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## Thermal Radiation Effect on MHD Stagnation Point flow of Williamson Fluid over a stretching Surface

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Abstract. Thermal radiation effects on MHD stagnation point flow of Williamson fluid over a stretching surface are studied. With the help of similarity transformation, the governing equations are converted to nonlinear ordinary differential equations and then solved numerically by Runge-Kutta-Fehlberg (RKF) technique. Numerical results for the reduced Nusselt number and reduced skin friction coefficient as well as the temperature and velocity profiles are elucidated through tables and graphs. The influence of Prandtl number, stretching parameter, Williamson fluid parameter, thermal radiation parameter and magnetic parameter are analyzed and discussed. It is found that, as Prandtl number and magnetic parameter increase, the temperature profiles decrease. Meanwhile, as Williamson fluid parameter and thermal radiation parameter because.

## 1. Introduction

In recent years, the study of non-Newtonian fluids has attracted attention of many researchers. In fact, there are many theoretical and technical applications both in industries and engineering processes, for example, in the aerodynamic extrusion of plastic sheets, glass fibre, paper production, manufacturing of polymer sheets [1], oil recovering and food processing [2]. In view of their difference with Newtonian fluids, many models of non-Newtonian fluid have been proposed such as the Jeffrey fluid [2], the second-grade fluid [3], the Casson fluid [4], the micropolar fluid [5] and the Williamson fluid [6].

Williamson [7] proposed the flow of pseudoplastic materials and developed a model equation to illustrate the flow of pseudoplastic fluid. Next, Nadeem et al. [6] started to investigate the flow of a Williamson fluid over a stretching sheet. The homotopy analysis was being used to solve the non-linear differential equation. After that, Khan et al. [8] and Nadeem and Hussain [9] investigated the boundary layer flow and heat transfer of Williamson fluid with chemical reactive species using scaling transformation approaches. It was found that the Williamson fluid model was very much similar to the blood and almost completely described the blood flow.

Next, the flow near the stagnation point refers to the vertical flow hit perpendicularly the horizontal surface which generated the stagnation line. Hiemenz [10] was the first lead to solve the problem involving stagnation point and managed to obtain the exact value for Navier-Stokes equations. In the conjunction with stretching surface, the external velocity was employed in the negative *y*-direction perpendicular to the flat plate while the stretching velocity was applied along the horizontal surface ([11];[12]). According to Wang [11], the utmost pressure, utmost heat transfer and the utmost rates of mass deposition are encountered by the stagnation region.

Problem associated with boundary layer flows on stagnation point and stretching surface has captivated many researchers such as Chao and Jeng [13], Nazar et al. [14], Ishak et al. [15], Khan et al. [16], Nandy and Mahapatra [17] and recently, Mehmood et al. [18] who studied on non-aligned stagnation point flow of radiating Casson fluid over a stretching surface. Comparison was made with previous published literature and good agreement was obtained. Next, Rehman et al. [19] investigated the thermo physical analysis for three-dimensional MHD stagnation-point flow of nano-material influenced by an exponential stretching surface. They found the one best example where flow caused by stretching plate near stagnation point could be detected through the spinning, floating and blowing of fibre glass. Thermal radiation and slip effects on MHD stagnation point flow of nano-fluid over stretching sheet had been deliberate by Haq et al. [20]. They used Runge-Kutta Fourth-fifth order method along with shooting technique to solve coupled ordinary differential equations. Sandeep et al. [21] presented stagnation-point flow of a Jeffery nano-fluid over a stretching surface with induced magnetic field and chemical reaction and the Runge-Kutta scheme were used to solve this problem.

The electrical components specifically deal with magnetic effects. The interaction of the fluid motion, the dynamics of fluids as good conductors of electricity with any ambient magnetic field coin as magnetohydrodynamic (MHD) effect [22]. Meanwhile, the thermal radiation effects play an important role in engineering applications such as high temperature plasmas, cooling of nuclear reactors and liquid metal fluids. Furthermore, the radiation heat transfer flow effects are also applied in space technology and in processes involving high temperatures [23]. Bataller [24] and Mukhopadhyay [25] studied the effects of radiation in both Blasius and Sakiadis flows and unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. Other researchers who considered the MHD and thermal radiation effects are Chen [26], Hayat et al. [27], Salleh et al. [28], Anwar et al. [29] and Elbashbeshy et al. [30]. Also, Makinde and Olanrewaju [31], Makinde and Aziz [32] and Olanrewaju et al. [33] considered MHD and thermal radiation effects in heat and mass transfer, mixed convection over a vertical plate in porous medium and viscous dissipation effects for Blasius and Sakiadis flow with convective boundary conditions. Thermal radiation and slip effects on magnetohydrodynamic (MHD) stagnation point flow of Casson fluid over a convective stretching sheet were examined by Raza [34]. It was found out that, there was an inverse relationship between magnetic parameter and stream wise velocity. Narayana and Babu [35] analyzed numerical study of MHD heat and mass transfer of a Jeffrey fluid over a stretching sheet with chemical reaction and thermal radiation. The result shows the effects of thermal radiation have caused the increase in the temperature of the thermal boundary layer and the reverse effect is seen by increasing the Prandtl number. Hayat et al. [36] addressed the simultaneous effects of heat generation/absorption and thermal radiation in magnetohydrodynamics (MHD) flow of Maxwell nanofluid towards a stretched surface. It was seen that, the thermal radiation parameter enhanced the

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temperature field and heat transfer rate. Recently, Gupta et al. [37] observed MHD mixed convection stagnation point flow and heat transfer of an incompressible nanofluid over an inclined stretching sheet with chemical reaction and radiation. The package BVPh 2.0 was used to solve the non-linear differential equation.

The aim of this study is to investigate the thermal radiation effects on MHD stagnation point flow of Williamson fluid over a stretching surface. From the literature studies, this problem has not been considered before, therefore, the results reported here are new.

### 2. Mathematical formulation

Consider the steady two-dimensional flow of a non-Newtonian Williamson fluid over a stretching plate as shown in Figure 1. The external and stretching velocities are  $u_e(x) = ax$  and  $u_w(x) = cx$ , where *a* and *c* are constants. The boundary layer equations ([6];[38]) are as follow



FIGURE 1: Physical model of the coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + v\frac{\partial^2 u}{\partial y^2} + \sqrt{2} v \Gamma \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (u_e - u)$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}$$
(3)

corresponds to the following conditions

$$u = u_w(x), \quad v = 0, \quad T = T_w \text{ at } y = 0$$
  
$$u \to u_e(x), \quad T \to T_\infty \quad \text{as } y \to \infty$$
(4)

where u and v are the velocity in the x and y-axis,  $T_w(x) = T_w + bx^2$  is the wall temperature with b as constant, v is the kinematic viscosity, T is the fluid temperature,  $B_o$  is the uniform magnetic field

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strength,  $\sigma$  is the electric conductivity k is the thermal conductivity,  $\rho$  is the fluid density,  $C_p$  is the specific heat,  $\Gamma$  is the time constant and  $\mu$  is the dynamic viscosity.

Using Rosseland approximation for radiation see Bataller [24], the radiative heat flux  $q_r$  in equation (3) may be simplified as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where  $\sigma^*$  and  $k^*$  are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow region, namely, the term  $T^4$  can be expressed as a linear function of temperature. Hence, expanding  $T^4$  in a Taylor series about  $T_{\infty}$  and neglecting higher-order terms, we get

$$T^4 \cong 4T^3_{\infty}T - 3T^4_{\infty} \tag{6}$$

Using equations (5) and (6), the equation (3) is reduced to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{k}{\rho C_p} + \frac{16\sigma * T_{\infty}^3}{k*}\right)\frac{\partial^2 T^2}{\partial y^2},$$
(7)

From the above equation it is seen that the effect of radiation is to enhance the thermal diffusivity. If we take  $Nr = \frac{4\sigma T_{\infty}^{3} \rho C_{p}}{k^{*}k}$  as the radiation parameter, equation (7) becomes

 $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left(1 + \frac{4}{3}Nr\right) \frac{\partial^2 T^2}{\partial y^2},$ (8)

Note that thermal radiation effects are absent when Nr = 0. Now, we introduce the following similarity variable

$$\eta = \left(\frac{c}{\nu}\right)^{\frac{1}{2}} y, \qquad \psi = \left(c\nu\right)^{\frac{1}{2}} xf(n), \quad \theta(\eta) = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}, \tag{9}$$

where  $\psi$  is the stream function defined by  $u = \frac{\partial \psi}{\partial y}$  and  $v = -\frac{\partial \psi}{\partial x}$ , which satisfies the equation (1).

Further, notice that

$$u = cxf'(\eta), \quad v = -(cv)^{\frac{1}{2}}f(\eta)$$
(10)

where prime represents differentiation with respect to  $\eta$ . By substituting equations (9) and (10), into equations (2) and (8), the transformed ordinary differential equation is obtained as follows:

$$f''' + ff'' + \varepsilon^2 - f'^2 + \lambda f''f''' + M(\varepsilon - f') = 0$$
(11)

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$$\left[1 + \frac{4}{3}Nr\right]\theta'' - \Pr\left[2f'\theta - f\theta'\right] = 0$$
(12)

where  $\Pr = \frac{v \rho C_p}{k}$  is the Prandtl number,  $\lambda = x \Gamma \sqrt{\frac{2c^3}{v}}$  is the non-Newtonian Williamson fluid parameter,  $M = \frac{\sigma B_o^2}{\rho c}$  is the magnetic parameter and  $\varepsilon = \frac{a}{c}$  is the stretching parameter.

Based on derivative by using equations (9) and (10) into boundary conditions (4) the outcomes are

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \text{ at } \eta = 0$$
  
$$f'(\infty) \to \varepsilon, \quad \theta(\infty) \to 0 \quad \text{as } \eta \to \infty$$
(13)

The skin friction coefficient  $C_f$  and the local Nusselt number  $Nu_x$  are given as

$$C_f = \frac{\tau_w}{\rho u_w^2} , \qquad N u_x = \frac{x q_w}{k (T_w - T_\infty)}$$
(14)

where,  $\tau_w$  the surface shear stress and  $q_w$  heat flux are defined as

$$\tau_{w} = \mu \frac{\partial u}{\partial y} \left[ 1 + \Gamma \sqrt{\frac{1}{2}} \frac{\partial u}{\partial y} \right] \quad , \qquad q_{w} = -k \frac{\partial T}{\partial y} + q_{r} \tag{15}$$

Using the similarity variables in (9), the outcomes are:

$$C_{f} \operatorname{Re}_{x}^{\frac{1}{2}} = f''(0) + \frac{\lambda}{2} (f''(0))^{2}, \qquad Nu_{x} \operatorname{Re}_{x}^{-\frac{1}{2}} = -\left[1 + \frac{4}{3} Nr\right] \theta'(0) \qquad (16)$$

where  $\operatorname{Re}_{x} = \frac{c x^{2}}{v}$  is the local Reynolds number.

#### 3. Results and discussion

Equations (11) and (12) with boundary conditions (13) were solved numerically by Runge-Kutta-Fehlberg technique using the MAPLE software. In order to study the flow characteristic, pertinent parameters, namely the Prandtl number Pr, the stretching parameter  $\varepsilon$ , the magnetic parameter M, the non-Newtonian Williamson fluid parameter  $\lambda$  and the thermal radiation parameter Nr considered. The validation for efficiency of the method used are shown in Tables 1 and 2. Notice that, results shown by Mahapatra and Gupta [39] in Table 1 are numerically calculated by using finite difference method known as Thomas algorithm while Nazar et al. [40] and Ishak et al. [15] consider the Keller-box method. From both Tables 1 and 2, it is found that the results presented are in an excellent agreement, therefore, we are confident with the accuracy of the result of the results in this problem.

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Е	Mahapatra and Gupta [39]	Nazar et al. [40]	Ishak et al. [15]	Present	
	f''(0)	f''(0)	f''(0)	f''(0)	
0.1	- 0.9694	- 0.9694	- 0.9694	- 0.969436	
0.2	- 0.9181	- 0.9181	- 0.9181	- 0.918113	
0.5	- 0.6673	- 0.6673	- 0.6673	- 0.667263	
2	2.0175	2.0176	2.0175	2.017502	
3	4.7293	4.7296	4.7294	4.729282	

**Table 1.** Comparison of the present results to those obtained in previous works when Pr = 1, Nr = 0,

Table 2. Comparison of the present results to those obtained in previous works when Pr = 3, Nr = 0,

$M = 0$ and $\mathcal{E} = 0$						
	Nadeem et al. [6]	Nadeem and	Present			
λ		Hussain [9]				
	f''(0)	f''(0)	f''(0)			
0.1	-1.03446		-1.034981			
0.2		-1.076	-1.076786			

Table 3 presents the values of reduced Nusselt number  $Nu_x \operatorname{Re}_x^{-1/2}$  and reduced skin friction coefficient  $C_f \operatorname{Re}_x^{1/2}$  for the various values of the non-Newtonian Williamson fluid parameter  $\lambda$ . It is found that as  $\lambda$  increases, the values of  $Nu_x \operatorname{Re}_x^{-1/2}$  decrease and the values of  $C_f \operatorname{Re}_x^{1/2}$  increase. It is physically found that the huge changes in  $\lambda$  gave small effect on  $Nu_x \operatorname{Re}_x^{-1/2}$ , while on  $C_f \operatorname{Re}_x^{1/2}$  gives huge effects. Further, the increase of  $\lambda$  promote to a decrease in convection capabilities in Williamson fluid.

**Table 3.** Values of  $Nu_x \operatorname{Re}_x^{-1/2}$  and  $C_f \operatorname{Re}_x^{1/2}$  for the various values of  $\lambda$  when  $\operatorname{Pr} = 7$ ,  $\varepsilon = 3$ , Nr = 1 and

M=1.					
$Nu_x \operatorname{Re}_x^{-1/2}$	$C_f \operatorname{Re}_x^{1/2}$				
8.74162	6.71948				
8.60367	7.62752				
8.37247	9.86503				
8.26885	11.32340				
8.20378	12.45849				
	$M = 1.$ $Nu_{x} \operatorname{Re}_{x}^{-1/2}$ 8.74162 8.60367 8.37247 8.26885 8.20378				

Table 4 presents the values of  $Nu_x \operatorname{Re}_x^{-1/2}$  and  $C_f \operatorname{Re}_x^{1/2}$  for various values of the stretching parameter  $\varepsilon$  when Nr = 1, M = 1,  $\lambda = 1$  and  $\operatorname{Pr} = 7$ , 10, 12. It is observed that, when  $\operatorname{Pr} = 7$ , an increase of  $\varepsilon$  makes values of  $Nu_x \operatorname{Re}_x^{-1/2}$  increase as well as the values of the values of  $C_f \operatorname{Re}_x^{1/2}$  and

for all values of Pr. Meanwhile, when  $\varepsilon$  is fixed, an increase of Pr results to the increase of  $Nu_x \operatorname{Re}_x^{-1/2}$  and  $C_f \operatorname{Re}_x^{1/2}$ .

**Table 4.** Values of  $Nu_x \operatorname{Re}_x^{-1/2}$  and  $C_f \operatorname{Re}_x^{1/2}$  for the various values of  $\mathcal{E}$  when  $\operatorname{Pr} = 7, 10, 12$ , Nr = 1, M = 1 and  $\lambda = 1$ .

	Pr = 7		Pr = 10		Pr = 12	
Е	$Nu_x \operatorname{Re}_x^{-1/2}$	$C_f \operatorname{Re}_x^{1/2}$	$Nu_x \operatorname{Re}_x^{-1/2}$	$C_f \operatorname{Re}_x^{1/2}$	$Nu_x \operatorname{Re}_x^{-1/2}$	$C_f \operatorname{Re}_x^{1/2}$
0.5	7.47424	0.10496	8.94838	0.14244	9.80769	0.16188
1	7.69706	0.74641	9.16790	0.77580	10.02533	0.79023
3	8.60040	7.59112	10.10785	7.60808	10.98352	7.61564
5	9.32551	21.43984	10.88287	21.45303	11.78427	21.45875
7	9.92813	42.32314	11.53398	42.33457	12.46078	42.33945

Table 5 presents the values of  $Nu_x \operatorname{Re}_x^{-1/2}$  and  $C_f \operatorname{Re}_x^{1/2}$  for various values of the magnetic parameter *M* when  $\operatorname{Pr} = 7$ ,  $\varepsilon = 3$ ,  $\lambda = 1$  and Nr = 0, 1, 7. An increase of *M* makes the values of  $Nu_x \operatorname{Re}_x^{-1/2}$  and  $C_f \operatorname{Re}_x^{1/2}$  increase. Meanwhile, it is noticed that the increasing values of Nr causes the increase the values of  $Nu_x \operatorname{Re}_x^{-1/2}$  and  $C_f \operatorname{Re}_x^{1/2}$ .

**Table 5.** Values of  $Nu_x \operatorname{Re}_x^{-1/2}$  and  $C_f \operatorname{Re}_x^{1/2}$  for the various values of M when  $\operatorname{Pr} = 7$ ,  $\varepsilon = 3$ , Nr = 0, 1, 7 and  $\lambda = 1$ .

	$N\mathbf{r} = 0$		Nr = 1		Nr = 7	
Μ	$Nu_x \operatorname{Re}_x^{-1/2}$	$C_f \operatorname{Re}_x^{1/2}$	$Nu_x \operatorname{Re}_x^{-1/2}$	$C_f \operatorname{Re}_x^{1/2}$	$Nu_x \operatorname{Re}_x^{-1/2}$	$C_f \operatorname{Re}_x^{1/2}$
0.5	4.74218	7.17957	8.56787	7.17957	21.56548	7.18817
1	4.75753	7.59112	8.60040	7.59112	21.68010	7.60808
3	4.81422	9.24307	8.72026	9.24307	22.09911	9.29144
5	4.86480	10.90265	8.82683	10.90265	22.46765	10.97967
7	4.91061	12.56830	8.92309	12.56830	22.79763	12.67173

Figure 2 points the temperature profiles for various values of Pr. Since the value of Pr rises, it is found that the value of the wall temperature and the thickness of the thermal boundary layer drop. Physically the Prandtl number indicates the ratio of momentum diffusivity to thermal diffusivity. Larger values of Pr have higher momentum diffusivity while smaller in thermal diffusivity. This higher momentum diffusivity corresponds to the thinning of thermal boundary layer thickness. Figure 3 presents the velocity profile and skin friction coefficient for several values of  $\varepsilon$  respectively. It is found that the velocity profile increases as  $\varepsilon$  increases which denoted to the rise of skin friction coefficient. Next, the velocity boundary layer thickness decreases as  $\varepsilon$  increases.

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**FIGURE 2:** Temperature  $\theta(\eta)$  for various values of Pr when  $\varepsilon = 3$ , Nr = 1, M = 1 and  $\lambda = 1$ 

**FIGURE 3**: Velocity profiles  $f'(\eta)$  for values of  $\varepsilon$  when  $\Pr = 7$ , Nr = 1, M = 1, and  $\lambda = 1$ 

Figure 4 present the temperature profile for various values of non-Newtonian Williamson fluid parameter  $\lambda$ . It is observed that the changes of  $\lambda$  results have a very small effect on the thermal boundary layer thickness. Figure 5 presents the temperature profile for various values thermal radiation parameter Nr respectively. From this figure, it is found that the temperature profiles and boundary layer thickness increase as Nr increases. The thermal radiation emits energy which raises the temperature therefore, enhances the energy spreading far away from plate surface. This increases the thickness of the thermal boundary layer.





**FIGURE 4:** Temperature  $\theta(\eta)$  for various values when Pr = 7, Nr = 1,  $\varepsilon = 3$  and M = 1

**FIGURE 5:** Temperature  $\theta(\eta)$  for several  $\lambda$  values of *Nr* when  $Pr = 7, \varepsilon = 3, M = 1$  and  $\lambda = 1$ 

## 4. Conclusion

In this paper, thermal radiation effects on MHD stagnation point flow of Williamson fluid over a stretching are numerically studied. The presented analysis leads to the following main results:

- The values of Nusselt number decrease and skin friction coefficient increases, as a non-Newtonian Williamson fluid parameter λ increases.
- The values of Nusselt number and skin friction coefficient increase, as stretching parameter  $\mathcal{E}$  and magnetic parameter M increase.
- The increase of Prandtl number Pr has resulted to a decrease of thermal boundary layer as well as skin friction coefficient.
- As stretching parameter  $\mathcal{E}$  increases, the velocity profile increases while the velocity boundary layer thickness decreases.
- As a non-Newtonian Williamson fluid parameter  $\lambda$  and thermal radiation parameter Nr increase, the thermal boundary layer also increases. The energy obtained from the thermal radiation has raised the temperature and led to the increase in energy spreading far from the plate surface.

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