# STRUCTURAL HEALTH MONITORING BASED TIME- DEPENDENT RELIABILITYANALYSIS OF CONCRETE BRIDGE

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**ABSTRACT**

The presence of chloride ions in concrete is the most important cause of steel reinforcing corrosion. Corrosion can lead to structural damage and needs to be managed effectively for better allocation of resources and effective bridge management. The application of de-icing salt or atmospheric exposure in marine environment could be the cause of corrosion initiation. This paper reviews chloride ingress prediction model and presents methodology to improve confidence in predicting corrosion concentration taking into account time dependent reliability analysis. Modeling uncertainty is often associated with limited knowledge which it can be reduced by increasing the availability of data. Additional information through bridge inspection and monitoring will increase confidence in prediction models. Monte Carlo simulation with Latin Hypercube Sampling is used to estimate prior and posterior performance prediction for chloride concentration. Bayesian Updating is used to incorporate prior beliefs about the condition and performance of the bridge together with data obtained through inspections and health monitoring to produce more quantitative data. The application of Bayesian Updating is shown to considerably reduce uncertainties associated with performance prediction. By using this approach, it will lead to the prediction of structural performance with increased confidence.

**Keywords**: Concrete Bridge, Corrosion, Chloride, Time- dependent Reliability, Monte- Carlo, Bayesian Updating

## 1.0 Introduction

Structural Health Monitoring System has been actively developed recently to monitor the corrosion of reinforcement. If the corrosion can be detected early, damage can be excluded or reduced significantly, hence the maintenance cost can be reduced.A few steps have been taken by the engineers such as design, construction and maintenance to ensure safe and durable services. Design consideration includes adoption of protective strategies (i.e increase concrete cover) and quality assurance during construction stage can increase the life span of the structure. The problem arise is the cost effectiveness of these and other measures is often unclear. Uncertainty associated with material, environmental load and structural effects are considered before decision making by bridge owner. Hence the need for probabilistic analysis expressing life cycle performance in reliability format [1][2]. In real life, engineers commonly need to make decision and solve a problem based on limited information.Probabilistic method can be used to deal with uncertainty exists. In this study, Fick's second law is used to mimic the chloride diffusion in concrete due to de-icing salt. This law takes initial values parameter as references to estimate future chloride content and these parameters tend to be statistical distributions of known moments through rigorous methods[3].Actual field data (with the availability of inspection and monitoring methods) collected as comparison with Fickian Model in predicting the real chloride concentration and moments to produce prior distribution. Data collected from health monitoring system need to be incorporated with prior distribution to produce posterior distribution hence improving confidence in predicting the future chloride content. Bayesian updating method is used to update belief by taking into account the prior belief given the likelihood that such event is known. Monte Carlo simulation is used to calculate the probability of failure for annual increment over the life time of the structure.

## 2. Background

Models based on the theory of diffusion have been developed to best represent the chloride ingress in concrete and are widely used in practice to predict the initiation of reinforcement corrosion in concrete [4]. Diffusion is mathematically represented by the partial differential equation using Fick's 2$^{nd}$ law of diffusion[5]:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z} \tag{1}$$

Where C represents the concentration of diffusing substance at time t at a location defined by the coordinates x, y and z, and D is the diffusion coefficient. According to Takewaka and Matsumoto [6] chloride penetration can be treated as diffusion process and seems to follow Fick's law of diffusion and they found that water cement ratio can give an effect to effective chloride diffusion coefficient. For a one dimensional diffusion process with constant diffusion coefficient, the Eq. 1 would be reduced to:

$$\frac{\partial y}{\partial x} = \left(D\frac{\partial^2 C}{\partial^2 y}\right) \tag{2}$$

The solution for the above equation has been derived in Crank[7]for a variety of scenarios (i. e. time dependent surface chloride concentration, and time dependent diffusion coefficient etc). Collepar.M[8]appears to be the first to apply Fick's second law to mimic chloride diffusion in concrete due to the de-icing salt. The model used for chloride ingress due to de-icing salt (based on solution Eq. 2) is as follows:

$$C(x,t) = C_O\left\{1 - erf\left(\frac{x}{2\sqrt{D.t}}\right)\right\} \tag{3}$$

Where $C_O$is the surface chloride concentration; $D$ is the effective diffusion coefficient; $x$ is the depth at which chloride concentration is required; t is the time of exposure; $C(x,t)$is the chloride concentration at depth $x$ and time $t$. Vu & Stewart [3] have made some improvement from existing deterioration model including the use of more accurate corrosion initiation and propagation model. Time-variant corrosion rates, time-variant loading model and shear failure limit state are considered in the analysis to study the effect of durability specifications.In general, it is assumed that chloride concentration $Cxcta\ (t_j)$ occur within the time interval $(0, t_L)$ at time $t_j$ $(j = 1,2,\ldots,n)$ then the cumulative probability of failure of chloride concentration anytime during this time interval is given by:

$$P_f(t_L) = 1 - Pr[Cxcta\,(t_1) < Cth \cap Cxcta\,(t_2) < Cth \cap \ldots\ldots \cap Cxcta(t_n) < Cth]$$

$$t_1 < t_2 < \cdots < t_n \leq t_L \tag{4}$$

Where $Cxcta\,(t_1)$ represents the initial distribution of chloride concentration and $Cxcta\,(t_2), Cxcta\,(t_n)$ represent the chloride concentration at time $t_j$ updated on survival of the previous load events. Technically, the updated chloride concentrations are influenced by time dependent changes in materials properties. Thus the cumulative probability of failure is dependent upon the prior and updated chloride concentration.

# 3.0 Probabilistic Modeling and Simulation

## 3.1 Probabilistic Modeling of Deterioration due to Chloride Ingress

Field and laboratory testing along with health monitoring system can be used to identify deterioration on the bridge. In particular, chloride profile generally tested in the laboratory to establish the concentration of chloride in the concrete samples. The effective diffusion coefficient and surface chloride concentration are derived by using non-linear regression analysis to fit the profiles to the diffusion based deterioration model. The objective of Bayesian updating procedure is to reduce the uncertainty (i.e. COV) in the predictive performance[9]. In this case, uncertainty is the probability of chloride concentration at given depth and cumulative time exceeds the threshold chloride concentration.

## 3.2 Simulation of Probabilistic Performance Prediction

In this study, Monte Carlo Simulation with Latin Hypercube Sampling is used to estimate prior and posterior performance prediction of chloride concentration. The cover depth (*Xc*) is set to be 40 mm and the time is set to be arbitrarily 20 years. The output of this simulation is in the form of probability density function of prior, likelihood and posterior distributions. The probability of corrosion initiation for a given time also presented. The parameters involved in chloride ingress model for typical concrete bridge elements (e.g. slab, beam, or cross beam etc) which are prone to de-icing salts are summarizedin table 1[9]:

*Table 1: Summary of parameters involved in chloride ingress model*

| Parameter | Mean | C.O.V | Distribution | References |
|---|---|---|---|---|
| $C_o$ | 3.5 kg/m$^3$ | 0.5 | Lognormal | Vu & Stewart (2000) |
| *D (Nominal)* | 5x10$^{-5}$m$^2$/yr | | | Vu & Stewart (2000) |
| *Model Error (D)* | 1.0 | 0.2 | Normal | Vu & Stewart (2000) |
| *Cth* | 0.9 kg/m$^3$ | 0.19 | Uniform (0.6-1.2 kg/m$^3$) | |
| *Xc* | 40 mm | 0.1 | Normal | Chryssanthopoulos& Sterrit (2002) |

## 4.0 Result and Discussion

### 4.1 Prior Probability of Failure

In this study, the failure probability is defined as the frequency of events that chloride concentration, *Cxt* at cover depth *Xc* at given cumulative time (e.g. 20 years in this study) exceed the threshold of chloride concentration, *Cth*.The prior probability of failure is determined based on previous inspection data. When the new inspection data become available, the updating procedure can be applied by incorporating the new inspection data together with the previous data to produce posterior probability of failure. Updating procedure can be performed using Bayesian framework. For the prior probability of failure of chloride concentration, the interval of simulation is one year (e.g. 20 years for this study) at given depth (e.g. 40mm for this study). The prior probability of failure for this particular study is based on time dependent analysis. The cumulative probability of failure is depending upon prior and updated failure margin of chloride concentration. For example, the probability of failure for year three should also consider for probability of failure for the previous year which is year two and one. Figure 1 shows point in time and cumulative time probability of failure for chloride concentration.

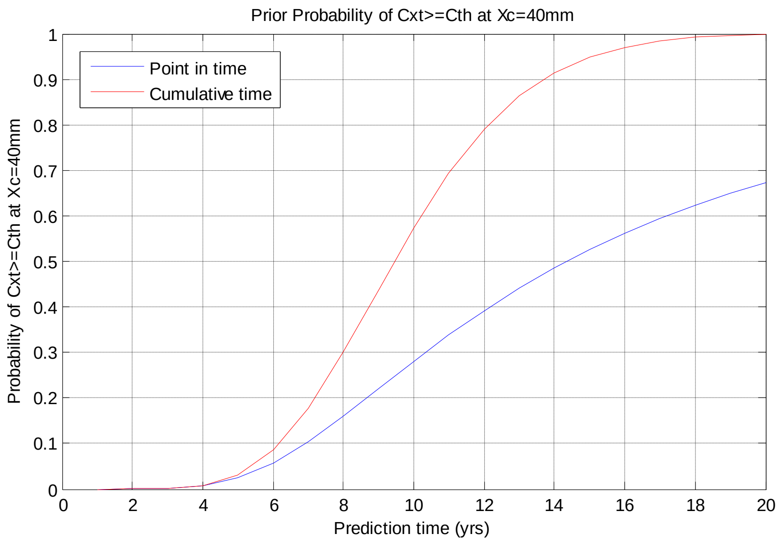


*Figure 1: Point in time and cumulative time probability of failure for chloride concentration*

From Figure 1, cumulative time probability of failure gives higher value of probability of failure as compared to the point in time probability of failure. For example, at year 12 the probability of failure for cumulative time is 0.8 and point in time is 0.4 respectively. This shows that, the probability of failure for cumulative time has increased by about 50% as compared to point in time. Table 2 summarizes the results for point in time and cumulative time probability of failure for year 6, 8 and 12 respectively. This shows that the result for cumulative time probability of failure is more reliable compared to point in time probability of failure.

*Table 2: Point in time and cumulative time probability of failure*

| Point in Time Probability of Failure (Cxt>= Cth) | | Cumulative Time Probability of Failure (Cxt>= Cth) | |
|---|---|---|---|
| *Year* | *Probability of failure(Pf)* | *Year* | *Probability of Failure (Pf)* |
| 6 | 0.06 | 6 | 0.09 |
| 8 | 0.15 | 8 | 0.30 |
| 12 | 0.40 | 12 | 0.80 |

## 4.2 Updating Procedure for Probability of Failure Based on Single Observation

In this study, event updating is adopted as a methodology to predict the posterior distribution. For simplification, prior distribution of chloride concentration follows normal distribution for certain value of mean and standard deviation for each year in twenty years. Thus, prior distribution of chloride concentration is given by:

$$p_{prior}\,(\theta) = \frac{1}{\sqrt{2\pi}\sigma_o} exp\left[-0.5\left(\frac{\theta-\theta_o}{\sigma_o}\right)^2\right] \qquad (5)$$

Figure 2ashows the prior distribution for chloride concentration for single observation at year 10.Since this paper is to produce more quantitative data by using Bayesian Updating, the monitoring data is known to follow the normal distribution. Thus, likelihood value for mean istaken as 30% decrease from prior value and standard deviation is30% increase from prior value.Figure 2b shows the likelihood distribution of chloride concentration with value of mean is 0.5019 and standard deviation is 0.5715.

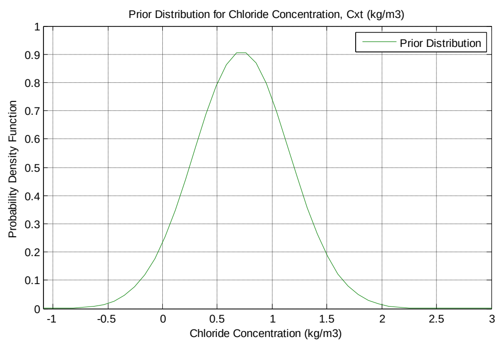


*Figure 2a*

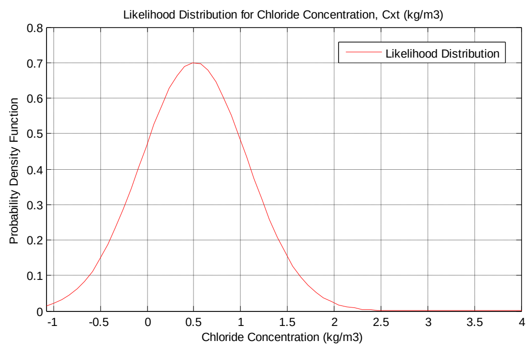


*Figure 2b*

*Figure 2a & b: Prior and likelihood distribution for chloride concentration at year 10*

Supposed that monitoring data is available and that an observation $y$ is made by this method to a sufficient approximation which is follows the normal distribution. If a single observation is made, the standardize likelihood function is represented by normal curve. If a prior $\theta \sim N(\theta_o, \sigma_o^2)$, and standardized likelihood function is represented by a normal curve centered at y with standard deviation $\sigma$. The posterior distribution of $\theta$ given $y$, $P_{post}(\theta|y)$, is the normal distribution $N(\bar{\theta}, \bar{\sigma}^2)$. Figure 3 illustrate the result of posterior distribution for chloride concentration after the process of updating the prior and likelihood distribution. It is shows that the variance for posterior distribution is slightly reduced compared to prior and likelihood distribution. It is shows that the uncertainty in posterior distribution is reduced hence increasing the confidence in predicting future probability of failure.

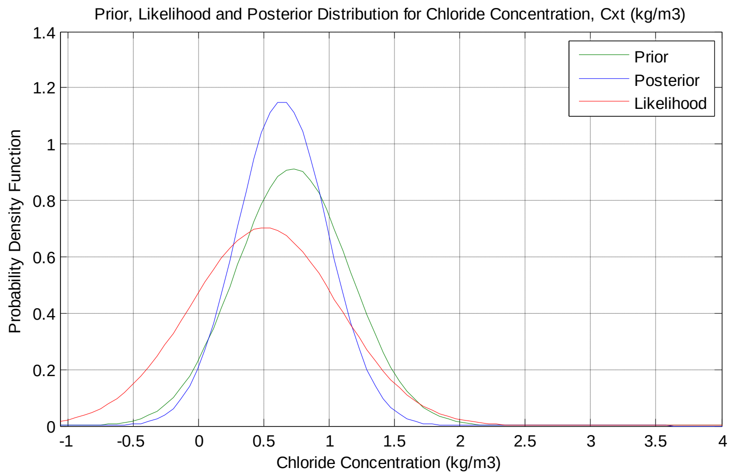


*Figure 3: Prior, likelihood and posterior distribution for single observation at year 10*

Figure 4 shows prior and posterior probability of failure for chloride concentration for single observation at year 10. Quantifying this Figure 4 in Table 3 shows that the posterior probability of failure causes a slightly reduced if compared to prior probability of failure. The result shows that at year 12 the posterior probability of failure is reduced for amount of 10% from prior probability of failure. By incorporating a new data and updating using Bayes Theorem, the probability of failure can be reduce thus increasing the confidence in predicting the future probability of failure.

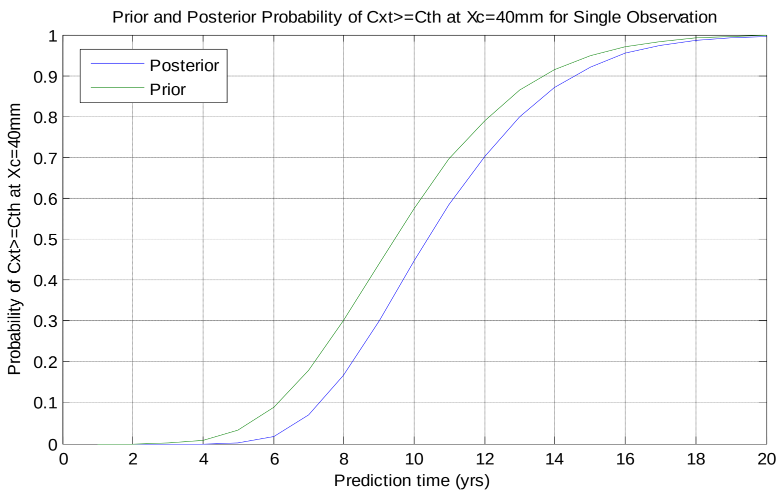


*Figure 4: Prior and posterior probability of failure for single observation*

*Table 3: Prior and posterior probability of failure*

| Prior Probability of Failure ($Cxt \geq Cth$) | | Posterior Probability of Failure ($Cxt \geq Cth$) | |
|---|---|---|---|
| *Year* | *Probability of failure(Pf)* | *Year* | *Probability of Failure (Pf)* |
| 6 | 0.10 | 6 | 0.02 |
| 8 | 0.30 | 8 | 0.17 |
| 12 | 0.80 | 12 | 0.70 |

## 5.0 Conclusion

Probabilistic modelling with various parameter defined as variables is presented in this paper. The probabilistic analysis by using Bayesian theory with considering time dependent reliability analysis is able to determine the probability of failure. In this case, the probability of failure is defined as the probability that chloride concentrations at future time have exceeds the threshold chloride concentrations. The analysis shown that by using Bayesian updating, the uncertainty can be reduced in the prediction of future performance which is has met theobjective. It is found that uncertainty of the posterior model is reduced hence increased confidence in predicting future performance. From the analysis, the probability of failure is reduced by incorporating a new data and updated using Bayesian Theory. The probabilistic modeling and simulation of chloride ingress in concrete bridge allows the bridge owner to manage the bridge effectively with better allocation of resources and effectively plan for bridge maintenance.

## 6.0 References


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