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MHD Flow and Heat Transfer of Ferrofluid on Stagnation Point along Flat Plate with Convective Boundary Condition and Thermal Radiation Effect

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Abstract. This theoretical study deals with numerical solution of MHD convection laminar boundary layer flow and heat transfer on stagnation point along a stationary horizontal flat plate. For this purpose, ferrofluid which contains magnetite, Fe_3O_4 as a ferroparticles and water as a base fluid is considered. Ferrofluid has shown a particular achievement when the effect of external magnetic field applied, such as helping to control the properties of physical and flow of ferrofluid. The study starts with the formulation of the mathematical equations that governed the ferrofluid flow and heat transfer. The governing equation which is in the form of dimensional nonlinear partial differential equations are reduced to nonlinear ordinary differential equations by using appropriate similarity transformation and then solved numerically by using the Keller-box method. Numerical result is discussed in terms of pertinent effects that influence the ferrofluid flow and heat transfer like magnetic parameter, ferroparticles volume fraction parameter, Biot number and radiation parameter on velocity and temperature profiles. It is found that the temperature profile increase with an increase volume fraction of ferroparticles parameter, radiation parameter and Biot number and decrease with increasing magnetic parameter.

1. Introduction

Nanofluid which is formed by magnetic nanoparticles is also known as the ferrofluid is experimentally proven to enhance thermal conductivity [1]. Ferrofluid is composed of ~3-15 nanometre magnetic particles [2] or known as ferroparticles, for example, magnetite (Fe_3O_4), hematite (Fe_2O_3), cobalt ferrite ($CoFe_2O_4$) or other compounds with iron dispersant and suspended in base fluid such as water, oil, ethylene glycol and so forth. Ferrofluid has special characteristics compared to the others nanofluid like alumina (Al_2O_3), titania (TiO_2) and copper (Cu) because ferrofluid is strongly magnetized in the presence of magnetic field which provides low viscosity, easy flowability and low energy [2]. In the absence of magnetic field, the ferroparticles are randomly distributed. When a magnetic field is applied to ferrofluid, the random motion of ferroparticles within base fluid becomes uniform and orient along the field.

The characteristics described ferrofluid as different than the other fluid as it has several applications in engineering technology such as heating controller in electric motors and speakers. Nowadays, most of the speakers and hearing aids such as earphone, headphone etc. employ ferrofluid to conduct heat away from speaker voice coil [3]. Besides that, ferrofluid is used in biomedical research for drug carrier in targeted therapy to provide better opportunities in cancer treatment and detection of membrane modification in malaria or Alzheimer's disease [4-6].

The characteristics and applications of ferrofluid have attracted many researches to investigate the fluid flow interaction over various surfaces in different assumptions such as slip flow, viscous dissipation, thermal radiation, direction of magnetic field and thermal boundary conditions. These have been conducted by [7-10] to enhance the convective heat transfer of ferrofluid. Magnetohydrodynamic (MHD) is one of the effects that must be considered in investigating flow and heat transfer of ferrofluid because it has iron components which interact with the magnetic applied. The body force acting in MHD on the fluid is called Lorentz force. In the presence of the magnetic field, the Lorentz force will rise when the electric current flows at an angle to the direction of an impressed magnetic field. According to Raju and Sandeep [11], the ferrofluid flow tends to improve the thermal boundary layer just as the momentum boundary layer thicknesses. They found that the magnetic field and radiation parameter have tendency to diminish the skin friction coefficient. Subsequently, radiation has significant impact in controlling the rate of heat transfer in boundary layer region [12-14]. The influence of thermal radiation on ferrofluid heat transfer in presence of Lorentz forces is investigated by Sheikholeslami and Shehzad [15]. It is discovered that the rate of heat transfer is enhanced with augment of radiation parameter. Recently, Reddy et al. [16] who concentrated the flow of ferrofluid along an exponentially stretching sheet found an increase in the radiation parameter is capable of enhancing the fluid temperature.

Meanwhile, the surface geometries also affect the convective heat transfer process [17]. The study of ferrofluid on a flat plate with stationary horizontal flat plate has been studied by [18, 19] found that the increase of ferroparticles volume fraction will in general increment the skin friction and heat transfer rate. The greater part of the investigations of ferrofluid flow and heat transfer were viewed as two cases thermal boundary conditions, which are constant wall temperature and constant heat flux. A few contributions of ferrofluid flow and heat transfer with convective boundary condition are that the surface heat transfer depends on the surface temperature. Recently, Ilias et al. [20, 21] solved the inclined plate and vertical plate with convective boundary conditions found that the velocity and temperature profile increment with the increase of Biot number which represents the ratio of conduction resistance within the geometry body to convection resistance at the surface geometry.

In this present study, we investigate the effects of ferroparticles volume fraction that impact the flow and heat transfer of ferrofluid on MHD stagnation point flow along a stationary flat plate with convective boundary condition and thermal radiation effect. The model proposed by Tiwari and Das (2007) will be adopted in mathematical modelling. This model is a suitable approach to study the behaviour of ferrofluid because it computes the coefficient physical properties like thermal conductivity, density and volume fraction of particles. The governing boundary layer equations are then transformed by using the similarity/non-similarity transformation to ordinary/partial differential equations. It will further be tackled numerically by using implicit finite difference scheme known as Keller-box method.

2. Mathematical Formulation

Consider a steady incompressible ferrofluid on a stagnation point past a flat plate with ambient temperature T_∞. Assume that the free stream velocity $U_\infty = bx$ where b are constant. Further, a uniform magnetic field of strength B_o is assumed to be applied in the positive y-direction normal to the horizontal flat plate. The magnetic Reynolds number is assumed to be small, and thus the induced magnetic field is negligible. The physical model and coordinate system of this problem is shown in figure 1. It is further assumed that the plate is subjected to convective boundary condition and no slip velocity condition is considered. The viscous dissipation and the chemical and chemical reaction effects are neglected. The boundary layer equations are [19, 22, 23]:

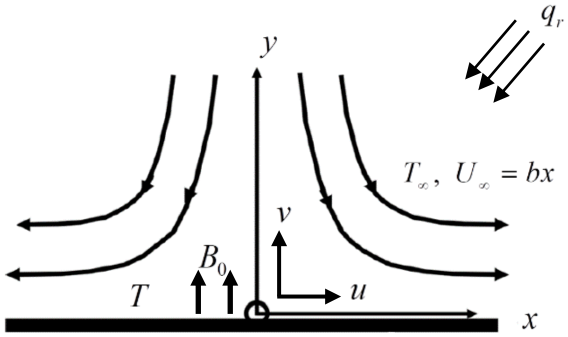

Figure 1. Physical model and coordinate system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\nu_{ff}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_{ff}B_o^2(x)}{\rho_{ff}}(u-U_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{ff}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\left(\rho C_p\right)_{ff}}\frac{\partial q_r}{\partial y}, \tag{3}$$

subject to the boundary conditions

$$u=0,\quad v=0,\quad -k\frac{\partial T}{\partial y}=-h_f(T_f-T)\quad \text{at } y=0,$$

$$u\to U_\infty,\ T\to T_\infty \quad \text{as } y\to\infty, \tag{4}$$

where u and v are the velocity components along the x and y directions, respectively. Further, T is the ferrofluid temperature in the boundary layer, T_f is the uniform temperature of the hot fluid, σ_{ff} is the electrical conductivity of ferrofluid, h_s is the heat transfer coefficient, $\left(\rho C_p\right)_{ff}$ is the heat capacity of ferrofluid, ρ_{ff} is the ferrofluid density, ν_{ff} is the kinematic viscosity of ferrofluid, α_{ff} is the thermal diffusivity of ferrofluid and μ_{ff} is the dynamic viscosity of ferrofluid. These parameters are often expressed as follows [19, 23-25]:

$$\nu_{ff}=\frac{\mu_{ff}}{\rho_{ff}},\ \rho_{ff}=(1-\phi)\rho_f+\phi\rho_s,\ \alpha_{ff}=\frac{k_{ff}}{\rho_{ff}\left(C_p\right)_{ff}},\ \mu_{ff}=\frac{\mu_f}{(1-\phi)^{2.5}},$$

$$\frac{\sigma_{ff}}{\sigma_f}=1+\frac{3\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi},\quad \left(\rho C_p\right)_{ff}=(1-\phi)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s. \tag{5}$$

where the thermal conductivity of ferrofluid, k_{ff} can be defined as

$$\frac{k_{ff}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}.$$

Note that k_{ff}, k_f and k_s are the thermal conductivity of the ferrofluid, base fluid and ferroparticles, respectively. As indicated by Turkyilmazoglu [26], the equations (5) are restricted to nanoparticles with spherical shape as follow the classical model by Brinkman [27] and Maxwell [28]. The radiative heat flux q_r are often simplified as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Using Rosseland approximation (see [29]) , the equation (3) is reduced to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\frac{k_{ff}}{\left(\rho C_p\right)_{ff}}+\frac{16\sigma^* T_\infty^3}{3k^*\left(\rho C_p\right)_{ff}}\right)\frac{\partial^2 T}{\partial y^2}, \tag{7}$$

From the above equation it is seen that the impact of radiation is to improve the thermal diffusivity. On the off chance that we take $N_R=\frac{4\sigma^* T_\infty^3}{k_{ff}k^*}$ as the radiation parameter, equation (7) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{ff}}{\left(\rho C_p\right)_{ff}}\left(1+\frac{4N_R}{3}\right)\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

The non-linear partial differential equations (1) - (3) then solved using the following similarity variables:

$$\eta=\left(\frac{b}{\nu_f}\right)^{1/2}y,\quad \psi=\left(b\nu_f\right)^{1/2}xf(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}, \tag{9}$$

where η, θ and ψ are non-dimensional similarity variable, temperature and stream function. The equation (1) satisfied by definition $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, respectively. Substitute the equations (5) and (9) into equations (2) and (8), then the following ordinary differential equations were obtained

$$\frac{1}{(1-\phi)^{2.5}\left[1-\phi+(\phi\rho_s)/(\rho_f)\right]}f'''+ff''-f'^2+1-\frac{\sigma_{ff}/\sigma_f}{(1-\phi)+\phi\left(\rho_s/\rho_f\right)}M(f'-1)=0, \tag{10}$$

$$\frac{k_{ff}/k_f}{(1-\phi)+\phi(\rho C_p)_s/(\rho C_p)_f}\left(1+\frac{4N_R}{3}\right)\theta''+\Pr f\theta'=0, \tag{11}$$

where $M=\frac{\sigma_f B_o^2(x)}{b\rho_f}$ is the magnetic parameter and $\Pr=\frac{\nu_f\left(\rho C_p\right)_f}{k_f}$ is the Prandtl number. The transformed boundary conditions are

$$f(0)=0,\ f'(0)=0,\ \theta'(0)=-Bi_x\left(1-\theta(0)\right),$$
$$f'(\eta)\to 1,\ \theta(\eta)\to 0 \text{ as } y\to\infty. \tag{12}$$

where $Bi_x = \frac{h_f}{k}\left(\frac{\nu_f}{b}\right)^{1/2}$ is the local Biot number. The physical quantity interests are the skin friction coefficient C_f and the local Nusselt number Nu_x which given by

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu_x = \frac{x q_w}{k_f (T_f - T_\infty)}, \tag{13}$$

with surface shear stress τ_w and the surface heat flux q_w given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k_{ff}\left(\frac{\partial T}{\partial y}\right)_{y=0} + q_r, \tag{14}$$

Using the similarity variables in (9) gives

$$C_f \operatorname{Re}_x^{1/2} = \frac{f''(0)}{(1-\phi)^{2.5}}, \quad Nu_x \big/ \operatorname{Re}_x^{1/2} = -\frac{k_{ff}}{k_f}\left(1 + \frac{4}{3}N_R\right)\theta'(0), \tag{15}$$

where $\operatorname{Re}_x = \frac{U_\infty x}{\nu_f}$ is the local Reynolds number.

3. Results and Discussion

The ordinary differential equations (10) and (11) which are subjected to the boundary conditions (12) were solved numerically by using Keller-box method, which is programmed in Matlab software. The black oxide of iron, magnetite (Fe_3O_4) as a ferroparticles and water as a base fluid are considered. Table 1 shows the thermophysical properties of base fluid and ferroparticles. The comparison presents result with the previously reported numerical results which has been made in table 2 to validate the numerical method accuracy. In order to make a good comparison results with the previous researchers, it is worth mentioning that the constant wall temperature results are recovered when a large value of Biot number ($Bi_x \to \infty$) is applied in boundary conditions. Table 2 shows, the present result of the values of the heat transfer coefficient $-\theta'(0)$ are found in good agreement with the previous published result conducted by White [30] and Khan et al. [31] when the case $\beta = 1$ and Mohamed et al. [22] for various values of the Prandtl number Pr when $M = \phi = N_R = 0$ and $Bi_x \to \infty$. Therefore, the practicality and effectiveness of Keller-box method is proven. The effect of pertinent parameter namely volume fraction of ferroparticles ϕ, magnetic M, Biot number and radiation N_R parameter on the velocity profile, temperature profile, reduced skin friction coefficient, $C_f \operatorname{Re}_x^{1/2}$ and reduced Nusselt number, $Nu_x / \operatorname{Re}_x^{1/2}$ were analysed. The value of Prandtl number of water is taken at 6.2 and the volume fraction effect of ferroparticles ϕ is studied in the range $0 \le \phi \le 0.1$ where $\phi = 0$ represent the pure fluid water.

Table 1 Thermophysical properties of base fluid and ferroparticles [32, 33]

Physical Properties	Water	Magnetite (Fe_3O_4)
ρ (kg/m^3)	997.1	5200
C_p (J/kg·K)	4179	670
k (W/m·K)	0.613	6
$\sigma\left(\Omega^{-1}\text{m}^{-1}\right)$	0.05	25000

Table 2 Comparison values of $-\theta'(0)$ with previously published result when $M = \phi = N_R = 0$ and $Bi \to \infty$ (CWT)

Pr	White [30]	Khan et al. [31]	Mohamed et al. [22]	Present
	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$	$-\theta'(0)$
1	0.5705	0.5705	0.5705	0.5705
2	0.7437	0.7437	-	0.7437
3	0.8652	0.8652	-	0.8652
6	1.1147	1.1147	-	1.1147
10	1.3388	1.3388	1.3389	1.3389

Table 3 Variation of θ, $-\theta'(0)$, $C_f \mathrm{Re}_x^{1/2}$ and $Nu_x/\mathrm{Re}_x^{1/2}$ for various values of M, ϕ and N_R when $\mathrm{Pr} = 6.2$ and $Bi_X = 0.1$

N_R	ϕ	$M = 1$				$M = 2$				$M = 5$			
		$\theta(0)$	$-\theta'(0)$	$C_f \mathrm{Re}_x^{1/2}$	$Nu_x/\mathrm{Re}_x^{1/2}$	$\theta(0)$	$-\theta'(0)$	$C_f \mathrm{Re}_x^{1/2}$	$Nu_x/\mathrm{Re}_x^{1/2}$	$\theta(0)$	$-\theta'(0)$	$C_f \mathrm{Re}_x^{1/2}$	$Nu_x/\mathrm{Re}_x^{1/2}$
0	0.001	0.0772	0.0923	1.5903	0.0925	0.0745	0.0926	1.8792	0.0928	0.0699	0.0930	2.5581	0.0932
	0.01	0.0776	0.0922	1.6352	0.0943	0.0750	0.0925	1.9306	0.0946	0.0703	0.0930	2.6253	0.0951
	0.1	0.0828	0.0917	2.1296	0.1139	0.0802	0.0920	2.5029	0.1142	0.0755	0.0924	3.3856	0.1148
1	0.001	0.1031	0.0897	1.5903	0.2098	0.0999	0.0900	1.8792	0.2105	0.0947	0.0905	2.5581	0.2117
	0.01	0.1037	0.0896	1.6352	0.2139	0.1006	0.0899	1.9305	0.2146	0.0953	0.0905	2.6253	0.2158
	0.1	0.1107	0.0889	2.1296	0.2576	0.1077	0.0892	2.5029	0.2585	0.1025	0.0897	3.3856	0.2600
3	0.001	0.1338	0.0866	1.5903	0.4341	0.1304	0.0870	1.8792	0.4358	0.1247	0.0875	2.5581	0.4386
	0.01	0.1346	0.0865	1.6352	0.4425	0.1312	0.0869	1.9305	0.4442	0.1257	0.0874	2.6253	0.4470
	0.1	0.1439	0.0856	2.1296	0.5315	0.1407	0.0859	2.5029	0.5335	0.1352	0.0865	3.3856	0.5369

Table 3 shows that the heat transfer will increase when the temperature decreases while the magnetic parameter increases. Physically, the characteristic of ferrofluid which consists of magnetic nanoparticles is very magnetized and respond to the magnetic applied. At the point when the magnetic field is applied, the cold ferrofluid with better cooling performance moves towards the magnetic field source near to the lower part of the hot wall which can prompt a superior cooling rate. This result is clearly illustrated in figure 2 and 3. It is found the velocity of ferrofluid will increase while the temperature will decrease when the magnetic parameter applied to the hot wall is increased. Consequently, the reduced skin friction and reduced Nusselt number is rising as shown in table 3.

Figure 4 shows the velocity of ferrofluid flow increases when the volume fraction of ferroparticles increases. It is known that the nanofluid viscosity is depending on the particle volume fraction and particle size [34]. Physically, the volume fraction of ferroparticles that increases will lead to an increase of the viscosity of the ferrofluid, hence causing the velocity to decrease as followed by the theoretical model proposed by Brinkman [27]. However, Figure 5 displays the influence of volume fraction of ferroparticles on temperature profile seen that an increase of the volume fraction of ferroparticles increased the ferrofluid temperature. Thus, the viscosity of the ferrofluid become low. The dispersed of ferroparticles (Fe_3O_4) in base fluid (water) increases the ferrofluid capabilities in thermal conductivity which is in line with ferroparticles physical behavior as presented in table 1. According to the experimental study conducted by Haiza et al. [35] the thermal conductivity ratio increases with an increase of the temperature. Besides that, the effect of volume fraction of ferroparticles which is observed in table 3 gives the significant result to the reduced Nusselt number which represents the ratio of amount of heat displaced by convection to conduction.

Figure 6 reveals that an increase of the radiation parameter will enhance the temperature of the ferrofluid flow and hence, increases the thermal boundary layer thickness. Physically, the thermal radiation will release the thermal energy to the fluid flow, hence the temperature and the reduced Nusselt number will be increased. However, the heat transfer is decreased and do not affect the skin friction as presents in table 3. Lastly, Figure 7 illustrates the effect of Biot number on temperature against the boundary layer thickness. Based on figure 7, ferrofluid temperature increases when Biot number is increased. The Biot number is the ratio of conduction resistance inside the body to the convection resistance at the surface. Generally, Biot number is less than 1 in simple geometry plates, cylinder and sphere. Therefore, the Biot number used in this study is less than 1 by assuming that it is the uniform temperature inside the body. The increase of ferrofluid temperature lead to an increase of the thermal boundary layer thickness.

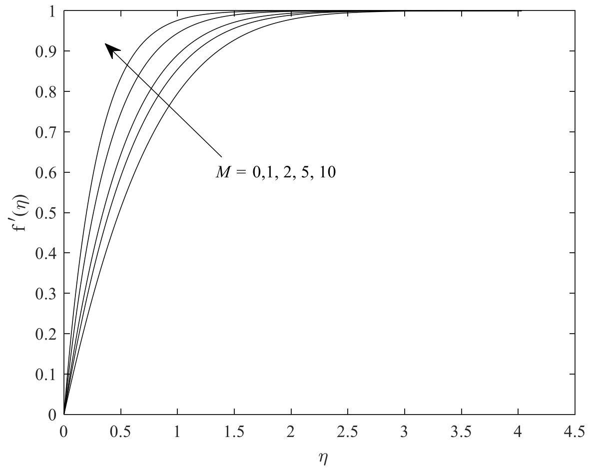

Figure 2 Velocity profile $f'(\eta)$, for increasing magnetic parameter, M when $N_R = 1$, $\phi = 0.1$ and $Bi_x = 0.1$

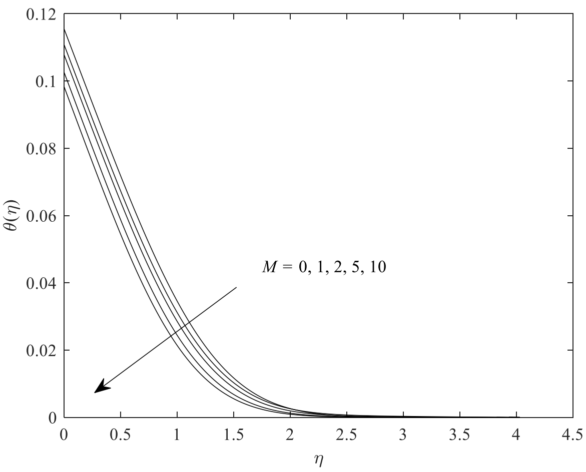

Figure 3 Temperature profile $\theta(\eta)$, for increasing magnetic parameter, M when $N_R = 1$, $\phi = 0.1$ and $Bi_x = 0.1$

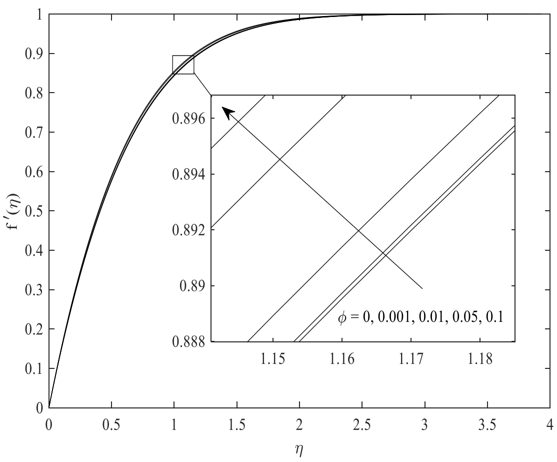

Figure 4 Velocity profile $f'(\eta)$, for increasing the ferroparticles volume fraction, ϕ when $M = N_R = 1$ and $Bi_x = 0.1$

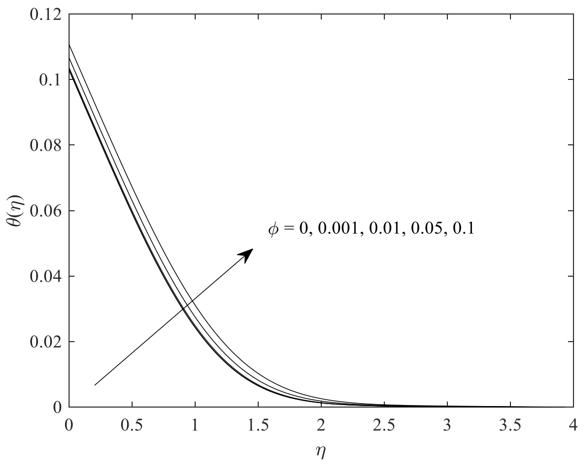

Figure 5 Temperature profile, $\theta(\eta)$ for increasing the ferroparticles volume fraction, ϕ when $M = N_R = 1$ and $Bi_x = 0.1$

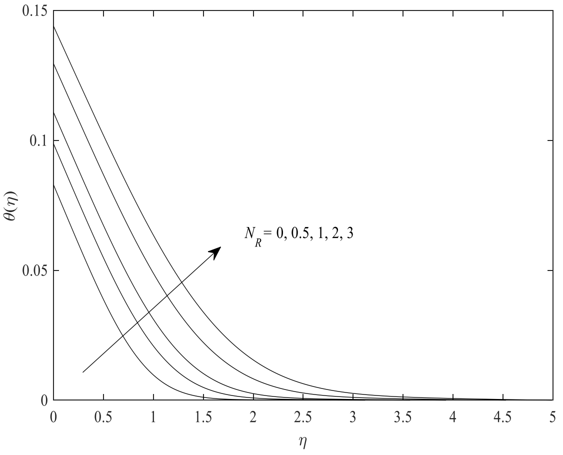

Figure 6 Temperature profile $\theta(\eta)$, for increasing radiation parameter, N_R when $M=1$, $\phi=0.1$ and $Bi_x=0.1$

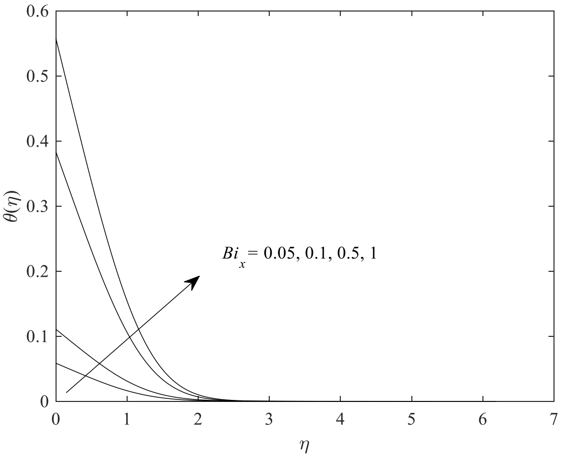

Figure 7 Temperature profile $\theta(\eta)$, for increasing Biot number, Bi_x when $M=N_R=1$ and $\phi=0.1$

4. Conclusions

The present study revealed the effects of some parameter influencing the magnetohydrodynamic (MHD) flow and heat transfer of ferrofluid (water as base fluid and magnetite, Fe_3O_4 as ferroparticles) on stagnation point along stationary flat plate with convective boundary condition and thermal radiation effect. The critical discoveries of the study are as below:

- The velocity of ferrofluid increases with an increase of the magnetic parameter and which reduced the momentum boundary layer thickness.
- The temperature of ferrofluid increases with an increase of the ferroparticles volume fraction, radiation parameter and Biot number which leads to an increase of the thermal boundary layer thickness.
- The reduced skin friction increases when the magnetic parameter and ferroparticles volume fraction is increased.
- The reduced Nusselt number increases when the volume fraction of ferroparticles, magnetic parameter and radiation parameter is increased.

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