IDENTIFICATION OF EXHAUST HANGER LOCATION BASED ON FINITE ELEMENT MODEL UPDATING TECHNIQUE

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MASTER OF SCIENCE

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SUPERVISOR'S DECLARATION

We hereby declare that We have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Master of Science.

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STUDENT’S DECLARATION

I hereby declare that the work in this thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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ABSTRAK

Pembangunan struktur ekzos ketika ini telah mendapat perhatian yang lebih oleh penyelidik dan jurutera dalam mengenalpasti lokasi penggantung yang sesuai untuk menggantung ekzos pada casis kenderaan. Ini kerana beban dinamik yang terhasil daripada keadaan jalan yang tidak rata dan gegaran enjin ketika pengoperasian telah dipindahkan melalui penggantung dan disebarkan ke seluruh struktur akan mempengaruhi prestasi serta jangka hayat struktur berkenaan. Oleh itu, kajian ini mencadangkan satu kaedah untuk mengenalpasti lokasi gantungan ekzos yang terbaik untuk memperbaiki tingkah laku dinamik struktur berdasarkan teknik pengemaskinian model unsur terhingga (FE) yang menggunakan analisa mod normal. Pada mulanya, struktur ekzos dimodelkan menggunakan perisian reka-bentuk terbantu komputer (CAD) dan dipindahkan ke dalam perisian analisa unsur terhingga (FEA) untuk prosedur pra-pemprosesan. Semasa fasa pra-pemprosesan, model FE telah melalui strategi pemodelan sambungan untuk mewakili struktur sebenar ekzos yang dikimpal kerana sambungan itu sendiri memberi pengaruh yang penting dalam tingkah laku dinamik sesebuah struktur. Melalui strategi pemodelan sambungan, beberapa unsur penyambung seperti unsur badan tegar jenis 2 (RBE2), unsur penyambung bar (CBAR), unsur penyambung rasuk (CBEAM), dan unsur penyambung spring (CELAS) yang terdapat dalam perisian FEA digunakan untuk meniru sambungan kimpalan sebenar. Kemudian, model FE dengan unsur penyambung yang paling dipercayai telah disahkan dengan menggunakan data dinamik yang diukur melalui analisa eksperimen modal (EMA) melalui kaedah analisa korelasi. Data dinamik diukur dengan menggunakan teknik ketukan serta pergerakkan sensor. Model FE dengan unsur penyambung CBAR didapati berjaya mewakili struktur sebenar ekzos yang dikimpal kerana ia mempunyai perbezaan terendah dalam analisa korelasi. Perbezaan antara hasil yang diramalkan dengan data yang diukur berlaku disebabkan permudahan yang dibuat semasa proses permodelan struktur yang kompleks dan andaian terhadap sifat bahan yang digunakan. Oleh itu, teknik pengemaskinian model FE telah digunakan untuk mengurangkan perbezaan ini dengan melaraskan parameter yang sesuai secara berulang. Di peringkat awal, analisa kepekaan dilakukan untuk memilih parameter yang benar-benar peka untuk dikemaskini. Prosedur pengemaskinian dapat mengurangkan perbezaan daripada 4.10% ralat kepada 3.74% ralat. Kemudian, model FE yang telah dikemaskini telah digunakan dalam mengenalpasti lokasi gantungan ekzos yang terbaik. Dalam mengenalpasti lokasi gantungan terbaik, beberapa kajian kes telah direka berdasarkan susun atur penggantung yang berkemungkinan. Susun atur penggantung ini dianalisa menggunakan tindakbalas frekuensi modal untuk menilai prestasi dinamiknya. Dengan ketaranya, sesaran maksimum lokasi asal penggantung yang melebihi 10 mm berjaya dikurangkan bawah 1 mm menggunakan susun atur yang ke-9 daripada 35 susun atur. Kesimpulannya, model FE ekzos yang dikimpal telah berjaya dibangunkan menggunakan strategi permodelan sambungan dan teknik pengemaskinian model. Model FE yang boleh dipercayai ini digunakan secara berkesan melalui kaedah numerik dalam mengenalpasti lokasi gantungan terbaik untuk meminimakan kesan gegaran dengan mengurangkan sesaran struktur berkenaan. Kaedah yang dicadangkan ini berguna untuk dimajukan kepada struktur lain dalam mengenalpasti lokasi terbaik tanpa melibatkan pengubahsuaian pada struktur fizikal yang menyebabkan pertambahan kos dan masa.

ABSTRACT

The development of exhaust structure to date has risen concerned among researchers and engineers due to the identification of suitable hanger location to suspend the structure on the vehicle's chassis. This is because dynamic loads produced from uneven road condition and engine operational vibration that are transferred *via* hangers and propagated along the structure will affect the performance and lifespan of the exhaust structure. Hence, the present study proposed an approach to identify the best exhaust hanger location in order to improve the dynamic behaviour of the structure based on finite element (FE) model updating technique using normal mode analysis. Initially, the exhaust structure was modelled in computer aided design (CAD) software and imported into finite element analysis (FEA) software for pre-processing procedure. During the pre-processing phase, the FE model was treated with joint modelling strategy to represent the real welded exhaust structure since the joint itself gives a significant influence in the dynamic behaviour of a structure. Through joint modelling strategy, several element connectors such as rigid body element type 2 (RBE2), bar element connector (CBAR), beam element connector (CBEAM), and spring element connector (CELAS) available in the FEA software were used to model the welded joints. In order to verify the most reliable FE model with element connectors, the measured dynamic data from experimental modal analysis (EMA) were used and compared with the predicted result computed in FEA through correlation analysis. The measured dynamic data in this study were obtained from EMA using impact excitation with roving accelerometer technique. It was found that the FE model with CBAR element connector is feasible to replicate the real welded exhaust structure since it has the lowest discrepancy in correlation analysis. The discrepancy between the predicted results and its measured counterpart is appeared due to simplification made in modelled the complex geometry of exhaust structure and assumptions of material properties used during modelling process. Hence, FE model updating technique was adopted to reduce the discrepancy by adjusting the parameters iteratively. Prior to the updating process, sensitivity analysis was performed to select only sensitive parameters to be updated. The updating procedure managed to reduce the discrepancy from 4.10 % of error to 3.74 % of error. Then, the updated FE model was used for further analysis in identifying the best exhaust hanger location. In identifying the best hanger location, several case studies were designed with 35 numbers of hanger configurations. These hanger configurations for the FE model were computed using modal frequency response to evaluate its dynamic performance. Significantly, the maximum displacement of the original hanger location which was above 10 mm was successfully reduced less than 1 mm obtained from the 9th configuration. As a conclusion, a reliable FE model of welded exhaust structure has been successfully developed using joint modelling strategy and model updating technique, which later was effectively used in identifying the best hanger location numerically in minimizing the vibration effect with the reduction of displacement of the structure. This proposed method is valuable to be extended for other types of exhaust structures in identifying the best hanger location without involving any modification on the physical structure, which requires extra costs, efforts and time.

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LIST OF SYMBOLS

A	Coefficient for steel bolt (1.67)
B	Coefficient for steel bolt (0.86)
C	Damping matrices
E_1	Young's Modulus
F	Load vector of nodes
J	Objective function
K	Stiffness matrices
M	Mass matrices
d	Diameter of bolt
t_1	Thickness of plate 1
t_2	Thickness of plate 2
ω	Natural frequency
$\ddot{x}$	Acceleration vector
$\dot{x}$	Velocity vector
x	Displacement vector
$(\phi_X)_q$	Experimental modal vector, mode q
$(\phi_A)_r$	Analytical modal vector, mode r
$(\phi_X)_q^T$	Transpose of experimental modal vector, mode q
$(\phi_A)_r^T$	Transpose of analytical modal vector, mode r
$\gamma_{xy}^2(f)$	Coherence function

LIST OF ABBREVIATIONS

1D	One Dimensional
2D	Two Dimensional
3D	Three Dimensional
ACM	Area Contact Model
ADDOFD	Average Driving Degree of Freedom Displacements
BDF	Bulk Data File
CAD	Computational Aided Design
CBAR	Bar Element connector
CBEAM	Beam Element connector
CBUSH	Fastener (bolt and nuts) Element connector
CELAS	Elastic (spring) Element connector
CPU	Computer Processing Unit
CTRIA3	Three triangular node shell element
CWELD	Weld Element connector
DAQ	Data Acquisition System
DOF	Degree of Freedom
EMA	Experimental Modal Analysis
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method
FFT	Fast Fourier Transform
FSW	Friction Stir Welding
ICE	Internal Combustion Engine
IRF	Impulse Response Function
MAC	Modal Assurance Criterion
NVH	Noise, Vibration, and Harshness
RBE	Rigid body element connector
RSW	Resistance spot weld

REFERENCES

Adel, F., Shokrollahi, S., Jamal-Omidi, M., & Ahmadian, H. (2017). A model updating method for hybrid composite/aluminum bolted joints using modal test data. *Journal of Sound and Vibration, 396*, 172-185. doi: http://dx.doi.org/10.1016/j.jsv.2017.02.035

Ahmadian, H., Mottershead, J. E., & Friswell, M. I. (1998). Regularisation methods for finite element model updating. *Mechanical Systems and Signal Processing, 12*(1), 47-64.

Avitable, P. (2000). Experimental modal analysis (a simple non-mathematical presentation). *Annual Technical Meeting-Institute of Environmental Sciences And Technology, 46*, 434-448.

Bathe, K.-J. (2016). Finite element procedures. K.J. Bathe, Watertown, MA: *Prentice Hall, Pearson Education, Inc.*(2nd edition).

Bi, S., Huang, L., & Tong, N.-H. (2019). Natural orbital-based Lanczos method for Anderson impurity models. *Computer Physics Communications, 235*, 196-209. doi: https://doi.org/10.1016/j.cpc.2018.09.002

Bötke, A., & Yazgaç, D. (2016). Exhaust mount placement optimisation with comparable methods. *23rd International Congress on Sound & Vibration (ICSV23)*, 1-8.

Bower, A. F. (2009). Applied mechanics of solids. *CRC press*.

Bringante, D., Rainieri, C., & Fabbrocino, G. (2017). The role of the Modal Assurance Criterion in the interpretation and validation of models for seismic analysis of architectural complexes. *Procedia Engineering, 199*, 3404-3409. doi: 10.1016/j.proeng.2017.09.484

Carne, T. G., & Stasiunas, E. C. (2006). Lessons learned in modal testing—part 3: Transient excitation for modal testing, more than just hammer impacts. *Experimental Techniques: Society for Experimental Mechanics, 30*(3), 69-79. doi: 10.1111/j.1747-1567.2006.00046.x

Chandregowda, S., & Reddy, G. R. C. (2018). Evaluation of fastener stiffness modelling methods for aircraft structural joints. *AIP Conference Proceedings, 1943*(1), 1-7. doi: https://doi.org/10.1063/1.5029577

Chen, H.-P., & Maung, T. S. (2014). Regularised finite element model updating using measured incomplete modal data. *Journal of Sound and Vibration, 333*(21), 5566-5582. doi: http://dx.doi.org/10.1016/j.jsv.2014.05.051

Chenn, X., Zhang, Y., Zhang, Y., & Jiang, M. (2014). Hanger location design and vibration isolation of an exhaust system. *SAE International*. doi: 10.4271/2014-01-1708.

D'Ambrogio, W., & Fregolent, A. (1998). On the use of consistent and significant information to reduce ill-conditioning in dynamic model updating. *Mechanical Systems and Signal Processing, 12*(1), 203-222.

Eiras, J. N., Payan, C., Rakotonarivo, S., & Garnier, V. (2018). Experimental modal analysis and finite element model updating for structural health monitoring of reinforced concrete radioactive waste packages. *Construction and Building Materials, 180*, 531-543. doi: https://doi.org/10.1016/j.conbuildmat.2018.06.004

El-Kafafy, M., Peeters, B., Geluk, T., & Guillaume, P. (2019). The MLMM modal parameter estimation method: A new feature to maximize modal model robustness. *Mechanical Systems and Signal Processing, 120*, 465-485. doi: https://doi.org/10.1016/j.ymssp.2018.10.015

Englund, T. L., Wall, J. E., Ahlin, K. A., & Broman, G. I. (2002). Significance of non-linearity and component-internal vibrations in an exhaust system. *2nd WSEAS International Conference on simulation, modelling and optimization*.

Eslaminejad, A., Ziejewski, M., & Karami, G. (2019). An experimental–numerical modal analysis for the study of shell-fluid interactions in a clamped hemispherical shell. *Applied Acoustics, 152*, 110-117. doi: https://doi.org/10.1016/j.apacoust.2019.03.029

Ewins, D. J. (2000). Modal Testing: Theory, Practice, and Application.

Fialko, S. Y., Kriksunov, E. Z., & Karpilovsky, V. S. (2003). A block Lanczos method with spectral transformations for natural vibrations and seismic analysis of large structures in SCAD software. *Computer Methods in Mechanics*, 1-9.

Friswell, M., & Mottershead, J. E. (2013). Finite element model updating in structural dynamics. *Springer Science & Business Media*(38).

Friswell, M. I., Mottershead, J. E., & Ahmadian, H. (2001). Finite element model updating using experimental test data: parametrization and regularization. *Mathematical, Physical & Engineering Sciences 359*, 169-186. doi: 10.1098/rsta.2000.0719

Gaonkar, C. D. (2015). Modal analysis of exhaust system to optimize mounting hanger location. *International Journal of Engineering Research & Technology (IJERT), 4*(03), 24-29.

George, J., Chen, Y., & Shih, H. R. (2013). A DOE based study to evaluate the dynamic performance of an automotive exhaust system. *SAE International*. doi: 10.4271/2013-01-1883

Gere, J. M. (2004). Mechanics of materials. Brooks/Cole-Thomson Learning, 10 Davis Drive, Belmont, CA 94002, USA: *Thomson Learning, Inc.*(Sixth Edition).

Geweth, C. A., Khosroshahi, F. S., Sepahvand, K., Kerkeling, C., & Marburg, S. (2017). Damage detection of fibre-reinforced composite structures using experimental modal analysis. *Procedia Engineering, 199*, 1900-1905. doi: 10.1016/j.proeng.2017.09.128

Girardi, M., Padovani, C., Pellegrini, D., Porcelli, M., & Robol, L. (2018). Finite element model updating for structural applications.

Green, D., & Unruh, W. G. (2006). The failure of the Tacoma Bridge: A physical model. *American Journal of Physics, 74*(8), 706-716. doi: 10.1119/1.2201854

Guo, Z.-C., Li, T., & Zhou, Y.-Y. (2018). Structure-preserving ΓQR and Γ-Lanczos algorithms for Bethe–Salpeter eigenvalue problems. *Journal of Computational and Applied Mathematics, 341*, 12-30. doi: https://doi.org/10.1016/j.cam.2018.03.010

Hanouf, Z., Faris, W. F., & Ahmad, K. (2017). An investigation into NVC characteristics of vehicle behaviour using modal analysis. *IOP Conference Series: Materials Science and Engineering, 184*(1), 1-13. doi: 10.1088/1757-899X/184/1/012027

Huňady, R., Pavelka, P., & Lengvarský, P. (2019). Vibration and modal analysis of a rotating disc using high-speed 3D digital image correlation. *Mechanical Systems and Signal Processing, 121*, 201-214. doi: https://doi.org/10.1016/j.ymssp.2018.11.024

Iranzad, M., & Ahmadian, H. (2012). Identification of nonlinear bolted lap joint models. *Computers and Structures, 96*, 1-8. doi: 10.1016/j.compstruc.2012.01.011

Jaishi, B., & Ren, W.-X. (2005). Structural finite element model updating using ambient vibration test results. *Journal of Structural Engineering, 131*(4), 617-628. doi: 10.1061/(ASCE)0733-9445(2005)131:4(617)

Jenny, C. L., & O'Brien, A. (2001). Welding science and technology. American Welding Society, 550 N.W. LeJeune Road, Miami, FL 33126: *American Welding Society (AWS).*

Kapidžić, Z., Nilsson, L., & Ansell, H. (2014). Finite element modeling of mechanically fastened composite-aluminum joints in aircraft structures. *Composite Structures, 109*, 198-210. doi: http://dx.doi.org/10.1016/j.compstruct.2013.10.056

Kihm, F., Langelier, A., & Munson, K. (2018). Influence of the modal damping on the estimated fatigue life. *Procedia engineering, 213*, 270-281. doi: 10.1016/j.proeng.2018.02.028

Komzsik, L. (2009). What every engineer should know about computational techniques of finite element analysis. 6000 Broken Sound Parkway NW, Suite 300 Boca Raton, FL 333487-2742: *CRC Press, Taylor & Francis Group, LLC.*(2nd edition).

Kumbhar, V., & Patil, D. C. (2017). Structural optimization & durability analysis of VW Bettle cross exhaust muffler system. *International Research Journal of Engineering and Technology (IRJET), 04*(07).

Langer, P., Sepahvand, K., Guist, C., & Marburg, S. (2017). Finite element modeling for structural dynamic analysis of bolted joints under uncertainty. *Procedia engineering, 199*, 954-959. doi: 10.1016/j.proeng.2017.09.199

Lauwagie, T., Strobbe, J., Dascotte, E., Clavier, J., & Monteagudo, M. (2009). Optimization of the dynamic response of a complete exhaust system. *Proceedings of the IMAC-XXVII Feb,* 9-12.

Lemler, K. J., & Semke, W. H. (2013). Application of modal testing and analysis techniques on a sUAV. *Special Topics in Structural Dynamics, 6*, 47-57.

Lupea, I. (2016). An exhaust system lumped model–identification and simulation. *ACTA Technica Napocensis-Series: Applied Mathematics, Mechanics, and Engineering, 59*(4), 359-364.

Maruthi, B. H., Rudresh, M., & Vikram, B. V. (2015). Evaluation of structural integrity of passenger car exhaust system. *International Journal of Research in Advent Technology 3*(2), 172-177.

Mbarek, A., Rincon, A. F. D., Hammami, A., Iglesias, M., Chaari, F., Viadero, F., & Haddar, M. (2018). Comparison of experimental and operational modal analysis on a back to back planetary gear. *Mechanism and Machine Theory, 124*, 226-247. doi: https://doi.org/10.1016/j.mechmachtheory.2018.03.005

Merry, R. J. E., & Steinbuch, M. (2003). Experimental modal analysis of the H-drive. *Report No. DCT 2003.78.*

Modak, S. V. (2014). Model updating using uncorrelated modes. *Journal of Sound and Vibration, 333*(11), 2297-2322. doi: http://dx.doi.org/10.1016/j.jsv.2014.01.013

Modak, S. V., Kundra, T. K., & Nakra, B. C. (2002). Comparative study of model updating methods using simulated experimental data. *Computers & structures, 80*(5-6), 437-447.

Mottershead, J. E., & Foster, C. D. (1991). On the treatment of ill-conditioning in spatial parameter estimation from measured vibration data. *Mechanical Systems and Signal Processing, 5*(2), 139-154.

Mottershead, J. E., & Friswell, M. I. (1993). Model updating in structural dynamics: a survey. *Journal of Sound and Vibration, 167*(2), 347-375.

Mottershead, J. E., Link, M., & Friswell, M. I. (2011). The sensitivity method in finite element model updating: A tutorial. *Mechanical Systems and Signal Processing, 25*(7), 2275-2296. doi: 10.1016/j.ymssp.2010.10.012

MSC Nastran User's Guide. (1999). Advanced CAE applications for professionals. *UAI/NASTRAN User's Guide, Version 20.1*.

MSC Nastran User's Guide. (2014). Element library reference. © *2014 Siemens Product Lifecycle Management Software Inc. All Rights Reserved.*

MSC Nastran User's Guide. (2017). Basic dynamic analysis user's guide. © *2017 Siemens Product Lifecycle Management Software Inc. All Rights Reserved.*

Myrén, M., & Olsson, J. (1999). Modal analysis of exhaust system. (Master of Science in Mechanical Engineering with emphasis of Structural Mechanics), University of Karlskrona/Ronneby, Karlskrona, Sweden.

Nazri, N. A., & Sani, M. S. M. (2017). Finite element normal mode analysis of resistance welding jointed of dissimilar plate hat structure. *IOP Conference Series: Materials Science and Engineering, 257*(1), 1-9. doi: 10.1088/1757-899X/257/1/012059

Nehete, D. V., Modak, S. V., & Gupta, K. (2015). Structural FE model updating of cavity systems incorporating vibro–acoustic coupling. *Mechanical Systems and Signal Processing, 50-51*, 362-379. doi: http://dx.doi.org/10.1016/j.ymssp.2014.05.028

Niu, D., Meng, J., & Li, H. (2018). A new shift strategy for the implicitly restarted refined harmonic Lanczos method. *Journal of Computational and Applied Mathematics, 336*, 458-467. doi: https://doi.org/10.1016/j.cam.2017.12.030

Noorazizi, M. S., Aminudin, B. A., & Faraziah, H. (2014). Technique for hanger location of vehicle exhaust system using finite element method. *Applied Mechanics & Materials, 663*. doi: 10.4028/www.scientific.net/AMM.663.485

Noorazizi, M. S., Aminudin, B. A., & Zetty, M. I. (2013). Systematic FEA study of vehicle exhaust system hanger location using ADDOFD method. *Applied Mechanics and Materials, 392*, 161-164. doi: 10.4028/www.scientific.net/AMM.392.161

Nyein, O., Abu, A. B., & Moe, A. L. (2015). Numerical analysis of vehicle exhaust system to determine hanger location using root mean square value. *ARPN Journal of Engineering and Applied Sciences, 10*(15), 6209-6213.

Omar, R., Rani, M. N. A., Yunus, M. A., Mirza, W. I. I. W. I., & Zin, M. S. M. (2018). Efficient finite element modelling for the investigation of the dynamic behaviour of a structure with bolted joints. *AIP Conference Proceedings, 1952*(1), 1-6. doi: https://doi.org/10.1063/1.5032044

Omar, R., Rani, M. N. A., Yunus, M. A., Starbuck, D. P., Mirza, W. I. I. W. I., & Zin, M. S. M. (2019). Different flexibility formulae for finite element modelling and analysis of the dynamic behaviour of a structure with bolted joints. *AIP Conference Proceedings, 2059*(1), 1-11. doi: https://doi.org/10.1063/1.5085947

Ouyang, H., Rani, M. N. A., Kasolang, S., Othman, M. H., Yunus, M. A., & Mirza, W. I. I. W. I. (2016). Finite element modelling and modal based updating for the dynamic behaviour of a laser spot welded structure. *23rd International Congress on Sound & Vibration (ICSV23)*, 1-8.

Palmonella, M., Friswell, M. I., Mottershead, J. E., & Lees, A. W. (2005). Finite element models of spot welds in structural dynamics: review and updating. *Computers & Structures, 83*(8-9), 648-661. doi: 10.1016/j.compstruc.2004.11.003

Pan, G. Y., & Cao, D. Q. (2016). Vibration characteristics analysis of exhaust system based on optimization of hanger position. *4th International Conference on Sustainable Energy and Enviromental Engineering (ICSEEE 2015)*, 679-685.

Panza, M. A. (2015). A review of experimental techniques for NVH analysis on a commercial vehicle. *Energy Procedia, 82*, 1017-1023. doi: 10.1016/j.egypro.2015.11.861

Pastor, M., Binda, M., & Harčarik, T. (2012). Modal assurance criterion. *Procedia Engineering, 48*, 543-548. doi: 10.1016/j.proeng.2012.09.551

Peter, S., Schreyer, F., & Leine, R. I. (2019). A method for numerical and experimental nonlinear modal analysis of nonsmooth systems. *Mechanical Systems and Signal Processing, 120*, 793-807. doi: https://doi.org/10.1016/j.ymssp.2018.11.009

Petersen, Ø. W., & Øiseth, O. (2017). Sensitivity-based finite element model updating of a pontoon bridge. *Engineering Structures, 150*, 573-584.

Prashant, S. W., Chougule, V. N., & Mitra, A. C. (2015). Investigation on modal parameters of rectangular cantilever beam using Experimental modal analysis. *Materials Today: Proceedings, 2*(4-5), 2121-2130. doi: 10.1016/j.matpr.2015.07.214

Raghu, S. (2010). Finite element modelling techniques in MSC. Nastran and LS/DYNA. *Maverick United Consulting Engineers*.

Rajadurai, S., Kavin, R., Rejinjose, Prabhakaran, & Rajeshraman. (2016). A system approach to dynamic characteristics of hanger rod in exhaust system. *International Journal of Innovative Science, Engineering & Technology (IJISET), 3*(5), 450-461.

Ramsey, K. A. (1983). Experimental modal analysis, structural modifications and FEM analysis on a desktop computer. *Sound and Vibration, 17*(2), 19-27.

Rao, S. S. (2011). Mechanical vibrations. *2004 Pearson Education, Inc., publishing as Prentice Hall*.(5th edition).

Richardson, M. H., & Formenti, D. L. (1985). Global curve fitting of frequency response measurements using the rational fraction polynomial method. *3RD IMAC Conference, Orlando, FL*, 390-397.

Sarmadi, H., Karamodin, A., & Entezami, A. (2016). A new iterative model updating technique based on least squares minimal residual method using measured modal data. *Applied Mathematical Modelling, 40*(23-24), 10323-10341. doi: http://dx.doi.org/10.1016/j.apm.2016.07.015

Schwarz, B. J., & Richardson, M. H. (1999). Experimental Modal Analysis. *CSI Reliability Week, Orlando, FL*, 1-12.

Shojaeifard, M. H., Ebrahimi-Nejad, R., & Kamarkhani, S. (2017). Optimization of exhaust system hangers for reduction of vehicle cabin vibrations. *International Journal of Automotive Engineering, 7*(1), 2314-2324. doi: 10.22068/ijae.7.1.2314

Solanki, A. S., & Jaju, Y. R. (2019). Test correlation of an exhaust system durability failure using enhanced CAE approach. *SAE International*. doi: 10.4271/2019-26-0293

Teughels, A., Maeck, J., & Roeck, G. D. (2002). Damage assessment by FE model updating using damage functions. *Computers & Structures, 80*(25), 1869-1879.

Turner, M. J., Clough, R. W., Martin, H. C., & Topp, L. J. (1956). Stiffness and deflection analysis of complex structures. *Journal of The Aeronautical Sciences, 23*(9), 805-823. doi: 10.2514/8.3664

Waghmare, V. S., Salunkhe, V., Nagrani, R., & Guttal, V. (2017). Analysis of exhaust system mounting scheme. *SAE International 1*(2017-26-0291), 89-96. doi: 10.4271/2017-26-0291

Wai, C. M., Rivai, A., & Bapokutty, O. (2013). Modelling optimization involving different types of elements in finite element analysis. *IOP Conference Series: Materials Science and Engineering, 50*(1), 1-8. doi: 10.1088/1757-899X/50/1/012036

Wei, Z., Das, S., Barr, R., Rohrs, G., Rebandt, R., Wu, X., & Kang, H. (2017). Development of lightweight hanger rods for vehicle exhaust applications. *SAE International* doi: 10.4271/2017-01-1709

Wenzhu, W., Gang, L., Mianhong, C., & Jun, W. (2015). Hanger location layout of automotive exhaust system. *SAE International*. doi: 10.4271/2015-01-2318

Yang, Y., Yang, Y., Sun, Y., Dong, Z., & Zhang, Y. (2016). Optimization of vehicle exhaust system hanger location. *SAE International*. doi: 10.4271/2016-01-0448

Yansong, W., Weiwei, W., & Jianghua, L. (2010). A new technique for hanger position optimization of vehicle exhaust system. *2010 International Conference on Intelligent Computation Technology and Automation, 1*, 246-249. doi: 10.1109/ICICTA.2010.291

Yu, E., Taciroglu, E., & Wallace, J. W. (2007). Parameter identification of framed structures using an improved finite element model-updating method-Part I: Formulation and verification. *Earthquake Engineering and Structural Dynamics, 36*, 619-639. doi: 10.1002/eqe.646

Zahari, S. N., Sani, M. S. M., & Ishak, M. (2017). Finite element modelling and updating of friction stir welding (FSW) joint for vibration analysis. *MATEC Web of Conferences, 90*. doi: 10.1051/ matecconf/20179001021

Zhang, P., & Song, X. (2012). Simulation research of vehicle exhaust system hanger location. *Advanced Materials Research, 482-484*, 347-353. doi: 10.4028/www.scientific.net/AMR.482-484.347

Zhu, T., Tian, W., Weng, S., Ge, H., Xia, Y., & Wang, C. (2018). Sensitivity-based finite element model updating using dynamic condensation approach. *International Journal of Structural Stability and Dynamics, 18*(8), 1-23. doi: 10.1142/S0219455418400047

Živanović, S., Pavic, A., & Reynolds, P. (2007). Finite element modelling and updating of a lively footbridge: The complete process. *Journal of Sound and Vibration, 301*(1-2), 126-145. doi: 10.1016/j.jsv.2006.09.024