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MHD boundary layer flow over a permeable flat plate in a ferrofluid with thermal radiation effect

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Abstract. Adaptation from the thermal management with limited gravitational acceleration and excesses of radiation such as in aerospace, present study investigated the magneto-hydrodynamic (MHD) boundary layer flow and heat transfer of ferrofluid past a permeable flat plate with thermal radiation effects. The mathematical model which is in non-linear partial differential equations are transformed to a more convenient form by similarity transformation approach before being solved numerically using the Runge-Kutta-Fehlberg (RKF45) method in MAPLE software. The characteristics and effects of pertinent parameters which are Prandtl number, magnetic parameter, thermal radiation parameter and the permeability parameter on the variation of Nusselt number and skin friction coefficient are analyzed and discussed. Three selected ferroparticles which are magnetite (Fe_3O_4), cobalt ferrite (CoFe_2O_4) and Mn-Zn ferrite ($\text{Mn-Zn Fe}_2\text{O}_4$) with water-based fluid are examined numerically.

1. Introduction

The study of the boundary layer flow in aerospace is rarely considered. In aerospace, the excessive of thermal radiation effects and zero gravity acceleration should be critically considered. Zero gravity conditions cause the fluid move in a random motion without any uniform direction. This made the heat transfer management via fluid convection in aerospace nearly impossible to be done. In order to counter this matter, scientist has modified the nanofluid from oxidize and metal nanoparticles to magnetite nanoparticles called as ferroparticles. This type of nanofluid called as ferrofluid. The ferrofluid is the propose fluid to be employed due to its magnetized characteristics so the fluid can be directional even without gravity acceleration for example in space shuttle engine combustion.

Experimental study on regarding to this topic is expensive to be considered, hence provided a limited finding and knowledge. Thus, the approach from a mathematical model is the alternative and relevant way to be considered so far. The numerical solution obtained from this approach is cheap, fast and provide the theoretical knowledge for the ferrofluid therefore suggest the researchers an early idea about the fluid flow and heat transfer characteristics. Recent study on ferrofluid included the study on



MHD and rotating flow on a stagnation point, stretching/shrinking surface and moving surface with the presence of thermal radiation, velocity slip factor and Newtonian heating boundary conditions by Zeeshan et al. [1], Ramli et al. [2], Rashad [3], Hussanan et al. [4], Jusoh et al. [5], Yasin et al. [6] and Mohamed et al. [7].

Thermal radiation act as an alternative energy transfer method rather than conduction and convection. Further, it is the only heat sources in aerospace which scientifically known has no medium transfer like air. It emitted energy onto surface which therefore increases the surface temperature. The effect of thermal radiation is important to study due to its presence may influence the heat transfer on surface. Related papers considered the thermal radiation effects include the works by Mohamed et al. [8], Chamkha et al. [9], Anwar et al. [10] and Zokri et al. [11] who investigated the problem of MHD and thermal radiation effects on a stagnation point flow towards a stretching sheet immersed in nanofluid, miropolar fluid and Jeffrey fluid with chemical reaction, viscous dissipation and convective boundary conditions. The fact that this study has never been considered before, thus the reported result in this paper is new.

2. Mathematical Formulation

By considering figure 1, a steady two-dimensional boundary layer flow over a flat plate embedded in a ferrofluid of ambient temperature, T_∞ is illustrates. Assuming that T_w is the wall temperature, U_∞ is the free stream velocity and q_r is the radiative heat flux. Next, a uniform magnetic field of strength B_0 is assumed to be applied in the positive y -direction normal to the flat plate. The Navier-Stoke equations that can be formed are [2]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(x)}{\rho_{nf}} (u - U_\infty), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{nf}} \frac{\partial q_r}{\partial y}, \quad (3)$$

subject to the boundary conditions

$$\begin{aligned} u = 0, \quad v = v_w, \quad T = T_w \quad \text{at } y = 0, \\ u = U_\infty, \quad T \rightarrow T_\infty, \quad \text{as } y \rightarrow \infty, \end{aligned} \quad (4)$$

where u and v are the velocity components along the x and y axes, respectively. ν_{nf} is the ferrofluid kinematic viscosity, ρ_{nf} is the ferrofluid density, σ is the electrical conductivity, T is the temperature inside the boundary layer, $(\rho C_p)_{nf}$ is the heat capacity of ferrofluid, k_{nf} is the thermal conductivity of ferrofluid and ν_w is the permeability of flat plate. Other properties which related to base fluid and the ferroparticles is denoted with subscript f and s respectively as follows:

$$\begin{aligned} \nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \alpha_{nf} = \frac{k_{nf}}{\rho_{nf}(C_p)_{nf}}, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \\ (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \quad \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2(k_f - k_s)}. \end{aligned} \quad (5)$$

Note that μ_{nf} is the ferrofluid dynamic viscosity and ϕ is ferroparticles volume fraction. Next, in order to solve the non-linear partial differential equations (1)-(3), the following similarity variables are considers [12]:

$$\eta = \left(\frac{U_\infty}{2\nu x}\right)^{1/2} y, \quad \psi = (2U_\infty \nu x)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{6}$$

where ψ and θ is a non-dimensional stream function and temperature, respectively. The used of similarity variables may transformed the dimensional equations to non-dimensional equations. Besides, this method reduces the number of dependent variables which make the equations easier to solve. Notice that the similarity variables (6) satisfy the continuity equation (1). Next, substitute the similarity variables (6) into governing equations (2) and (3) gives the following transformed ordinary differential equations:

$$\frac{1}{(1-\phi)^{2.5} [1-\phi + (\phi\rho_s)/(\rho_f)]} f''' + ff'' - M(f' - 1) = 0 \tag{7}$$

$$\frac{1}{\text{Pr} (1-\phi) + \phi(\rho C_p)_s / (\rho C_p)_f} \left(1 + \frac{4}{3} N_R\right) \theta'' + f\theta' = 0. \tag{8}$$

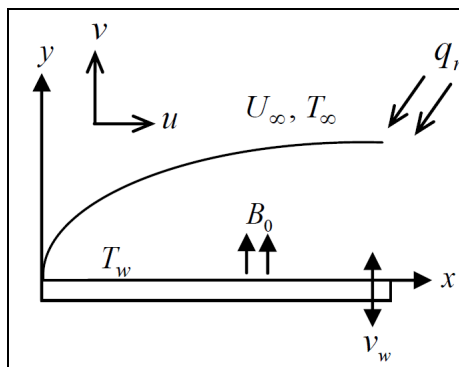


Figure 1. Physical model and the coordinate system

By definition, $M = \frac{2x\sigma B_o^2(x)}{U_\infty \rho_{nf}}$ is the magnetic parameter, $\text{Pr} = \frac{\nu_f (\rho C_p)_f}{k_f}$ is Prandtl number which will be set as 6.2 in calculation with respect to water-based fluid and $N_R = \frac{4\sigma^* T_\infty^3}{k^* k_{nf}}$ is thermal radiation parameter. In order that the similarity solution for equations (1) to (4) exist, it is assume [13]:

$$\nu_w = -\left(\frac{U_\infty \nu_f}{2x}\right)^{1/2} \lambda, \quad B_0 = Bx^{-1} \tag{9}$$

where B and λ are constants. λ measures the permeability rate at the flat plate surface, with $\lambda > 0$ and $\lambda < 0$ corresponds for suction and injection, respectively. The boundary conditions become

$$\begin{aligned} f(0) &= \lambda, \quad f'(0) = 0, \quad \theta(0) = 1, \\ f'(\eta) &\rightarrow 1, \quad \theta(\eta) \rightarrow 0, \quad \text{as } \eta \rightarrow \infty. \end{aligned} \tag{10}$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x are given by:

$$C_f = \frac{\tau_w}{\rho_f u_\infty^2}, \quad Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)}, \quad (11)$$

with the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0}, \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial \bar{y}} \right)_{\bar{y}=0} + q_r, \quad (12)$$

Using variables in (6) and equation (11) give

$$C_f (2Re)^{1/2} = \frac{f''(0)}{(1-\phi)^{2.5}} \text{ and } Nu_x (Re/2)^{-1/2} = -\frac{k_{nf}}{k_f} \left[1 + \frac{4}{3} N_R \right] \theta'(0) \quad (13)$$

3. Results and Discussion

The system of ordinary differential equations (7) and (8) with boundary conditions (10) were solved numerically using the RKF45 technique in Maple. The numerical results obtained for the reduced Nusselt number $Nu_x (Re/2)^{-1/2}$ and the reduced skin friction coefficient $C_f (2Re)^{1/2}$ for a various values of pertinent parameter namely as the Prandtl number Pr , the suction/injection parameter λ , the thermal radiation parameter N_R , ferroparticles volume fraction ϕ and the magnetic parameter M . In computing the results, the boundary layer thickness from 2 to 8 is sufficient to provide the accurate numerical results for water-based ferrofluid with magnetite Fe_3O_4 , Cobalt Ferrite $CoFe_2O_4$ and Mn-Zn Ferrite $Mn-ZnFe_2O_4$ particles. The values of thermophysical properties of a ferroparticles consider are tabulated in table 1.

In order to validate the numerical results obtained, the comparison has been made. Table 2 shows the comparison values of $-\theta'(0)/\sqrt{2}$ with previous results in Bataller [14] and Roşca and Pop [15]. It is found that both numerical results are in good agreement. Further, table 2 included results for $-\theta'(0)$ as well as the value of $f''(0)$. It is found that the increase of Pr results to the increase in $-\theta'(0)$ while gives no effect on $f''(0)$. Noticed that the value of $f''(0) = 0.469600$ obtained is similar as reported by Blasius [12].

Table 1. Thermophysical properties of water and ferroparticles.

| Physical properties | Water | Ferroparticles | | |
|-----------------------------|----------------------|-------------------------|--------------------------------|----------------------------------|
| | | Magnetite, Fe_3O_4 | Cobalt ferrite, $CoFe_2O_4$ | Mn-Zn Ferrite, $Mn-ZnFe_2O_4$ |
| ρ (kg/m ³) | 997 | 5180 | 4907 | 4900 |
| C_p (J/kg·K) | 4179 | 670 | 700 | 800 |
| k (W/m·K) | 0.613 | 9.7 | 3.7 | 5 |
| β (K ⁻¹) | 2.1×10^{-5} | 1.3×10^{-5} | 1.3×10^{-5} | 1.2×10^{-5} |

Table 2. Values of $-\theta'(0)/\sqrt{2}$, $-\theta'(0)$ and $f''(0)$ for various values of Pr when $N_R = \lambda = M = \phi = 0$.

| Pr | $-\theta'(0)/\sqrt{2}$ | | $-\theta'(0)$ (CWT) | | $f''(0)$ |
|-----|------------------------|--------------------|---------------------|----------|----------|
| | Bataller [14] | Roşca and Pop [15] | Present | Present | |
| 0.7 | 0.29268 | 0.29268 | 0.292680 | 0.413912 | 0.469600 |
| 0.8 | | 0.30691 | 0.306917 | 0.434046 | 0.469600 |
| 1 | | 0.33205 | 0.332057 | 0.469600 | 0.469600 |
| 5 | 0.57669 | 0.57668 | 0.576689 | 0.815561 | 0.469600 |
| 10 | 0.72814 | 0.72814 | 0.728141 | 1.029747 | 0.469600 |
| 30 | 1.05173 | | 1.051693 | 1.487319 | 0.469600 |

Figure 2 shows the variation of reduced Nusselt number $Nu_x(Re/2)^{-1/2}$ for various values of the suction/injection parameter λ . It is found that the presence of suction effect ($\lambda > 0$) results to the increase of $Nu_x(Re/2)^{-1/2}$ while injection effects ($\lambda < 0$) does the contrary. From the figure, it is shown that large injection factor may reduce $Nu_x(Re/2)^{-1/2}$ to $Nu_x(Re/2)^{-1/2} \approx 0$ which physically termed as pure conduction process. Further, small effects are recorded for a variety of ferroparticles considered. Fe_3O_4 results a little bit higher in $Nu_x(Re/2)^{-1/2}$ compared to $CoFe_2O_4$ and $Mn-ZnFe_2O_4$.

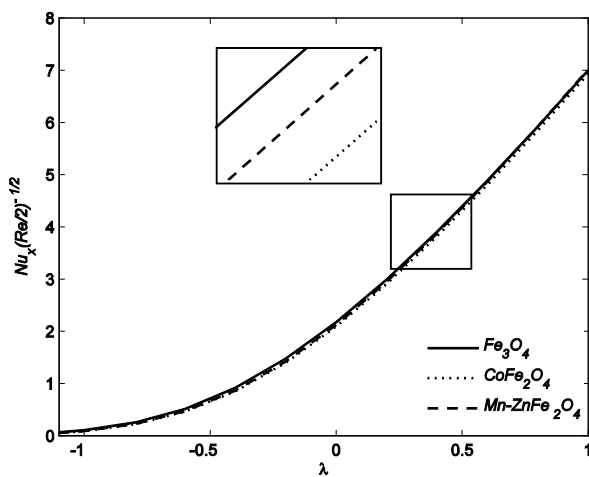


Figure 2. Variation values of $Nu_x(Re/2)^{-1/2}$ with various values of λ when Pr=6.2, $M = N_R = 1$ and $\phi = 0.1$

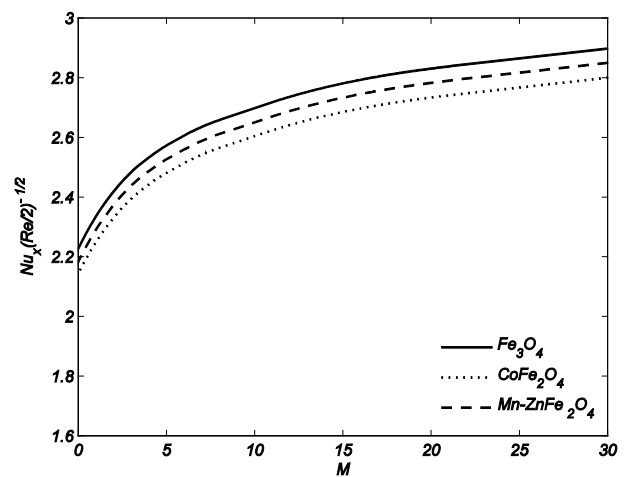


Figure 3. Variation values of $Nu_x(Re/2)^{-1/2}$ with various values of M when Pr=6.2, $\lambda = 0, N_R = 1$ and $\phi = 0.1$

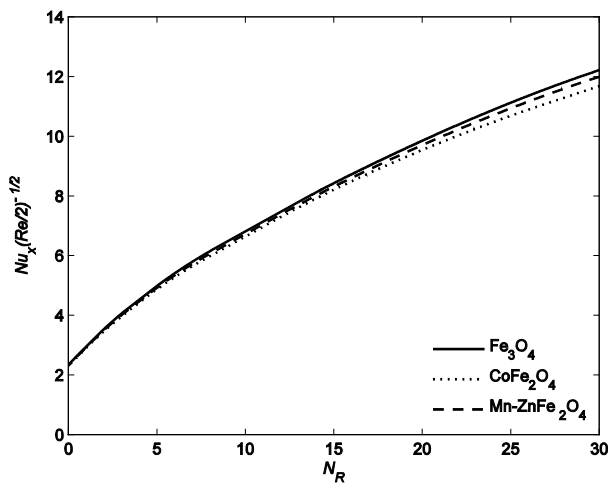


Figure 4. Variation values of $Nu_x(Re/2)^{-1/2}$ with various values of N_R when $Pr = 6.2$, $\lambda = 0, M = 1$ and $\phi = 0.1$

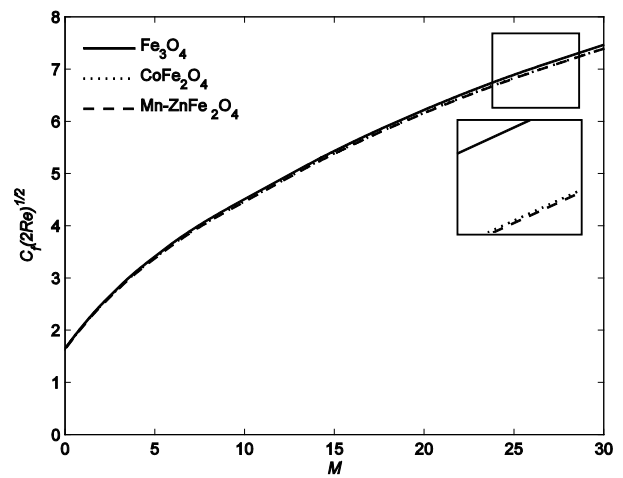


Figure 6. Variation values of $C_f(2Re)^{1/2}$ with various values of M when $Pr = 6.2$, $\lambda = 0, N_R = 1$ and $\phi = 0.1$

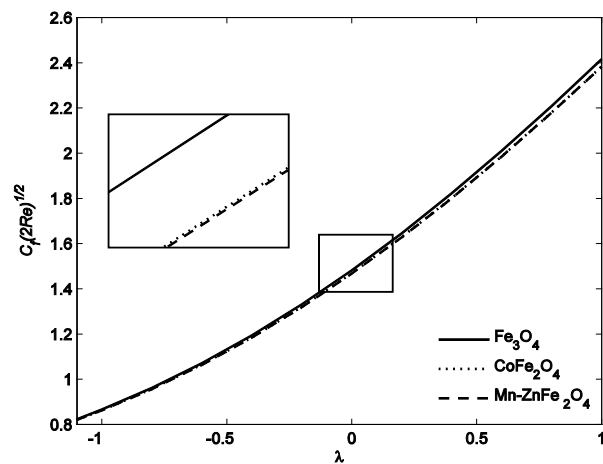


Figure 5. Variation values of $C_f(2Re)^{1/2}$ with various values of λ when $Pr = 6.2$, $M = N_R = 1$ and $\phi = 0.1$

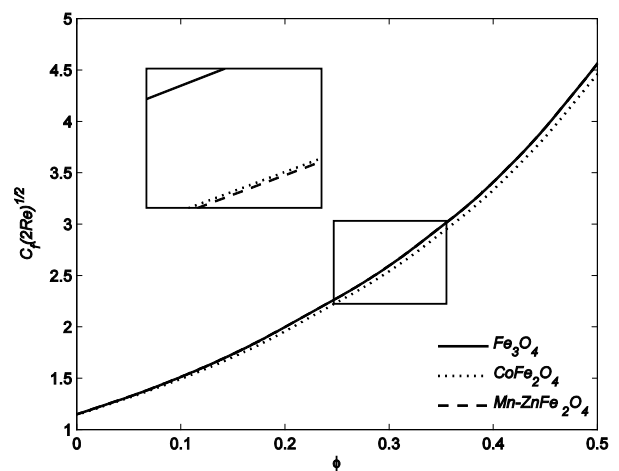


Figure 7. Variation values of $C_f(2Re)^{1/2}$ with various values of ϕ when $Pr = 6.2$, $M = N_R = 1$ and $\lambda = 0$.

Next, the variation of $Nu_x(Re/2)^{-1/2}$ for various values of the magnetic parameter M and the thermal radiation parameter N_R are illustrates in figure 3 and 4. From both figures, the increase of both parameter raises the values of $Nu_x(Re/2)^{-1/2}$ which also denoted as the rise of convective heat transfer of process. This result is significant to the fact that the presence of N_R emitted energy in electromagnetic waves then promoted to the increase in temperature and its gradient which enhanced the heat transfer capabilities from plate surface to fluid flow. Further, the high thermal conductivity of

Fe_3O_4 has contributed to its high $Nu_x (Re/2)^{-1/2}$ values for figure 3. However, in figure 4, these effects are only significant at large values of N_R .

The effect of λ, M and ϕ on reduced skin friction coefficient $C_f (2Re)^{1/2}$ variation are shown in figures 5 to 7, respectively. From figure 5, the presence of suction effects raised the values of $C_f (2Re)^{1/2}$ while injection effects reduced the quantity. In considering the effects of M and ϕ , the increase of M and ϕ has results to the increase of $C_f (2Re)^{1/2}$. This is due to the fact that, the increase of M has enhanced the ferrofluid velocity (which reflect to magnetic effect) then increase the velocity different between fluid flow and plate surface, therefore results to the increase in $C_f (2Re)^{1/2}$. Further, the increase in ϕ reflects to the increase in ferrofluid momentum and its velocity due to the increase in ferrofluid viscosity, thus lead to the increase of $C_f (2Re)^{1/2}$. Lastly, from figure 5 to 7, it is noticed that the thermophysical of ferroparticles properties gave a very small effect on $C_f (2Re)^{1/2}$. Fe_3O_4 particles which have high-density results slightly higher values of $C_f (2Re)^{1/2}$, while $CoFe_2O_4$ and $Mn - ZnFe_2O_4$ ferrofluid is observed have a quite similar value of $C_f (2Re)^{1/2}$ as well as their ferroparticles density.

4. Conclusion

As a conclusion, it is found that the increase of magnetic parameter and suction effect leads to the increase of Nusselt number and skin friction coefficient while the injection effects does contrary. Large injection effect may promote to pure conduction heat transfer process. Next, the presence of thermal radiation enhances the convective heat transfer capability but give no effects on fluid velocity as well as the skin friction coefficient. Lastly, it is observed that the high thermal conductivity and density of magnetite particles has results to the highest values of Nusselt number as well as the skin friction coefficient, compared to cobalt ferrite and Mn-Zn ferrite.

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