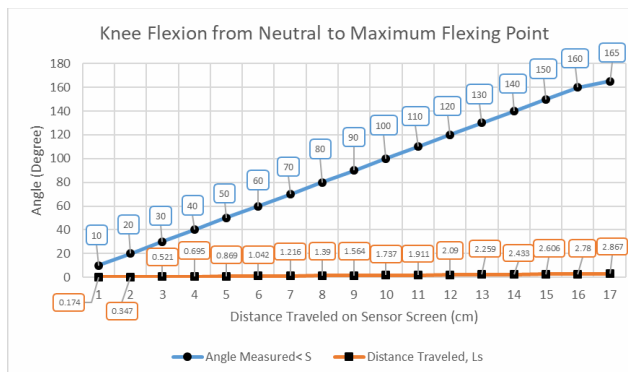| Angle Measured, $\angle S$ (deg.)	10	20	30	40	50	60	70	80
Distance Travelled, Ls (cm)	0.174	0.347	0.521	0.695	0.869	1.042	1.216	1.39
$\angle S$ (deg.)	90	100	110	120	130	140	150	160	165
Ls (cm)	1.564	1.737	1.911	2.09	2.259	2.433	2.606	2.78	2.867

Figure 10. Result Outcome of knee flexion from neutral to maximum flexing point.
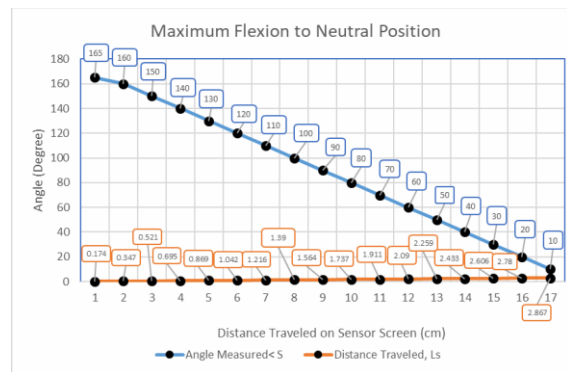

Figure 11. Result of knee movement from maximum flexion to neutral position.

Result shown in Figure 10 is the outcome of moving the knee from straight/neutral condition to a maximum inward rotation till shank touches the hamstring. Hence, when the knee is stretched back to its neutral position again, it should produce an exact inverse of previous one as the knee will rotate the opposite direction as shown in Figure 11.
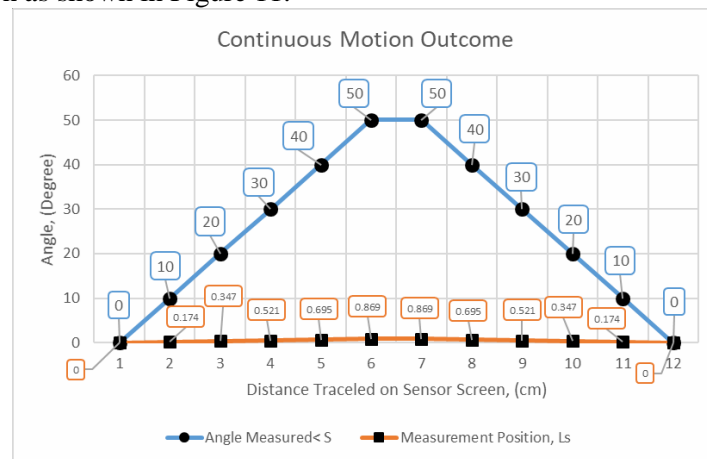

Figure 12. Continuous motion outcome graph.

When the knee is in continuous motion as in walking condition to analyse gait movement condition of the knee, outcome result is in continuous form as shown in Figure 12 for one cycle. Data have been taken from knee neutral position (standing) until 50° and returns back to its neutral condition again, data as shown is Table 2. The sensor output for static body positions (i.e. sitting, standing and lying) and dynamics body movement (i.e. walking, running and cycling) can be represented by the proposed sensor based on the repetitive knee joint movement as illustrated in Figure 12.

Table 2. Data used to analyse a simple gait motion

(A) Moving from neutral position to 50°.

Angle Measured, $\angle S$ (deg.)	0	10	20	30	40	50
Distance Travelled, Ls (cm)	0	0.174	0.347	0.521	0.695	0.869

(B) Moving from 50° to neutral position.

Angle Measured, $\angle S$ (deg.)	50	40	30	20	10	0
Distance Travelled, Ls (cm)	0.869	0.695	0.521	0.347	0.174	0

7. Discussion and Summary

The proposed optical fibre sensor arrangement for human joint monitoring in this study is based on the use of single light source (LED), multiple photodiodes (to be realized using linear array photodiode) and plastic optical fibre. Mathematical estimation is applied to observe the suitable sensor components such as gear radius, gear perimeter and maximum gear rotation angle that suits the proposed knee application. From the simulated output response, it is possible to detect an angle of up to 165 degrees, using a set of gear and shaft arrangement that moves along linear array photodiode with a maximum distance of 2.867 cm. There is a linear relationship between the angle of the joint (rotation of the gear) and the distance travelled by the gear along the photodiode array (linear movement). Thus, it is possible to design a high accuracy and repeatability knee monitoring sensor using the proposed optical fibre sensor arrangement. For the time being, the output resolution (in degree) of the proposed sensor is yet to be verified. Other factors that could influence the sensor performance such as the pixel number of the photodiode, the length of photodiode sensitive are and the diameter of the light beam that touch the light sensitive surface is also not consider at this time.

It can be summarized that the implementation of optic fibre together with the linear array photodiode sensor is theoretically possible based on the findings in this study. The proposed optical fibre sensor has the required characteristics for continuous knee joint movement and gait activity applications, which is beneficial for early detection of abnormal knee condition or for rehabilitation process. The inherit advantages of optical fibre sensor will be added values for the proposed sensor especially when safety and health issue is one of the critical factors for sensor implementation in various human health applications. Finally, it is clearly possible to achieve a feasible sensor outcome for knee application using optical fibre sensor for the purpose of laboratory-based and field test measurement.

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