

TWO-PHASE MIXED CONVECTION FLOW  
OVER A VERTICAL STRETCHING SHEET  
WITH ALIGNED MAGNETIC FIELD AND  
NEWTONIAN HEATING

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We hereby declare that We have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Doctor of Philosophy.

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### **STUDENT'S DECLARATION**

I hereby declare that the work in this thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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TWO-PHASE MIXED CONVECTION FLOW OVER A VERTICAL STRETCHING  
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NUR SYAMILAH BINTI ARIFIN

Thesis submitted in fulfillment of the requirements  
for the award of the degree of  
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In dedication to:

My beloved parents,  
Encik Arifin Ahmad and Puan Suriyani Abdullah

My loving sisters,  
Afifah, Nasriah and Alia Sofiah

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## **ABSTRAK**

Fenomena sistem dua fasa (bendalir dan pepejal) boleh didapati dalam banyak aplikasi, contohnya, pencemaran udara atau air, aliran darah dalam arteri, pengaliran di dalam tiub roket, pemendapan dan tilam bercecair. Penyelidikan berkaitan system dua fasa ini telah menarik perhatian penyelidik kerana potensinya yang penting dalam penyiasatan dinamik bendalir dengan zarah pepejal terampai. Daripada hanya memberi tumpuan kepada aliran bendalir (satu fasa), model dua fasa ini mengkaji taburan bagi kedua-dua bendalir dan zarah pepejal. Ini menunjukkan bahawa masalah bagi model aliran bendalir satu fasa diubahsuai kepada model dua fasa dengan menambah unsur interaksi antara bendalir dan zarah pepejal. Dicatatkan bahawa, zarah pepejal diandaikan berbentuk abu, jelaga dan debu. Dalam tesis ini, model yang telah diubahsuai disiasat secara teori dengan mempertimbangkan bendalir Newtonan dan bendalir bukan Newtonan yang berdebu, di mana penerbitan bagi model yang dicadangkan telah dirumuskan berdasarkan model aliran dua fasa. Khususnya, aliran dan pemindahan haba bagi kedua-dua bendalir Newtonan dan bendalir bukan Newtonan (Casson, Williamson dan Jeffrey) dengan kehadiran zarah-zarah debu dipertimbangkan. Selain itu, tesis ini memberi perhatian kepada pengaruh medan magnet sejajar dan olakan campuran yang berkaitan dengan syarat sempadan terma pemanasan Newtonan (NH) yang mana merentasi lembaran regangan menegak. Dalam keadaan ini, aliran bendalir Newtonan tanpa zarah debu juga diberi perhatian dalam mengkaji sifat aliran bendalir sebelum meneruskan dengan kajian bagi kedua-dua bendalir dan zarah habuk. Persamaan menakluk bagi model yang dicadangkan iaitu dalam bentuk persamaan pembezaan separa dijelmakan kepada persamaan pembezaan biasa dengan menggunakan penjelmaan keserupaan yang sesuai. Pengiraan berangka untuk persamaan yang diperolehi kemudian diselesaikan dengan menggunakan kaedah kotak-Keller yang diatur cara dalam perisian MATLAB. Keputusan dipaparkan untuk taburan halaju dan suhu bersama-sama dengan pekali geseran kulit dan nombor Nusselt. Beberapa parameter fizikal seperti parameter interaksi bendalir-zarah, sudut sejajar, medan magnet, olakan campuran, parameter konjugat untuk NH, nombor Prandtl serta parameter perwakilan untuk setiap model bendalir dikaji dengan mendalam. Didapati bahawa, kelakuan halaju dan suhu bagi semua jenis bendalir dipengaruhi oleh kehadiran zarah debu di mana ia mempunyai kecenderungan untuk mengurangkan taburan halaju dan suhu bagi bendalir. Selain itu, parameter sudut sejajar diperaku sebagai parameter kawalan, yang dapat mengawal keamatian medan. Sementara itu, parameter yang mewakili model bendalir yang diberi perhatian dalam thesis ini, merangkumi Casson, Williamson dan nisbah santaian kepada masa rencatan meningkatkan taburan suhu bendalir dan habuk, manakala trend sebaliknya berlaku untuk taburan halaju. Keputusan perbandingan untuk semua model yang dicadangkan menunjukkan bahawa medan aliran untuk bendalir Newtonan berdebu mempunyai taburan tertinggi berbanding bendalir bukan Newtonan berdebu. Walaubagaimanapun, bendalir Casson berdebu mempunyai ciri pemindahan haba yang lebih tinggi berbanding bendalir Williamson berdebu dan bendalir Jeffrey berdebu.

## ABSTRACT

The phenomena of two-phase system (fluid and solid) can be found in many applications, for instance, the air or water pollution, blood flow in arteries, flows in rocket tubes, sedimentation and fluidized bed. The research on this two-phase system has been given considerable attention by many researchers due to its significant potential in investigating the fluid dynamics with the suspension of the solid particles. Instead of only focused on the fluid flow (one phase), this two-phase model examines the distributions of both fluid and solid particles. This implies that the single phase model of fluid flow problem is modified into the two-phase model by the addition of the element of interaction between the fluid and solid particles. Note that, the solid particle is assumed to be in the form of ash, soot and dust. In this thesis, the modified model is investigated theoretically by considering the dusty Newtonian fluid and dusty non-Newtonian fluid, in which the derivation for the particular proposed model have been formulated based on two-phase flow model. Specifically, the flow and heat transfer of both Newtonian and non-Newtonian (Casson, Williamson and Jeffrey) fluids are considered in the presence of dust particles. Besides, this thesis concerns on the influences of aligned magnetic field and mixed convection associated with the thermal boundary conditions of Newtonian heating (NH) where it passed along a vertical stretching sheet. Under these conditions, the single phase flow of Newtonian fluid without the dust particles is also given attention in order to study the characteristic of fluid flow independently before proceeding further with investigation for both fluid and dust particles. The governing equations of all proposed models in the form of partial differential equations are transformed into the ordinary differential equations by employing the suitable similarity transformation. The numerical computations for the obtained equations are then computed using the Keller-box method which is programmed in MATLAB software. The results are presented for the velocity and temperature distribution together with the skin friction coefficient and Nusselt number. Several physical parameters such as fluid-particle interaction, aligned angle, magnetic field, mixed convection, conjugate parameter for NH, Prandtl number, as well as the representative parameters for each fluid model are investigated in details. It is found that, the velocity and temperature behavior of all types of fluid are affected by the presence of dust particles, in which it has the tendency to decrease velocity and temperature distribution of fluid. Moreover, the aligned angle parameter is acknowledged as the controlling parameter, which can control the intensity of magnetic field. Meanwhile, the representative parameters for the fluid models concentrated in this thesis, which encompass of Casson, Williamson and ratio of relaxation to retardation times improved the temperature distribution of fluid as well as dust phase, whereas the reverse trend occurs for velocity distributions. Comparative results of all proposed models show that dusty Newtonian fluid has the highest distribution in the flow field in comparison to dusty non-Newtonian fluid. However, dusty Casson fluid has the higher heat transfer characteristic compared to dusty Williamson fluid and dusty Jeffrey fluid.

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## LIST OF SYMBOLS

<b>a</b>	acceleration
<b>A</b> <sub>1</sub>	first Rivlin-Erickson tensor
<i>A</i>	Casson parameter
<i>a</i>	positive constant
<b>B</b>	total magnetic field
<b>B</b> <sub>0</sub>	uniform magnetic field
<b>b</b> <sub>1</sub>	induced magnetic field
<i>b</i>	conjugate parameter for NH
<i>C</i> <sub>f</sub>	local skin friction coefficient
<i>c</i> <sub>p</sub>	specific heat of fluid phase
<i>c</i> <sub>s</sub>	specific heat of particle phase
<b>D</b>	rate of strain tensor
<i>D</i> <sub>ij</sub>	total stress component
<i>De</i>	Deborah number
<b>E</b>	electric field
<i>e</i> <sub>ij</sub>	( <i>i, j</i> ) – th components of deformation rate
<b>F</b>	force
<b>F</b> <sub>b</sub>	body force
<b>F</b> <sub>p</sub>	fluid-particle interaction force
<b>F</b> <sub>s</sub>	surface force
<i>f</i> <sub>x</sub>	<i>x</i> – component of body force
<i>f</i> <sub>y</sub>	<i>y</i> – component of body force
<i>f</i> <sub>z</sub>	<i>z</i> – component of body force
<b>g</b>	gravitational force
<i>Gr</i> <sub>x</sub>	local Grashof number
<i>h</i> <sub>s</sub>	heat transfer coefficient
<b>I</b>	identity vector
<b>J</b>	current density
<i>k</i>	thermal conductivity
<i>M</i>	magnetic parameter
<i>N</i>	mass concentration of particle phase
<i>Nu</i> <sub>x</sub>	local Nusselt number
<i>n</i>	number density of particle
<i>p</i>	pressure of fluid phase
<i>p</i> <sub>a</sub>	hydrostatic pressure
<i>p</i> <sub>d</sub>	dynamic pressure
<i>p</i> <sub>p</sub>	pressure of dust phase
<b>Pr</b>	Prandtl number
<i>Q</i> <sub>p</sub>	heat transfer from a particle to fluid
<i>q</i> <sub>w</sub>	surface heat flux

$\text{Re}_x$	Reynolds number
$r_p$	radius of the particle
$\mathbf{T}$	stress tensor
$T_{ij}$	stress tensor in index notation
$T$	temperature of fluid
$T_p$	temperature of dust particles
$T_\infty$	ambient temperature of fluid
$t$	time
$\mathbf{u}$	velocity of fluid in $x$ -direction
$v$	velocity of fluid in $y$ -direction
$u_p$	velocity of particles in $x$ -direction
$v_p$	velocity of particles in $y$ -direction
$\mathbf{V}$	velocity vector field of fluid phase
$\mathbf{V}_p$	velocity vector field of dust phase
$x$	Dimensionless coordinate axis along the flow
$y$	Dimensionless coordinate axis normal to the flow

#### Greek letters

$\alpha_1$	aligned angle
$\nu$	kinematic viscosity
$\mu$	dynamic viscosity
$\mu_B$	plastic dynamic viscosity of non-Newtonian fluid
$\pi$	product of component of deformation rate with itself
$\pi_c$	critical value of product based on non-Newtonian model
$\sigma$	electric conductivity of fluid
$\sigma_x$	normal stress in $x$ -direction
$\sigma_y$	normal stress in $y$ -direction
$\sigma_z$	normal stress in $z$ -direction
$\tau_{xx}$	shear stress in $x$ -direction exerted on a face normal to $x$ -axis
$\tau_{xy}$	shear stress in $y$ -direction exerted on a face normal to $x$ -axis
$\tau_{xz}$	shear stress in $z$ -direction exerted on a face normal to $x$ -axis
$\tau_{yx}$	shear stress in $x$ -direction exerted on a face normal to $y$ -axis
$\tau_{yy}$	shear stress in $y$ -direction exerted on a face normal to $y$ -axis
$\tau_{yz}$	shear stress in $z$ -direction exerted on a face normal to $y$ -axis
$\tau_{zx}$	shear stress in $x$ -direction exerted on a face normal to $z$ -axis
$\tau_{zy}$	shear stress in $y$ -direction exerted on a face normal to $z$ -axis
$\tau_{zz}$	shear stress in $z$ -direction exerted on a face normal to $z$ -axis
$\boldsymbol{\tau}$	viscous stress tensor
$\tau_v$	velocity relaxation time of particles
$\tau_T$	thermal relaxation time of particles

$\tau_w$	wall shear stress
$\beta^*$	thermal expansion coefficient
$\beta$	fluid-particle interaction parameter
$\gamma$	specific heat ratio of mixture
$\delta$	boundary layer thickness
$\delta_{ij}$	Kronecker Delta
$\lambda$	mixed convection parameter
$\lambda_1$	retardation time
$\lambda_2$	ratio of relaxation to retardation times
$\lambda_3$	Williamson parameter
$\theta$	dimensionless of temperature for fluid phase
$\theta_p$	dimensionless of temperature for dust phase
$\rho$	density of fluid
$\rho_p$	density of particles
$\rho_y$	fluid yield stress
$\psi$	stream function
$\eta$	similarity variable
$\Delta\eta$	step size
$\Gamma$	time constant

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