

IDENTIFICATION OF EXHAUST HANGER
LOCATION BASED ON FINITE ELEMENT
MODEL UPDATING TECHNIQUE

MOHD SAHRIL BIN MOHD FOUZI

MASTER OF SCIENCE

UNIVERSITI MALAYSIA PAHANG



SUPERVISOR'S DECLARATION

We hereby declare that We have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Master of Science.

A handwritten signature in black ink, appearing to read 'Shahrir', is written above a horizontal line.

(Supervisor's Signature)

Full Name : ASSOC. PROF. DR. MOHD SHAHRIR BIN MOHD SANI

Position : SENIOR LECTURER

Date :

A handwritten signature in black ink, appearing to read 'Zamri', is written above a horizontal line.

(Co-supervisor's Signature)

Full Name : IR. DR. ZAMRI BIN MOHAMED

Position : SENIOR LECTURER

Date :



STUDENT'S DECLARATION

I hereby declare that the work in this thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

A handwritten signature in black ink, appearing to read 'Mohd Sahril Bin Mohd Fouzi', is written above a horizontal line.

(Student's Signature)

Full Name : MOHD SAHRIL BIN MOHD FOUZI

ID Number : MMA17001

Date :

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ABSTRAK

Pembangunan struktur ekzos ketika ini telah mendapat perhatian yang lebih oleh penyelidik dan jurutera dalam mengenalpasti lokasi penggantung yang sesuai untuk menggantung ekzos pada casis kenderaan. Ini kerana beban dinamik yang terhasil daripada keadaan jalan yang tidak rata dan gegaran enjin ketika pengoperasian telah dipindahkan melalui penggantung dan disebarkan ke seluruh struktur akan mempengaruhi prestasi serta jangka hayat struktur berkenaan. Oleh itu, kajian ini mencadangkan satu kaedah untuk mengenalpasti lokasi gantungan ekzos yang terbaik untuk memperbaiki tingkah laku dinamik struktur berdasarkan teknik pengemaskinian model unsur terhingga (FE) yang menggunakan analisa mod normal. Pada mulanya, struktur ekzos dimodelkan menggunakan perisian reka-bentuk terbantu komputer (CAD) dan dipindahkan ke dalam perisian analisa unsur terhingga (FEA) untuk prosedur pra-pemprosesan. Semasa fasa pra-pemprosesan, model FE telah melalui strategi pemodelan sambungan untuk mewakili struktur sebenar ekzos yang dikimpal kerana sambungan itu sendiri memberi pengaruh yang penting dalam tingkah laku dinamik sesebuah struktur. Melalui strategi pemodelan sambungan, beberapa unsur penyambung seperti unsur badan tegar jenis 2 (RBE2), unsur penyambung bar (CBAR), unsur penyambung rasuk (CBEAM), dan unsur penyambung spring (CELAS) yang terdapat dalam perisian FEA digunakan untuk meniru sambungan kimpalan sebenar. Kemudian, model FE dengan unsur penyambung yang paling dipercayai telah disahkan dengan menggunakan data dinamik yang diukur melalui analisa eksperimen modal (EMA) melalui kaedah analisa korelasi. Data dinamik diukur dengan menggunakan teknik ketukan serta pergerakan sensor. Model FE dengan unsur penyambung CBAR didapati berjaya mewakili struktur sebenar ekzos yang dikimpal kerana ia mempunyai perbezaan terendah dalam analisa korelasi. Perbezaan antara hasil yang diramalkan dengan data yang diukur berlaku disebabkan kemudahan yang dibuat semasa proses permodelan struktur yang kompleks dan andaian terhadap sifat bahan yang digunakan. Oleh itu, teknik pengemaskinian model FE telah digunakan untuk mengurangkan perbezaan ini dengan melaraskan parameter yang sesuai secara berulang. Di peringkat awal, analisa kepekaan dilakukan untuk memilih parameter yang benar-benar peka untuk dikemaskini. Prosedur pengemaskinian dapat mengurangkan perbezaan daripada 4.10% ralat kepada 3.74% ralat. Kemudian, model FE yang telah dikemaskini telah digunakan dalam mengenalpasti lokasi gantungan ekzos yang terbaik. Dalam mengenalpasti lokasi gantungan terbaik, beberapa kajian kes telah direka berdasarkan susun atur penggantung yang berkemungkinan. Susun atur penggantung ini dianalisa menggunakan tindakbalas frekuensi modal untuk menilai prestasi dinamikinya. Dengan ketaranya, sesaran maksimum lokasi asal penggantung yang melebihi 10 mm berjaya dikurangkan bawah 1 mm menggunakan susun atur yang ke-9 daripada 35 susun atur. Kesimpulannya, model FE ekzos yang dikimpal telah berjaya dibangunkan menggunakan strategi permodelan sambungan dan teknik pengemaskinian model. Model FE yang boleh dipercayai ini digunakan secara berkesan melalui kaedah numerik dalam mengenalpasti lokasi gantungan terbaik untuk meminimalkan kesan gegaran dengan mengurangkan sesaran struktur berkenaan. Kaedah yang dicadangkan ini berguna untuk dimajukan kepada struktur lain dalam mengenalpasti lokasi terbaik tanpa melibatkan pengubahsuaian pada struktur fizikal yang menyebabkan pertambahan kos dan masa.

ABSTRACT

The development of exhaust structure to date has risen concerned among researchers and engineers due to the identification of suitable hanger location to suspend the structure on the vehicle's chassis. This is because dynamic loads produced from uneven road condition and engine operational vibration that are transferred *via* hangers and propagated along the structure will affect the performance and lifespan of the exhaust structure. Hence, the present study proposed an approach to identify the best exhaust hanger location in order to improve the dynamic behaviour of the structure based on finite element (FE) model updating technique using normal mode analysis. Initially, the exhaust structure was modelled in computer aided design (CAD) software and imported into finite element analysis (FEA) software for pre-processing procedure. During the pre-processing phase, the FE model was treated with joint modelling strategy to represent the real welded exhaust structure since the joint itself gives a significant influence in the dynamic behaviour of a structure. Through joint modelling strategy, several element connectors such as rigid body element type 2 (RBE2), bar element connector (CBAR), beam element connector (CBEAM), and spring element connector (CELAS) available in the FEA software were used to model the welded joints. In order to verify the most reliable FE model with element connectors, the measured dynamic data from experimental modal analysis (EMA) were used and compared with the predicted result computed in FEA through correlation analysis. The measured dynamic data in this study were obtained from EMA using impact excitation with roving accelerometer technique. It was found that the FE model with CBAR element connector is feasible to replicate the real welded exhaust structure since it has the lowest discrepancy in correlation analysis. The discrepancy between the predicted results and its measured counterpart is appeared due to simplification made in modelled the complex geometry of exhaust structure and assumptions of material properties used during modelling process. Hence, FE model updating technique was adopted to reduce the discrepancy by adjusting the parameters iteratively. Prior to the updating process, sensitivity analysis was performed to select only sensitive parameters to be updated. The updating procedure managed to reduce the discrepancy from 4.10 % of error to 3.74 % of error. Then, the updated FE model was used for further analysis in identifying the best exhaust hanger location. In identifying the best hanger location, several case studies were designed with 35 numbers of hanger configurations. These hanger configurations for the FE model were computed using modal frequency response to evaluate its dynamic performance. Significantly, the maximum displacement of the original hanger location which was above 10 mm was successfully reduced less than 1 mm obtained from the 9th configuration. As a conclusion, a reliable FE model of welded exhaust structure has been successfully developed using joint modelling strategy and model updating technique, which later was effectively used in identifying the best hanger location numerically in minimizing the vibration effect with the reduction of displacement of the structure. This proposed method is valuable to be extended for other types of exhaust structures in identifying the best hanger location without involving any modification on the physical structure, which requires extra costs, efforts and time.

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LIST OF SYMBOLS

A	Coefficient for steel bolt (1.67)
B	Coefficient for steel bolt (0.86)
C	Damping matrices
E_1	Young's Modulus
F	Load vector of nodes
J	Objective function
K	Stiffness matrices
M	Mass matrices
d	Diameter of bolt
t_1	Thickness of plate 1
t_2	Thickness of plate 2
ω	Natural frequency
\ddot{x}	Acceleration vector
\dot{x}	Velocity vector
x	Displacement vector
$(\phi_X)_q$	Experimental modal vector, mode q
$(\phi_A)_r$	Analytical modal vector, mode r
$(\phi_X)_q^T$	Transpose of experimental modal vector, mode q
$(\phi_A)_r^T$	Transpose of analytical modal vector, mode r
$\gamma_{xy}^2(f)$	Coherence function

LIST OF ABBREVIATIONS

1D	One Dimensional
2D	Two Dimensional
3D	Three Dimensional
ACM	Area Contact Model
ADDOFD	Average Driving Degree of Freedom Displacements
BDF	Bulk Data File
CAD	Computational Aided Design
CBAR	Bar Element connector
CBEAM	Beam Element connector
CBUSH	Fastener (bolt and nuts) Element connector
CELAS	Elastic (spring) Element connector
CPU	Computer Processing Unit
CTRIA3	Three triangular node shell element
CWELD	Weld Element connector
DAQ	Data Acquisition System
DOF	Degree of Freedom
EMA	Experimental Modal Analysis
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method
FFT	Fast Fourier Transform
FSW	Friction Stir Welding
ICE	Internal Combustion Engine
IRF	Impulse Response Function
MAC	Modal Assurance Criterion
NVH	Noise, Vibration, and Harshness
RBE	Rigid body element connector
RSW	Resistance spot weld

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