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Abstract. In domain $\Omega \subset E_n$ it is considered a set of smooth lines such that through a point $X \in \Omega$ passed one line of given set. The moving frame $\Re = \left(X, \vec{e_i}\right)$ $\left(i, j, k = \overline{1, n}\right)$ is frame of Frenet for the line ω^i of the given set. Integral lines of the vector fields $\vec{e_i}$ are formed net Σ_n of Frenet. There is exist the point $F_i^n \in \left(X, \vec{e_1}\right)$ on the tangent of the line ω^i . When the point X is shifted in the domain Ω, the point F_i^n describes it's domain Ω_i^n in E_n. It is defined the partial mapping $f_i^n : \Omega \to \Omega_i^n$ such that $f_i^n\left(X\right) = F_i^n$ Necessary and sufficient conditions of of quasi-double lines of the partial mapping f_i^n of space E_n are proved.

1. Introduction

In domain $\Omega \subset E_n$ it is considered a set of smooth lines such that through a point $X \in \Omega$ passed one line of given set. The moving frame $\Re = \left(X, \vec{e_i}\right)$ $\left(i, j, k = \overline{1, n}\right)$ is frame of Frenet for the line ω^1 of the given set of smooth lines. Derivation formulas of the frame $\Re$ have a form:

$$d\overrightarrow{X} = \omega^i \vec{e}_i \ , \ d\vec{e_i} = \omega_i^k \vec{e}_k \tag{1}$$

The forms ω^i, ω_i^k satisfied structure equations of Euclidean space:

$$D\omega^i = \omega^k \wedge \omega_k^i, \ D\omega_i^k = \omega_i^j \wedge \omega_j^k, \ \omega_i^j + \omega_j^i = 0 \tag{2}$$

Integral lines of vector fields $\vec{e}_i$ are formed the net Σ_n of Frenet for the line ω^1 of the given set of lines. Since frame $\mathfrak{R}$ is constructed on tangent of lines of the net Σ_n, the forms ω_i^k are principal forms [1]; in other words

$$\omega_i^k = \Lambda_{ij}^k \omega^j \tag{3}$$

Using (3) with combination of the equation (2) it follows that

$$\Lambda_{ij}^k = -\Lambda_{kj}^i \tag{4}$$

If we differentiate equation (3) externally, then we have:

$$D\omega_i^k = d\Lambda_{ij}^k \wedge \omega^j + \Lambda_{ij}^k D\omega^j .$$

By using equation (2)

$$\omega_i^j \wedge \omega_j^k = d\Lambda_{ij}^k \wedge \omega^j + \Lambda_{ij}^k \wedge \omega^\ell \wedge \omega_\ell^j .$$

If we note the formula (3), then from the latter formula it follows that

$$\omega_i^j \wedge \Lambda_{j\ell}^k \omega^\ell = d\Lambda_{ij}^k \wedge \omega^j - \Lambda_{ij}^k \omega_\ell^j \wedge \omega^\ell$$

or

$$\Lambda_{j\ell}^k \omega_i^j \wedge \omega^\ell = d\Lambda_{ij}^k \wedge \omega^j - \Lambda_{ij}^k \wedge \omega_\ell^j \wedge \omega^\ell .$$

From here we found:

$$d\Lambda_{ij}^k \wedge \omega^j - \Lambda_{i\ell}^k \omega_j^\ell \wedge \omega^j - \Lambda_{j\ell}^k \omega_i^j \wedge \omega^\ell = 0$$

or

$$\left(d\Lambda_{ij}^k - \Lambda_{i\ell}^k \omega_j^\ell - \Lambda_{\ell j}^k \omega_i^\ell \right) \wedge \omega^j = 0 .$$

By using Lemma of Cartan [2] we have:

$$d\Lambda_{ij}^k - \Lambda_{i\ell}^k \omega_j^\ell - \Lambda_{\ell j}^k \omega_i^\ell = \Lambda_{ijm}^k \omega^m$$

or

$$d\Lambda_{ij}^{\ k} = \left(\Lambda_{ijm}^{\ k} + \Lambda_{il}^{\ k} \Lambda_{jm}^{\ l} + \Lambda_{lj}^{\ k} \Lambda_{im}^{\ l} \right) \omega^m , \tag{5}$$

where

$$B_{ikm}^j = \left(\Lambda_{ijm}^k + \Lambda_{il}^k \Lambda_{im}^l + \Lambda_{lj}^k \Lambda_{im}^l \right) \tag{6}$$

The system of variable $\left\{ \Lambda_{ij}^k , \Lambda_{ijm}^k \right\}$ is formed geometrical object of second order. The formulas of Frenet (see for details to [3]) for the line ω^1 of the given set have a form

$$d_1\vec{e}_1 = \Lambda_{11}^2 \vec{e}_2 ,$$
$$d_1\vec{e}_2 = \Lambda_{21}^1 \vec{e}_1 + \Lambda_{21}^3 \vec{e}_3 ,$$
$$d_1\vec{e}_3 = \Lambda_{31}^2 \vec{e}_2 + \Lambda_{31}^4 \vec{e}_4 ,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$d_1\overrightarrow{e_{n-1}} = -\Lambda_{n-2,1}^{\ n-1} \overrightarrow{e_{n-2}} + \Lambda_{n-1,1}^{\ n} \overrightarrow{e_n} ,$$
$$d_1\overrightarrow{e_n} = -\Lambda_{n-1,1}^{\ n} \overrightarrow{e_{n-1}} ,$$

where d_1 – symbol of differentiation along the line ω^1, $K_i^{(1)} = \Lambda_{i1}^{\ i+1}$ – i –curvature of the line ω^1 of given set ,

$$\Lambda_{i1}^{\ i} = 0 \quad \left(i < j,\ i = 1, 2, ..., n-2, ;\ j = 3, 4, ..., i+1, ..., n \right) \tag{7}$$

and

$$\Lambda_{i1}^{i+1} \neq 0 \quad (i < j,\ i = 1, 2, ..., n-2) \tag{8}$$

(here symbol Λ from above noted the meaning which cannot take index j).

A pseudo focus [5] F_i^j $(i \neq j)$ of tangent of the line ω^i of the net Σ_n is defined by radius-vector:

$$\vec{F}_i^{\,j} = \vec{X} - \frac{1}{\Lambda_{ij}^{j}}\vec{e}_i = \vec{X} + \frac{1}{\Lambda_{jj}^{i}}\vec{e}_i \tag{9}$$

There exist $n-1$ pseudo focuses on each tangent $(X, \vec{e}_i)$. Let net Σ_n is cycle net of Frenet. The net Σ_n in $\Omega \subset E_n$ is called a cycle net of Frenet [4] if the frames $\Re_1 = (X, \vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4, \vec{e}_5)$, $\Re_2 = (X, \vec{e}_2, \vec{e}_3, \vec{e}_4, \vec{e}_5, \vec{e}_1)$,…, $\Re_n = \left(X, \overrightarrow{e_n}, \overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}, ..., \overrightarrow{e_{n-1}}\right)$, are frames of Frenet for lines ω^1, ω^2 , ω^3,…, ω^n respectively of net Σ_n simultaneously.

We will denote it by $\tilde{\Sigma}_n$. Pseudo focus $F_1^n \in \left(X, \overrightarrow{e_i}\right)$ defined by radius -vector:

$$\overrightarrow{F_i^n} = \overrightarrow{X} - \frac{1}{\Lambda_{in}^{n}}\overrightarrow{e_i} = \overrightarrow{X} + \frac{1}{\Lambda_{nn}^{i}}\overrightarrow{e_i} \tag{10}$$

When the point X is moving in the domain $\Omega \subset E_n$, pseudo focus F_i^n describes it's domain Ω_i^n. Such defined the partial mapping $f_i^n : \Omega \to \Omega_i^n$ such that $f_i^n(X) = F_i^n$.

If differentiate the equation (10) we have:

$$d\overrightarrow{F_i^n} = d\overrightarrow{X} - \frac{d\Lambda_{in}^{n}}{\left(\Lambda_{in}^{n}\right)^2}\overrightarrow{e_i} - \frac{1}{\Lambda_{in}^{n}}d\overrightarrow{e_i}\ .$$

Considering equations (1), (2) and (5) we derive:

$$d\overrightarrow{F_i^n} = \omega^i\overrightarrow{e_j} + \frac{B_{inj}^{n}\omega^j}{\left(\Lambda_{in}^{n}\right)^2}\overrightarrow{e_i} - \frac{1}{\Lambda_{in}^{n}}\Lambda_{ij}^{k}\omega^j\overrightarrow{e_k}$$

or

$$d\overrightarrow{F_i^n} = \left\{\overrightarrow{e_j} + \frac{B_{inj}^{n}}{\left(\Lambda_{in}^{n}\right)^2}\overrightarrow{e_i} - \frac{\Lambda_{ij}^{k}}{\Lambda_{in}^{n}}\overrightarrow{e_k}\right\}\omega^j$$

The vector $\overrightarrow{c_i}$ is following:

$$\overrightarrow{c_j} = \frac{B_{inj}^{n}}{\left(\Lambda_{in}^{n}\right)^2}\overrightarrow{e_i} + \overrightarrow{e_j} - \frac{\Lambda_{ij}^{k}}{\Lambda_{in}^{n}}\overrightarrow{e_k} \tag{11}$$

We will join to $\Omega_i^n \in E_n$ with the moving frame $\Re' = \left(F_i^n, \overrightarrow{c_j}\right)$.

Definition 1. Lines ω^i, $g(\omega^i) = \overline{\omega^i}$ are called quasi-double lines of a mapping g, if tangents of this lines in the points $X, g(X)$ are parallel or intersect [7] .

2. A line ℓ is called a double line of a pair (g, Δ_p), if a line ℓ is a double line of a mapping g and belonging to a distribution Δ_p.

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3. A line ℓ is called a quasi-double line of a pair $\left(g, \Delta_p\right)$ if a line ℓ is a quasi-double line of a mapping g and belonging to a distribution Δ_p.

2. Main Results

Theorem. The line ℓ belonging to p – dimensional distribution Δ_p, is quasi-double line of the pair $\left(f_i^n, \Delta_p\right)$ if and only if when realized the conditions

$\ell^a \Lambda_{ia}^{\tilde{k}} = 0, \tilde{i}, \tilde{j}, \tilde{k} = a+1, \dots, n.$

Proof. Let the line ℓ is belonging to a distribution $\Delta_p = \left(X, \vec{e_1}, \vec{e_1}, \vec{e_2}, \dots, \vec{e_p}\right)$ and with tangent vector $\vec{\ell} = \ell^a \vec{e_a}$ $\left(a, b, c = 1, 2, \dots, p, \quad p < n\right)$.

It is found that the tangent vector $\vec{\bar{\ell}}$ of the line $\bar{\ell} = f_i^n\left(\ell\right) : \vec{\bar{\ell}} = \ell^a \vec{c_a}$.

By applying (10) it gives

$$\vec{\bar{\ell}} = \ell^a \left(\frac{B_{ina}^n}{\left(\Lambda_{in}^n\right)^2} \vec{e_i} + \vec{e_a} - \frac{\Lambda_{ia}^k}{\Lambda_{in}^n} \vec{e_k} \right).$$

From a condition $\vec{\ell}, \vec{\bar{\ell}}, \overrightarrow{XF_i^n} \in \Delta_p$ we obtain

$$\ell^a \Lambda_{ia}^{\tilde{k}} = 0 \quad \left(\tilde{i}, \tilde{j}, \tilde{k} = a+1, \dots, n\right) \tag{12}$$

Inversely, if take place conditions (12) then the line ℓ is quasi-double line of the pair $\left(f_i^n, \Delta_p\right)$.

The geometrical meaning of the equation (12) is following: $\vec{\ell} \perp \vec{\theta_{\tilde{k}}}$,

where we used following notations: $\vec{\theta_{\tilde{k}}} = \sum_a \Lambda_{ia}^{\tilde{k}} \vec{e_a}$ and $\Lambda_{ia}^{\tilde{k}} = \vec{e_{\tilde{k}}} d_a \vec{e_i}$.

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