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# Magnetohydrodynamic micropolar nanofluid flow over a wedge with chemical reaction

**S N Zulkifli[1], N M Sarif[1], M Z Salleh[1] and A Samsudin[2]**

[1] Pusat Sains Matematik, Universiti Malaysia Pahang, 26300 Kuantan, Pahang, Malaysia
[2] Fakulti Teknologi Kejuruteraan Elektrik dan Elektronik, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia

Corresponding email: norhafizah@ump.edu.my

**Abstract.** In this paper, the flow of electrically conducting micropolar nanofluid flow over a wedge with the effect of chemical reaction is considered. A new boundary condition that assumed nanoparticle flux at the surface to be zero is incorporated in this study. This boundary condition is more physically acceptable due to the condition of nanoparticle volume fraction at the boundary that is passively controlled. In order to obtain the numerical solution, a set of similarity transformation variable was applied to reduce the governing non-linear partial differential equations to non-dimensional ordinary differential equations. The following non-linear ordinary differential equations are solved numerically using Runge-Kutta-Fehlberg (RKF45) method. Based on our findings, concentration profile is enhanced with uprising value of chemical reaction parameter.

## 1. Introduction

The study of boundary layer flow over a wedge has a vital role in many industrial and engineering process especially in geothermal systems, crude oil extraction, storage of nuclear waste, heat exchangers and ground water pollution [1]. The problem was first scrutinized by Falkner and Skan [2] in which the author introduced similarity transformations known as Falkner-Skan solutions that can be applied for fluid with similar flows to reduce the differential boundary layer equations to an ordinary differential equation. Afterwards, the problem of Falkner-Skan flow was evaluated numerically by Hartree [3]. In the study, the author highlighted that an appropriate flow conditions was depended on the mass transfer strength at the wall, $\beta$. Further, the problem of flow field over a wedge was extended to the problem of non-Newtonian fluid by Rajagopal et al. [4] where fluid of a second grade is considered. The study of flow field over a wedge was then considered in many other literatures [5-8]. From the study that has been discussed, the idea of non-Newtonian fluid flow over a wedge is certainly an interesting subject to be deliberated as the fluid could describe different types of fluid with complex nature that is unable to be explain by Newtonian fluid. In the current study, a type of non-Newtonian fluid namely micropolar fluid is considered.

The study of micropolar fluid flow has been analysed in many literatures due to the importance of the investigation in explaining fluid with microsturucture for instance liquid crystal, animal blood, fluid containing additives, polymeric, suspensions and colloidal solutions. The study of micropolar fluid has been introduced by Eringen [9] that takes into account the microscopic effect arises from micromotion and microstructure of the element in the fluid. The microstructure particles are known to be randomly oriented and rotate independently to the motion of the fluid [10]. There have been several studies reported on the flow of micropolar fluid over a wedge for the last decades. Kim [11] and Kim and Kim [12] investigated a laminar flow of micropolar fluid past a wedge with constant wall temperature and constant heat flux. The model presented by Kim [12] was extended and revised by Ishak et al. [13] in their study that considered the effect of variable magnetic field and variable surface temperature. In a different study, Ishak et al. [14] extended the same model to the case of constant wall heat flux and solved the problem using Keller Box method. More recently, Roy and Gorla [15] examined unsteady mixed convection flow of micropolar fluid over a vertical wedge with the influence of thermal radiation and heat absorption or generation. Further, Singh and Kumar [1] determined the effect of chemical reaction on the forced convection flow of an electrically conducting micropolar fluid over a wedge. The study found that the raise of chemical reaction parameter and Schmidt number will increase the rate of mass transfer.

Due to the high demand from industry that require a working fluid with better transfer rate, a new working fluid is employed to enhance the heat transfer rate of the base fluid [16]. In the present study, micropolar fluid is consider as a base fluid and the new working fluid, nanofluid that is known to has high thermal conductivity will increase the transfer rate of micropolar fluid. It is important to note that the study of micropolar nanofluid over a wedge is quite new thus the number of studies on the subject is limited. In a recent study by Zaib and Haq [17] the effect of titanium oxides (TiO2) nanoparticles on mixed convection flow of micropolar fluid that is flowing through a static wedge was observed. The study by them highlighted enhancement in microrotation profiles for the first and second solutions while nanofluid velocity diminishing in the first solution and enhance in second solution. In another study by Zaib et al. [18] the effects viscosity, thermal conductivity and effective Prandtl number on mixed convection of a micropolar liquid is taken into account. This study claimed that multiple solutions is obtainable only for opposing boundary layer flow. In contrast to the previous studies of micropolar nanofluid over a wedge, this study applied Buongiorno's model of nanofluid along with assumption that nanoparticle flux at the boundary is zero.

Generally, chemical reaction is classified into two types of reaction namely homogeneous and heterogeneous reaction. The reaction that occur in a single phase is defined as homogeneous reaction while the reaction that occur at the interphase or includes two or more phase is known as heterogeneous reaction [19]. Potential applications of this effect can be seen during the transfer of energy in a wet cooling tower, drying, flow in a desert cooler and cooling of nuclear reactors [20]. Hence, the study of chemical reaction effect in heat transfer has attracted many researchers over the years. Das and Duari [21] investigated the effect of chemical reaction on micropolar nanofluid that flows over a stretching sheet. In the study, nanofluid concentration is shown to be diminishing with uprising chemical reaction parameter. Another study by Maripala and Naikoti [22] consider the problem of variable chemical reaction and heat source/sink on magnetohydrodynamic micropolar nanofluid flow past a stretching sheet. From the study, it is indicated that temperature profile declines with elevating value of chemical reaction parameter.

In view of the previous literatures, this paper aims to examine the micropolar nanofluid flow over a wedge considering the influence of zero nanoparticle flux at the boundary and chemical reaction. It appears from the aforementioned literatures that investigations on micropolar fluid over a wedge as well as micropolar nanofluid with chemical reaction effect has been done. However, to the best of our knowledge, an extensive study on micropolar nanofluid over a wedge with chemical reaction effect are still not been attempted. Further, the present study is solved numerically using Runge-Kutta-Fehlberg (RKF45) method.

## 2. Mathematical formulation

A two-dimensional, steady and incompressible micropolar nanofluid flow over a wedge was considered in this study. It is assumed that a variable magnetic field $B = B_0 x^{\left(\frac{m-1}{2}\right)}$ is normal to the $x$-axis. The physical representation of this problem can be seen in Figure 1. The magnetic Reynolds number is considered small hence the induced magnetic field can be neglected. Further, the boundary condition utilized an assumption that the nanoparticle volume fraction is passively control besides temperature at the surface is constant. Based on the assumptions, the approximated governing bounding layer equations are [4]:

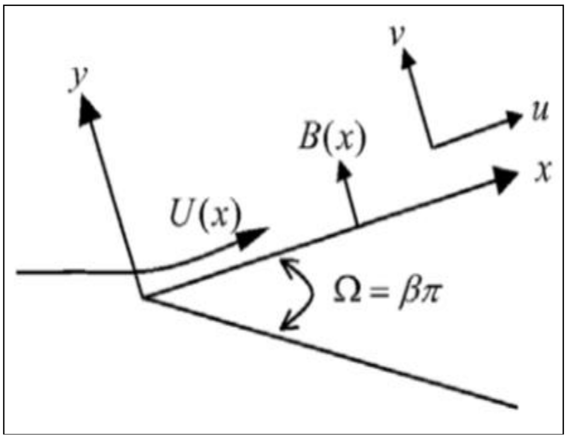

**Figure 1.** Physical model and coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \left(\frac{\mu + k}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \left(\frac{k}{\rho}\right)\frac{\partial N}{\partial y} + \frac{\sigma B^2(x)}{\rho}\left(U - u\right) \tag{2}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \left(\frac{\gamma}{j\rho}\right)\frac{\partial^2 N}{\partial y^2} - \left(\frac{k}{j\rho}\right)\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - K(C - C_\infty) \tag{5}$$

subjected to the boundary conditions

$$u = 0,\quad v = 0,\quad N = -\frac{1}{2}\frac{\partial u}{\partial y},\quad T = T_w,\quad D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0 \ \text{ at } \ y = 0$$

$$u \to U(x),\quad N \to 0,\quad T \to T_\infty,\quad C \to C_\infty \ \text{ as } \ y \to \infty \tag{6}$$

where $u$ and $v$ represents velocity components along $x$ and $y$ direction and $U(x)=ax^m$ denoted external velocity of the fluid. Further, $\sigma, \rho$ and $\alpha$ can be defined as electrical conductivity, density of the base fluid and thermal diffusivity while $\tau, D_B$ and $D_T$ refers to heat capacity, Brownian motion, and thermophoresis respectively. In order to reduce the nonlinear governing partial differential equations to nonlinear ordinary differential equations, a set of similarity solutions is introduced:

$$\psi=\left[\frac{2\nu xU(x)}{m+1}\right]^{\frac{1}{2}} f(\eta),\ \eta=\left[\frac{(m+1)U(x)}{2\nu x}\right]^{\frac{1}{2}} y,\ N(\eta)=U(x)\left[\frac{(m+1)U(x)}{2\nu x}\right]^{\frac{1}{2}} h(\eta)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_\infty} \tag{7}$$

where $\psi$ is the stream function, $\theta$ is dimensionless temperature, $N$ is microrotation or angular velocity and $\phi$ is dimensionless nanofluid concentration. Next, $m=\frac{\beta}{2-\beta}$ with $\beta$ represents Hatree pressure gradient parameter corresponds to $\beta=\Omega\pi$ where $\Omega$ represents angle of the wedge. After employing similarity solution variable (7) to the equations (1) to (5), equation (1) is identically satisfied while equations (2) to (5) are reduced to the following nonlinear ordinary differential equations:

$$(1+K)f'''+f''+\frac{2m}{m+1}(1-f'^2)+Kh'+M(1-f')=0 \tag{8}$$

$$\left(1+\frac{K}{2}\right)h''+h'f-\frac{3m-1}{m+1}hf'-\frac{2}{m+1}K(2h+f'')=0 \tag{9}$$

$$\frac{1}{\Pr}\theta''+f\theta'+N_b\theta'\phi'+N_t\theta'^2=0 \tag{10}$$

$$\phi''+\frac{N_t}{N_b}\theta''+Le(f\phi'-\gamma\phi)=0 \tag{11}$$

subjected to boundary conditions

$$f(0)=0,\ f'(0)=0,\ h(0)=-\frac{1}{2}f''(0),\ \theta(0)=1,\ N_b\phi'(0)+N_t\theta'(0)=0$$

$$f'(\infty)\to 1,\ h(\infty)\to 0,\ \theta(\infty)\to 0,\ \phi(\infty)\to 0 \tag{12}$$

where $\gamma=\frac{K}{a}$ mathematically defined chemical reaction parameter, $M=\frac{2\sigma {B_0}^2}{a\rho(m+1)}$ is magnetic parameter, $\nu=\frac{\mu}{\rho}$ is kinematic viscosity, $\Pr=\frac{\nu}{\alpha}$ is Prandtl number, $N_b=\frac{\tau D_B C_\infty}{\nu}$ is Brownian motion, and $N_t = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu}$ is thermophoresis. Next, dimensionless viscosity ratio or material parameter is represented by $K = \frac{\kappa}{\mu}$. The quantity of physical interest are the local skin friction coefficient that can be defined as

$$C_f = \frac{\tau_w}{\rho U^2(x)/2} \tag{13}$$

where $\tau_w$ is defined as

$$\tau_w = \left[ (\mu + K)\frac{\partial u}{\partial y} + \kappa N \right] \tag{14}$$

By using equations (7), (13) and (14), we obtain

$$\frac{1}{2} C_f \operatorname{Re}_x^{1/2} = \sqrt{\frac{m+1}{2}} \left( 1 + \frac{K}{2} \right) f''(0) \tag{15}$$

## 3. Results and discussion

The transformed nonlinear ordinary differential Equations (8) to (11) subjected to boundary equation (12) are solved numerically using Runge-Kutta-Fehlberg method. The graphical results for several parameters namely chemical reaction parameter $\gamma$, magnetic parameter $M$, Brownian motion $N_b$, thermophoresis $N_t$ and wedge angle parameter $m$ on velocity profile, angular velocity distribution, temperature profiles and concentration profiles were obtained. The parameter Brownian motion describe the random motion of the particles in the fluid, thermophoresis describe the motion of the particles that is induced by temperature gradient, chemical reaction parameter is the dimensionless quantity for the reaction rate that occur in the fluid and magnetic parameter signify the magnetic field that was induced to the flowing fluid. In this study, the value of the parameter are as follows: $\gamma = 0.1,\ Le = 5,\ m = 1/3,\ N_b = Nt = 0.2,\ K = M = \Pr = 1.$

Table 1 displays the comparative skin coefficient friction $C_f \operatorname{Re}_x^{1/2}$ for various value of $m$ between previous study conducted by Chamkha et al. [23] and Ishak et al. [13] and the present investigation. As can be seen, the comparative values are found to be in an excellent agreement thus validate the accuracy of the present results.

Figure 2 and 3 display the velocity profile for various values of $M$ and $m$ respectively. It appears that the velocity profile increase with increasing values of $M$ and $m$. The result found is consistent with the findings from previous literature published by Ishak et al. [14]. This is because, with gradual increase in magnetic parameter, there are a reduced amount of suppression of Lorenz force thus improving the velocity distribution [24]. Next, it is apparent that the velocity profile increase as the value of wedge angle is elevated. This shows that the velocity profile is moving closer to the wedge surface and hydrodynamic boundary layer is lessened [25]. Further, Figure 4 depicts the angular velocity distribution $h(\eta)$ with increasing values of wedge angle parameter $m$. It is observed that as the value of wedge angle parameter rises, the angular velocity distribution increases. Further, Figure 5 show that as the value of chemical reaction parameter is raised, the concentration profile declines. This trend was found to be similar with a results reported by [26-29]. According to Krishnamurthy et al. [30], the declination of concentration profile occur due to the consumption of chemical during the chemical reaction that occur in the system.

Figure 6 indicates that as Brownian motion parameter enhance, concentration profile is found to be decreasing. Next, the effect of thermophoresis parameter on temperature and concentration profile is presented in Figure 7 and Figure 8. Temperature profile and concentration profile are found to be increasing with thermophoresis parameter. The reason is thermophoresis explain the movement of particle from the heated region to the cold region. A large number of nanoparticles shifted from the hot region causes the temperature of the fluid to escalate.

**Table 1.** Comparison value of $C_f \, \mathrm{Re}_x^{1/2}$ for various value of $m$.

| $m$ | Chamkha et al. [23] Implicit Finite-Difference | Ishak et al. [14] Keller-Box | Present Results RKF |
|---|---|---|---|
| 0 | 0.3322 | 0.3321 | 0.3321 |
| 1/3 | 0.7574 | 0.7575 | 0.7575 |
| 1 | 1.2326 | 1.2326 | 1.2326 |

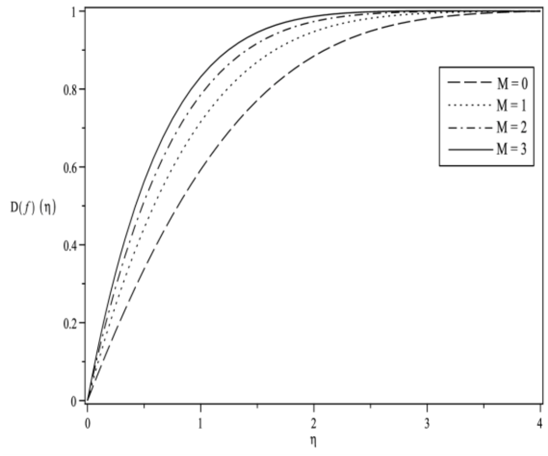

**Figure 2.** Velocity profile for various values of magnetic parameter

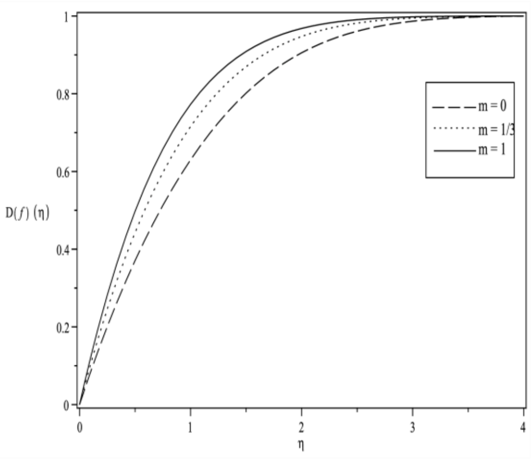

**Figure 3.** Velocity profile for various values of wedge angle parameter

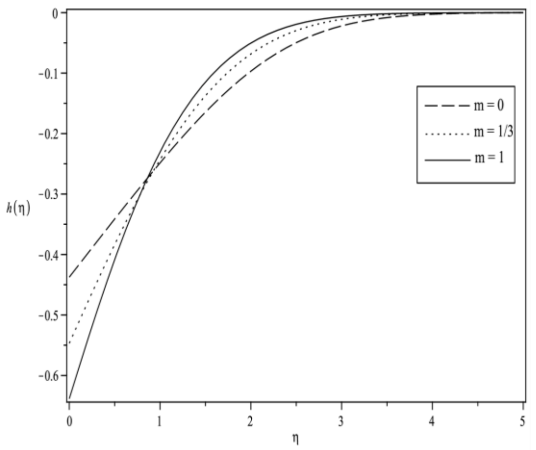

**Figure 4.** Angular velocity distribution for various value of wedge angle parameter

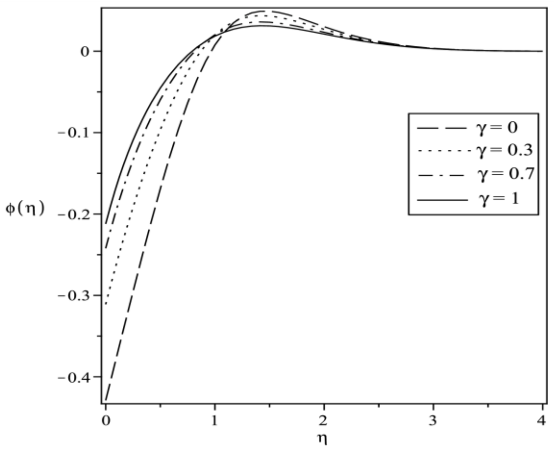

**Figure 5.** Concentration profile for various value of chemical reaction parameter

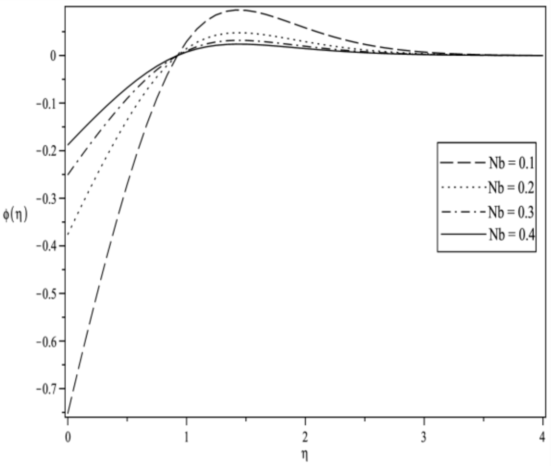

**Figure 6.** Concentration profile for various value of Brownian motion

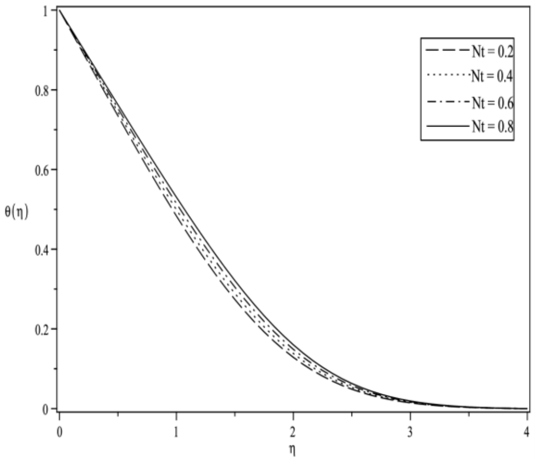

**Figure 7.** Temperature profile for various value of thermophoresis

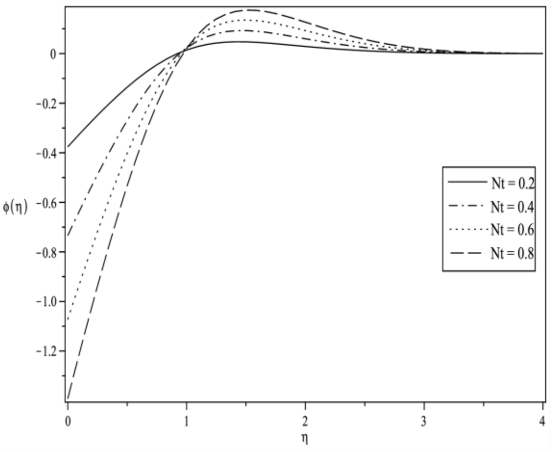

**Figure 8.** Concentration profile for various value of thermophoresis

## 4. Conclusions

The objective of this study is to analyse the influence of chemical reaction on micropolar nanofluid flow over a wedge. A boundary condition that consider nanoparticle flux is zero at the boundary is applied in this study. In conclusion, the present study found that velocity profile increase when the value of magnetic parameter $M$ and wedge angle $m$ are enhanced. Next, angular velocity distribution is observed to be increasing when the value of wedge angle parameter $m$ is increased. Then, temperature profile is shown to be inclining with increasing value of thermophoresis parameter $N_t$. Further, concentration profiles is illustrated to be increasing with thermophoresis parameter $N_t$, while decreasing when the chemical reaction parameter $\gamma$ and Brownian motion parameter $N_b$ enhanced.

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