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Existence of immovability lines of a partial mapping of Euclidean space E5

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Abstract. It is considered a set of smooth lines such that through a point $X \in \Omega$ passed one line of given set in domain $\Omega \subset E_5$. The moving frame $\Re = \left(X, \vec{e}_{i,}\right)$ $\left(i, j, k = \overline{1,5}\right)$ is frame of Frenet for the line ω^1 of the given set. Integral lines of the vector fields $\overrightarrow{e_i}$ are formed net Σ_5 of Frenet. There exists a point $F_5^4 \in \left(X, \vec{e}_5\right)$ on the tangent of the line ω^5. When a point X is shifted in the domain Ω, the point F_5^4 describes it's domain Ω_5^4 in E_5. It is defined the partial mapping $f_5^4 : \Omega \to \Omega_5^4$, such that $f_5^4(X) = F_5^4$. Necessary and sufficient conditions of immovability and degeneration of lines $\left(X, \vec{e}_1\right)$, $\left(X, \vec{e}_2\right)$ and $\left(X, \vec{e}_3\right)$ in partial mapping f_5^4 are obtained.

Key words: partial mapping, cyclic net of Frenet, Frenet frame, pseudofocus, immovability of line.

1. Introduction

In domain $\Omega \subset E_5$ it is considered a set of smooth lines such that through a point $X \in \Omega$ passed one line of given set. The moving frame $\Re = \left(X, \vec{e}_{i,}\right)$ $\left(i, j, k = \overline{1,5}\right)$ is frame of Frenet for the line ω^1 of the given set of smooth lines. Derivation formula of the frame $\Re$ have a form:

$$d\overrightarrow{X} = \omega^i \vec{e}_i \ , \ d\overrightarrow{e_i} = \omega_i^k \vec{e}_k \qquad (1)$$

The forms ω^i, ω_i^k satisfied structure equations of Euclidean space:

$$D\omega^i = \omega^k \wedge \omega_k^i, \ D\omega_i^k = \omega_i^j \wedge \omega_j^k, \ \omega_i^j + \omega_j^i = 0\,. \qquad (2)$$

Integral lines of vector fields $\vec{e}_i$ are formed the net Σ_5 of Frenet for the line ω^1 of the given set of lines. Since frame $\Re$ is constructed on tangent of lines of the net Σ_5, the forms ω_i^k are principal forms [3]; in orther words:

$$\omega_i^k = \Lambda_{ij}^k \omega^j \tag{3}$$

Taking into account (3) from (2) we have:

$$\Lambda_{ij}^k = -\Lambda_{kj}^i \tag{4}$$

If we differentiate externally (3) then we have:

$$D\omega_i^k = d\Lambda_{ij}^k \wedge \omega^j + \Lambda_{ij}^k D\omega^j .$$

By using formula (2) from here we have got:

$$\omega_i^j \wedge \omega_j^k = d\Lambda_{ij}^k \wedge \omega^j + \Lambda_{ij}^k \wedge \omega^\ell \wedge \omega_\ell^j .$$

By applying the latter with combination of the formula (3) we obtain:

$$\omega_i^j \wedge \Lambda_{j\ell}^k \omega^\ell = d\Lambda_{ij}^k \wedge \omega^j - \Lambda_{ij}^k \omega_\ell^j \wedge \omega^\ell$$

or

$$\Lambda_{j\ell}^k \omega_i^j \wedge \omega^\ell = d\Lambda_{ij}^k \wedge \omega^j - \Lambda_{ij}^k \wedge \omega_\ell^j \wedge \omega^\ell .$$

From here we found:

$$d\Lambda_{ij}^k \wedge \omega^j - \Lambda_{i\ell}^k \omega_j^\ell \wedge \omega^j - \Lambda_{j\ell}^k \omega_i^j \wedge \omega^\ell = 0$$

or

$$\left(d\Lambda_{ij}^k - \Lambda_{i\ell}^k \omega_j^\ell - \Lambda_{\ell j}^k \omega_i^\ell \right) \wedge \omega^j = 0 .$$

By using lemma of Cartan from [1] we conclude that:

$$d\Lambda_{ij}^k - \Lambda_{i\ell}^k \omega_j^\ell - \Lambda_{\ell j}^k \omega_i^\ell = \Lambda_{ijm}^k \omega^m$$

or

$$d\Lambda_{ij}^k = \left(\Lambda_{ijm}^k + \Lambda_{il}^k \Lambda_{jm}^l + \Lambda_{lj}^k \Lambda_{im}^l \right) \omega^m \tag{5}$$

The system of variable $\left\{ \Lambda_{ij}^k , \Lambda_{ijm}^k \right\}$ is formed geometrical object of second order. The formulas of Frenet for the line ω^1 of given set have a form

$$\begin{aligned} d_1\vec{e}_1 &= \Lambda_{11}^2 \vec{e}_2 , \\ d_1\vec{e}_2 &= \Lambda_{21}^1 \vec{e}_1 + \Lambda_{21}^3 \vec{e}_3 , \\ d_1\vec{e}_3 &= \Lambda_{31}^2 \vec{e}_2 + \Lambda_{31}^4 \vec{e}_4 , \\ d_1\vec{e}_4 &= \Lambda_{41}^3 \vec{e}_3 + \Lambda_{41}^5 \vec{e}_5 , \\ d_1\vec{e}_5 &= \Lambda_{51}^4 \vec{e}_4 , \end{aligned}$$

and

$$\begin{aligned} \Lambda_{11}^3 &= -\Lambda_{11}^3 = 0, \\ \Lambda_{11}^4 &= -\Lambda_{41}^1 = 0, \\ \Lambda_{11}^5 &= -\Lambda_{51}^1 = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} \Lambda_{21}^5 &= -\Lambda_{51}^2 = 0, \\ \Lambda_{21}^4 &= -\Lambda_{41}^2 = 0, \\ \Lambda_{31}^5 &= -\Lambda_{51}^3 = 0 \end{aligned} \tag{7}$$

There $k_1^1 = \Lambda_{11}^2$, $k_2^1 = \Lambda_{21}^3$, $k_3^1 = \Lambda_{31}^4$, $k_4^1 = \Lambda_{41}^5 = -\Lambda_{51}^4$ – a first, a second, a third and a fourth curvature of line ω^1 respectively (there d_1 – symbol of differentiation along line ω^1).

A pseudofocus [4] F_i^j $(i \neq j)$ of tangent of the line ω^i of the net Σ_5 is defined by radius- vector:

$$\vec{F}_i^j = \vec{X} - \frac{1}{\Lambda_{ij}^j}\vec{e}_i = \vec{X} + \frac{1}{\Lambda_{jj}^i}\vec{e}_i. \tag{8}$$

There are exist four pseudofocuses on each tangent $(X, \vec{e}_i)$:
on the straight line $(X, \vec{e}_1)$ - $F_1^2, F_1^3, F_1^4, F_1^5$;
on $(X, \vec{e}_2)$ – $F_2^1, F_2^3, F_2^4, F_2^5$;
on $(X, \vec{e}_3)$ – $F_3^1, F_3^2, F_3^4, F_3^5$;
on $(X, \vec{e}_4)$ – $F_4^1, F_4^2, F_4^3, F_5^4$;
on $(X, \vec{e}_5)$ – $F_5^1, F_5^2, F_5^3, F_5^4$.

Definition. The net Σ_5 in $\Omega \subset E_5$ is called a cycle net of Frenet ([5]) if the frames
$\Re_1 = (X, \vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4, \vec{e}_5)$,
$\Re_2 = (X, \vec{e}_2, \vec{e}_3, \vec{e}_4, \vec{e}_5, \vec{e}_1)$,
$\Re_3 = (X, \vec{e}_3, \vec{e}_4, \vec{e}_5, \vec{e}_1, \vec{e}_2)$,
$\Re_4 = (X, \vec{e}_4, \vec{e}_5, \vec{e}_1, \vec{e}_2, \vec{e}_3)$
and
$\Re_5 = (X, \vec{e}_5, \vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4)$
are frames of Frenet for lines ω^1, ω^2, ω^3, ω^4, ω^5 respectively of net Σ_5 simultaneously.

2. Main results

In this section we formulate and prove the main results of the paper.

Theorem 1. Partial mapping f_5^4 is degenerate if and only if it is related to the one of the conditions

1) $D_{541}^4 = 0$, where $D_{541}^4 = -\overrightarrow{e_5}\, d_1 \overrightarrow{k_{54}}$.

2) $\dfrac{\Lambda_{54}^1}{\Lambda_{55}^1} = \dfrac{D_{544}^4}{\left(\Lambda_{54}^4\right)^2 + D_{545}^4}$.

Proof. Let net Σ_5 is cycle net of Frenet. We will denote it by $\tilde{\Sigma}_5$. Pseudofocus $F_5^4 \in (X, \vec{e}_5)$ defined by radius -vector:

$$\overrightarrow{F_5^4} = \vec{X} - \frac{1}{\Lambda_{54}^4}\vec{e}_5 = \vec{X} + \frac{1}{\Lambda_{44}^5}\vec{e}_5. \tag{9}$$

When the point X is moving in the domain $\Omega \subset E_5$, pseudofocus F_5^4 describes it's domain Ω_5^4 Thus defined the partial mapping $f_5^4 : \Omega \to \Omega_5^4$ such that $f_5^4(X) = F_5^4$. We will join to $\Omega_5^4 \subset E_5$ the moving frame $\Re' = \left(F_5^4, \vec{d}_i\right)$, where vectors $\vec{d}_i$ have a form [8]:

$$\vec{d}_1 = \vec{e}_1 - \frac{\Lambda^4_{51}}{\Lambda^4_{54}}\vec{e}_4 + \frac{D^4_{541}}{\left(\Lambda^4_{54}\right)^2}\vec{e}_5,$$
$$\vec{d}_2 = -\frac{\Lambda^1_{52}}{\Lambda^4_{54}}\vec{e}_1 + \vec{e}_2 - \frac{\Lambda^4_{52}}{\Lambda^4_{54}}\vec{e}_4 + \frac{D^4_{542}}{\left(\Lambda^4_{54}\right)^2}\vec{e}_5,$$
$$\vec{d}_3 = -\frac{\Lambda^1_{53}}{\Lambda^4_{54}}\vec{e}_1 - \frac{\Lambda^4_{53}}{\Lambda^4_{54}}\vec{e}_4 + \vec{e}_3 + \frac{D^4_{543}}{\left(\Lambda^4_{54}\right)^2}\vec{e}_5, \tag{10}$$
$$\vec{d}_4 = -\frac{\Lambda^1_{54}}{\Lambda^4_{54}}\vec{e}_1 + \frac{D^4_{544}}{\left(\Lambda^4_{54}\right)^2}\vec{e}_5,$$
$$\vec{d}_5 = -\frac{\Lambda^1_{55}}{\Lambda^4_{54}}\vec{e}_1 + \left[1 + \frac{D^4_{545}}{\left(\Lambda^4_{54}\right)^2}\right]\vec{e}_5.$$

In general case vectors $\vec{d}_i$ are linearly independent. We shall demand that vectors $\vec{d}_i$ are linearly dependent. From here we have:

$$1)\ D^4_{541} = 0 \tag{11}$$

or

$$2)\ \frac{\Lambda^1_{54}}{\Lambda^1_{55}} = \frac{D^4_{544}}{\left(\Lambda^4_{54}\right)^2 + D^4_{545}}\ , \tag{12}$$

where

$$D^4_{541} = -\overrightarrow{e_5}\, d_1 \overrightarrow{k_{54}}\,,$$

d_1 – symbol of differentiation along of direction $\overrightarrow{e_1}$, Λ^1_{55}– first curvature of the line ω^5 of the net $\tilde{\Sigma}_5$, Λ^1_{54} – second curvature of the line ω^4 of the net $\tilde{\Sigma}_5$. Inversely, if satisfied one of conditions (11), (12), then the partial mapping f^4_5 is degenerate. Thus, we have obtained a statement of the Theorem 1. The proof of Theorem 1 is completed.

Theorem 2 Let f^4_5 be partial mapping. Then

1) Straight line $\left(X,\vec{e}_3\right)$,$\left(X,\vec{e}_2\right)$,$\left(X,\vec{e}_1\right)$ is immovable in the partial mapping f^4_5 if and only if $\Lambda^1_{53} = 0,\ \Lambda^4_{53} = 0,\ D^4_{543} = 0.$

2) Straight line $\left(X,\vec{e}_3\right)$,$\left(X,\vec{e}_2\right)$,$\left(X,\vec{e}_1\right)$ is immovable in the partial mapping f^4_5 if and only if $\Lambda^1_{52} = 0,\ \Lambda^4_{52} = 0,\ D^4_{542} = 0.$

3) Straight line $\left(X,\vec{e}_3\right)$,$\left(X,\vec{e}_2\right)$,$\left(X,\vec{e}_1\right)$ is immovable in the partial mapping f^4_5 if and only if $\Lambda^4_{51} = 0,\ D^4_{541} = 0.$

Proof. Let us take straight line $\left(X,\vec{e}_3\right)$ is immovable in the partial mapping f^4_5. From (10) we derive:

$$\Lambda^{1}_{53} = 0,$$
$$\Lambda^{4}_{53} = 0, \qquad (13)$$
$$D^{4}_{543} = 0,$$

where $\Lambda^{1}_{53} = -\Lambda^{5}_{13}$–third; $\Lambda^{4}_{53} = -\Lambda^{5}_{43}$–second curvature of the line ω^{3} of the net $\tilde{\Sigma}_{5}$, $D^{4}_{543} = -\vec{e}_{5} \cdot d_{3}\vec{k}_{54}$ (where $\vec{k}_{54}$–a vector of first curvature of the line ω^{4} of the net $\tilde{\Sigma}_{5}$).
Inversely, if take place conditions (13), then the straight line $(X, \vec{e}_{3})$ is immovable in the partial mapping f^{4}_{5}.

Analogously, we have necessary and sufficient conditions of the immovability of the straight lines $(X, \vec{e}_{2})$, $(X, \vec{e}_{1})$ in the partial mapping f^{4}_{5} respectively:

$$\Lambda^{1}_{52} = 0,$$
$$\Lambda^{4}_{52} = 0,$$
$$D^{4}_{542} = 0,$$

and

$$\Lambda^{4}_{51} = 0,$$
$$D^{4}_{541} = 0,$$

where $\Lambda^{1}_{52} = -\Lambda^{5}_{12}$–fourth, $\Lambda^{4}_{52} = -\Lambda^{5}_{42}$ – third curvature of the line ω^{2} of the net $\tilde{\Sigma}_{5}$. The proof of the Theorem 2 is completed.

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