NEWTON LOOP-NODE ANALYSIS STUDY ON GAS PIPELINE NETWORK

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ABSTRACT

The gas network consists of tree type network and loop type network. There are three types of network analysis, Newton Nodal Method, Newton Loop Method and Newton Loop-Node Method. This research covers on the Newton Loop-Node method. Furthermore, there are many numerical methods that can be used in performing gas network analysis. The hypothesis is Gauss Elimination is more accurate and less iteration number than Gauss Seidel. The objectives of this research are to study on the pressure and flowrate at nodes and pipeline, to determine the suitable flow equation and to compare the iteration number between Gauss Elimination and Gauss Seidel. This research focuses more on on the steady state low pressure compressible fluid and piping route in Gebeng Industrial Area. The analysis involves 5 nodes low pressure gas pipeline network. Lastly, the manual calculation of the network analysis is compare with Fortran 90 program.

ABSTRAK

Rangkaian gas yang digunakan terdiri daripada rangkaian jenis pokok dan rangkaian jenis pusingan. Terdapat tiga jenis analisis rangkaian, Nodal Kaedah Newton, Pusingan Kaedah Newton dan Pusingan-Nodal Kaedah Newton. Kajian ini merangkumi Pusingan-Nodal Kaedah Newton. Selanjutnya, terdapat pelbagai jenis kaedah berangka yang dapat digunakan dalam analisa rangkaian gas. Hipotesis adalah Eliminasi Gauss lebih tepat dan kuarang jumlah iterasi dari Gauss Seidel. Tujuan penyelidikan ini adalah untuk mengkaji tekanan dan laju alir pada node dan saluran paip, menentukan persamaan aliran yang paling sesuai and untuk membandingkan jumlah iterasi antara Eliminasi Gauss dan Gauss Seidel. Penyelidikan ini lebih memfokus kepada bendalir yg berada dalam keadaan stabil dan bertekanan rendah dan paip laluan gas die Kawasan Perindustrian Gebeng. Kajian ini melibatkan talian paip gas 5 node yang bertekanan rendah. Akhir sekali perhitungan manual mengenai analisis rangkaian akan dibandingkan dengan program Fortran 90.

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LIST OF ABBREVIATION

T_n	-	Standard Temperature	K
P_n	-	Pressure at Standard Temperature	bar/kPa
Q	-	Flow rate	m^3/hr
p_1	-	Pressure at Sending Node	bar/kPa
p_2	-	Pressure at Receiving Node	bar/kPa
D	-	Pipe Diameter	mm
K	-	Constant Flow Equation	
S	-	Specific Gravity of Gas	
L	-	Length	m
T	-	Temperature	K
$\varepsilon_{a,i}$	-	Percentage Approximation	

CHAPTER 1

INTRODUCTION

1.0 Background of Study

Nowadays, the effective and more efficient way to transfer gas to consumers is via pipeline. Pipelines are the least understood and least appreciated mode of transport. Pipelines are poorly understood by the general public because they are most often underground and invisible – out of sight, out of mind! Despite the low degree of recognition by the public, pipelines are vitally important to the economic well being and security of most nations. All modern nations rely almost exclusively on pipelines commodities (Liu, 2003).

Liu (2003) pointed out that for the transport of large quantities of fluid (liquid or gas), a pipeline is undisputedly the most favored mode of transportation. The first advantage of pipelines is economical in many circumstances. It can be installed in hostile environment. The cost of using pipelines is also lower than other modes of transportation such as truck or train. Next is low energy consumption. The energy intensiveness of large pipelines is much lower than that of trucks, and even lower than that of rail. The energy intensiveness is defined as the energy consumed in transporting unit weight of cargo over unit distance, in units such as Btu per ton-mile (Liu, 2003).

There are also other advantages of using pipelines, for instance, friendly to the environment, safe for humans, high reliability, convenience, efficient land use and lastly high degree of security.

Distribution is the final step in delivering natural gas to end users. The delivery of natural gas to its point of end use by a distribution utility involves moving smaller volumes of gas at much lower pressures over shorter distances to a great number of individual users. Small diameter pipe is used to transport natural gas from the city gate to individual consumers (Natural Gas Supply Association, 2004).

Gas distribution network consists of two different types that is tree or loop. A tree type network is when the main pipe is not looping. This type of network can be solved directly using straightforward pressure drop calculations for each pipe segment. Another one is loop type network. Obviously, this type of network is a system with loops and because of that, gas flow and direction in each pipe cannot be easily calculated. The solution is only by trial and error method to solve the network calculation (Woh, 2007).

1.1 Problem Statement

The purpose of this research is to study on the gas pipeline network and obtain the most optimum and accurate calculation manually and verify by Fortran 90 program.

1.2 Objectives

The first objective of this research is to study the pressure and flowrate at nodes and pipeline. Next is to determine the suitable flow equation. Finally is to determine the accuracy and to compare the iteration number between Gauss Elimination and Gauss Seidel.

1.3 Scope of Research

Scopes that need to be focused to complete this research:

a) Steady state and low pressure compressible fluid.
b) Piping route in Gebeng industrial area.
c) Verify with Fortran 90 program.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Literature review is the crucial part in this research. It helps to define the most suitable way to analyze the gas piping network. In order to study on gas network, studies on the concept of steady state compressible flow equation, method of loop type network analysis, numerical method of solving linear algebraic equation by using Gauss Elimination and Gauss Seidel and Fortran 90 programming.

2.2 Steady State Compressible Flow Equations

The compressibility of a fluid is, basically, a measure of the change in density that will be produced in the fluid by a specified change in pressure. Gases are, in general, highly compressible whereas most liquids have a very low compressibility. Now, in a fluid flow, there are usually changes in pressure associated, for example, with changes in the velocity in the flow. These pressure changes will, in general, induce density changes which will have an influence on the flow, i.e., the compressibility of the fluid involved will have an influence on the flow. Although the density changes in a flow field can be very important, there exist many situations of great practical importance in which the effects of these density and temperature changes are negligible (Oosthuizen & Carscallen, 1997).

Flow equations are required to calculate the pressure drop in the gas network. There are many flow equations that can be used in gas industry. Thus, they are only capable to a limited range of flow and pipe surface conditions (Poh, 2007). Table 2.1 shows the guidelines for the selection of flow equation for the calculation and Table 2.2 shows the K value calculation for every equation.

Table 2.1: Guidelines for the Selection of Flow Equation

Pressure Range (bar gauge)	**Type of Flow**	**Equation**
High pressure (main supply) > 7.0	Partially turbulent	Panhandle A $Q = 7.57 \times 10^{-4} \frac{T_n}{P_n} \sqrt{\frac{(p_1^2 - p_2^2) D^5}{fSLTZ}}$
High pressure (main supply) > 7.0	Fully turbulent	Weymouth $Q = 11854124.6 \frac{T_n}{P_n} D^{8/3} \sqrt{\frac{(p_1^2 - p_2^2)}{SLT}}$
Medium and high pressure (distribution) > 7.0	Partially turbulent	Panhandle A $Q = 7.57 \times 10^{-4} \frac{T_n}{P_n} \sqrt{\frac{(p_1^2 - p_2^2) D^5}{fSLTZ}}$

Medium and high pressure (distribution) > 7.0	Partially turbulent	Weymouth $$Q = 11854124.6 \frac{T_n}{P_n} D^{8/3} \sqrt{\frac{(p_1^2 - p_2^2)}{SLT}}$$
Medium and high pressure (distribution) 0.75 – 7.0	Partially turbulent	Cox's $$Q = 1.69 \times 10^{-3} \sqrt{\frac{(p_1^2 - p_2^2) D^5}{SL}}$$
Low pressure (distribution) 0 – 0.075	Partially turbulent	Lacey's $$Q = 7.1 \times 10^{-3} \sqrt{\frac{(p_1 - p_2) D^5}{SL}}$$

Table 2.2: K Value Calculation for Every Equation

Equation	**K value calculation**
Lacey's	$K = 11.7 \times 10^3 \frac{L}{D^5}$
Cox's	$K = 206.22527 \times 10^3 \frac{L}{D^5}$
Panhandle A	$K = 18.43 \frac{L}{E^2 D^{4.854}}$
Weymouth	$K = 2590000 \frac{L}{D^{16/3}}$

There is no friction factor for Weymouth, Cox's and Lacey's equation. The specific gravity of gas is assumed to be 0.589. This specific gravity value is subjected to change, depend on the properties of natural gas. The temperature and pressure is assumed as standard pressure, normally used by international gas users in solving network analysis.

2.3 Method of Loop Type Network Analysis

Method use for this loop network analysis is Newton Loop – Node Method. This method is the combination of Kirchhoff's first and second law. Kirchhoff's first law states that the algebraic sum of the flows at any node is zero. This means that the load at any node is equal to the sum of the branch flows into and out of the node. However for the Kirchhoff's second law, states that the pressure drop around any closed loop is zero. A closed loop starts and finishes at the same node so there can be no pressure around the loop.

It is applied where loop equations are transformed into nodal equation, which means that the loop equations have to be transformed to an equivalent set of nodal equation. Therefore the advantage of Nodal and Loop method can be maintained. Newton loop – node equation is

$$J^k x(P)^k = V^k \tag{1}$$

Where J^k is Jacobi matrix, P^k is nodal pressure and V^k is vector nodal. The Jacobi matrix formula as below

$$J = -\begin{bmatrix} (R_1^{-1} + R_4^{-1}) & -R_4^{-1} & 0 \\ -R_4^{-1} & (R_2^{-1} + R_4^{-1} + R_5^{-1}) & -R_5^{-1} \\ 0 & -R_5^{-1} & R_3^{-1} + R_5^{-1} \end{bmatrix} \tag{2}$$

The Vector nodal for each load node is given by the equation

Vector Node 2: $V = (\Delta P_1 - P_1) + \Delta P_4$ (3)

Vector Node 3: $V = (\Delta P_2 - P_1) + \Delta P_4$ (4)

Vector Node 4: $V = (\Delta P_3 - P_1) + \Delta P_5$ (5)

2.4 Numerical Method of Solving Linear Algebraic Equation

2.4.1 Gauss Elimination

There are two methods being use in this research. The first one is Gauss Elimination. It involves combining equations to eliminate unknowns. Although it is one of the earliest methods for solving simultaneous equation, it remains among the most important algorithms in use today and it is the basis for linear equation solving on many popular software packages (Chapra & Canale, 2006).

The procedure consisted of two steps:

1. The equations were manipulated to eliminate one of the unknowns from the equations. The result of this elimination step was that we had one equation with one unknown.
2. Consequently, this equation could be solved directly and the result back-substituted into one of the original equations to solve for the remaining unknown.

This section includes the systematic techniques for forward elimination and back substitution that comprise Gauss elimination. The approach is designed to solve a general set of n equations:

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \ldots + a_{1n} x_n = b_1 \tag{6}$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \ldots + a_{2n} x_n = b_2 \tag{7}$$

$$a_{n1} x_1 + a_{n2} x_2 + a_{n3} x_3 + \ldots + a_{nn} x_n = b_n \tag{8}$$

As was the case with the solution of two equations, the technique for n equations consists of two phases: The first one is Forward Elimination of unknowns. In this step, the unknown is eliminated in each equation. This way, the equations are reduced to one equation and one unknown in each equation. Next is back substitution. In this step, starting from the last equation, each of the unknowns is found.

The first unknown, x_1 is eliminated from all rows below the first row in the first step of forward elimination. The first equation is selected as the pivot equation to eliminate x_1. So, to eliminate x_1 in the second equation, one divides the first equation by a_{11} (hence called the pivot element) and then multiplies it by a_{21}. This is the same as multiplying the first equation by a_{21} / a_{11} to give

$$a_{21}x_1 + \frac{a_{21}}{a_{11}}a_{12}x_2 + ... + \frac{a_{21}}{a_{11}}a_{1n}x_n = \frac{a_{21}}{a_{11}}b_1 \tag{9}$$

Now, this equation can be subtracted from equation (19) to give

$$\left(a_{22} - \frac{a_{21}}{a_{11}}a_{12}\right)x_2 + ... + \left(a_{2n} - \frac{a_{21}}{a_{11}}a_{1n}\right)x_n = b_2 - \frac{a_{21}}{a_{11}}b_1 \tag{10}$$

or

$$a'_{22}x_2 + ... + a'_{2n}x_n = b'_2 \tag{11}$$

where

$$\begin{aligned} a'_{22} &= \quad a_{22} - \frac{a_{21}}{a_{11}}a_{12} \\ &\vdots \\ a'_{2n} &= \quad a_{2n} - \frac{a_{21}}{a_{11}}a_{1n} \end{aligned} \tag{12}$$

This procedure of eliminating x_1 is now repeated for the next equation to the n^{th} equation to reduce the set of equations as

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n = b_1 \tag{13}$$

$$a'_{22}x_2 + a'_{23}x_3 + ... + a'_{2n}x_n = b'_2 \tag{14}$$

$$a'_{32}x_2 + a'_{33}x_3 + ... + a'_{3n}x_n = b'_3 \tag{15}$$

$$a'_{n2}x_2 + a'_{n3}x_3 + ... + a'_{nn}x_n = b'_n \tag{16}$$

This is the end of the first step of forward elimination. Now, for the second step of forward elimination, we start with the second equation as the pivot equation and a'_{22} as the pivot element. So, to eliminate x_2 in the third equation, one divides the second equation by a'_{22} (the pivot element) and then multiply it by a'_{32}. This is the same as multiplying the second equation by a'_{32}/a'_{22} and subtracting it from the third equation. This makes the coefficient of x_2 zero in the third equation. The same procedure is now repeated for the fourth equation till the n^{th} equation to give

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n = b_1 \tag{17}$$

$$a'_{22}x_2 + a'_{23}x_3 + ... + a'_{2n}x_n = b'_2 \tag{18}$$

$$a''_{33}x_3 + ... + a''_{3n}x_n = b''_3 \tag{19}$$

$$a''_{n3}x_3 + ... + a''_{nn}x_n = b''_n \tag{20}$$

The next steps of forward elimination are conducted by using the third equation as a pivot equation and so on. That is, there will be a total of $n-1$ steps of forward elimination. At the end of $n-1$ steps of forward elimination, we get a set of equations that look like

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \tag{21}$$

$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2 \tag{22}$$

$$a''_{33}x_3 + \ldots + a''_{3n}x_n = b''_3 \tag{23}$$

$$\begin{matrix} . & & . \\ . & & . \\ . & & . \end{matrix}$$

$$a_{nn}^{(n-1)}x_n = b_n^{(n-1)} \tag{24}$$

Now the equations are solved starting from the last equation as it has only one unknown.

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}} \tag{25}$$

Then the second last equation, that is the $(n-1)^{\text{th}}$ equation, has two unknowns: x_n and x_{n-1}, but x_n is already known. This reduces the $(n-1)^{\text{th}}$ equation also to one unknown. Back substitution hence can be represented for all equations by the formula

$$x_i = \frac{b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)}x_j}{a_{ii}^{(i-1)}} \quad \text{for } i = n-1, n-2, \ldots, 1 \tag{26}$$

and

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}} \tag{27}$$

2.4.2 Gauss Seidel

The second method used is Gauss Seidel method, also known as the Liebmann method or the method of successive displacement. It is an iterative method a linear system of equations. Assume that

$$[A][X] = [B] \tag{28}$$

Suppose that for conciseness we limit ourselves to a 3 × 3 set of equations. If the diagonal elements are all nonzero, the first equation can be solved for x_1, the second for x_2, and the third for x_3 to yield (Chapra & Canale, 2006)

$$x_1 = \frac{b_1 - a_{12}x_2 - a_{13}x_3}{a_{11}} \tag{29}$$

$$x_2 = \frac{b_2 - a_{21}x_1 - a_{23}x_3}{a_{22}} \tag{30}$$

$$x_3 = \frac{b_3 - a_{31}x_1 - a_{32}x_2}{a_{33}} \tag{31}$$

Chapra and Canale (2006, p. 290) explained that start the solution process by choosing guesses for the *x*'s. A simple way to obtain initial guesses is to assume that they are all zero. The zeros can be substituted into equation (38), which can be used to calculate a new value for $x_1 = b_1/a_{11}$. Then, substitute this new value of x_1 along with the previous guess of zero for x_3 into equation (39) to compute a new value for x_2. The process is repeated for equation (40) to calculate a new estimate for x_3. After that, return to the first equation and repeat the entire procedure until solution converges closely enough to the true values. Convergence can be checked using the criterion

$$|\varepsilon_{a,i}| = \left|\frac{x_i^j - x_i^{j-i}}{x_i^j}\right| 100\% < \varepsilon_s \tag{32}$$

where j and $j - i$ are the present and previous iterations

The convergence properties of the Gauss Seidel method are dependent on the matrix *A*. Namely, the procedure is known to converge if either, *A* is symmetric positive-definite or *A* is strictly or irreducibly diagonally dominant. However, the Gauss Seidel method will sometimes converge even if these conditions are not satisfied.

2.5 Modeling and Simulation of Pipeline Networks for Compressible Fluids

In this research, for the steady state modeling and simulation of pipeline networks for compressible fluids that is gas is using Fortran 90. Fortran is the most widely use programming language in the world for numerical applications. It has achieved this position partly by being on the scene earlier than any of the other major languages and partly because it seems gradually to have evolved the features which its users especially scientists and engineers, found most useful. One of the most important features of Fortran programs is their portability, that is the ease with which they can be moved from one computer system to another (Page, 2005).

As stated by Page (2005), Fortran has become popular and widespread because of its unique combination of properties. Its numerical and input/output facilities are almost unrivalled while those for logic and character handling are as good as most other languages. Fortran is simple enough that you do not need to be a computer specialist to become familiar with it fairly quickly, yet it has features, such as the independent compilation of program units, which allow it to be used on very large applications. Finally, the ease with which existing procedures can be incorporated into new software makes it especially easy to develop new programs out of old ones. However, there are also drawbacks and weaknesses. Such as lacks various control and data structures (Page, 2005).

CHAPTER 3

METHODOLOGY

3.1 Introduction

The methodology for this research computes of selection of flow equation for the analysis, Newton Loop-Node Method network analysis, compare the iteration number between Newton Gauss Elimination and Newton Gauss Seidel, modeling and simulation of the network analysis in Fortran 90 and compare the result of manual calculation and program with Fortran 90.

3.2 Selection of Flow Equation

The gas flow rate in a pipe can be described by many formulae, but none are universal. The effects of friction are difficult to quantity and are the main reason for variations in the flow formulae. A different equation is used depending on the working pressure of the system. Many gas flow equations has been developed and used by the gas industry. Majorities are based on the result of gas flow experiments. For low pressure network operating between 0 – 0.075 bar gauge, Lacey's equation is applied. However, for operating pressure more than 29.4 kPa gauge, Cox's equation is used.

3.3 Newton Loop-Node Method for Network Analysis

The Newton Loop-Node method is applied for the simple low pressure gas network. Figure 3.1 shows the methods for Newton Loop-Node method.

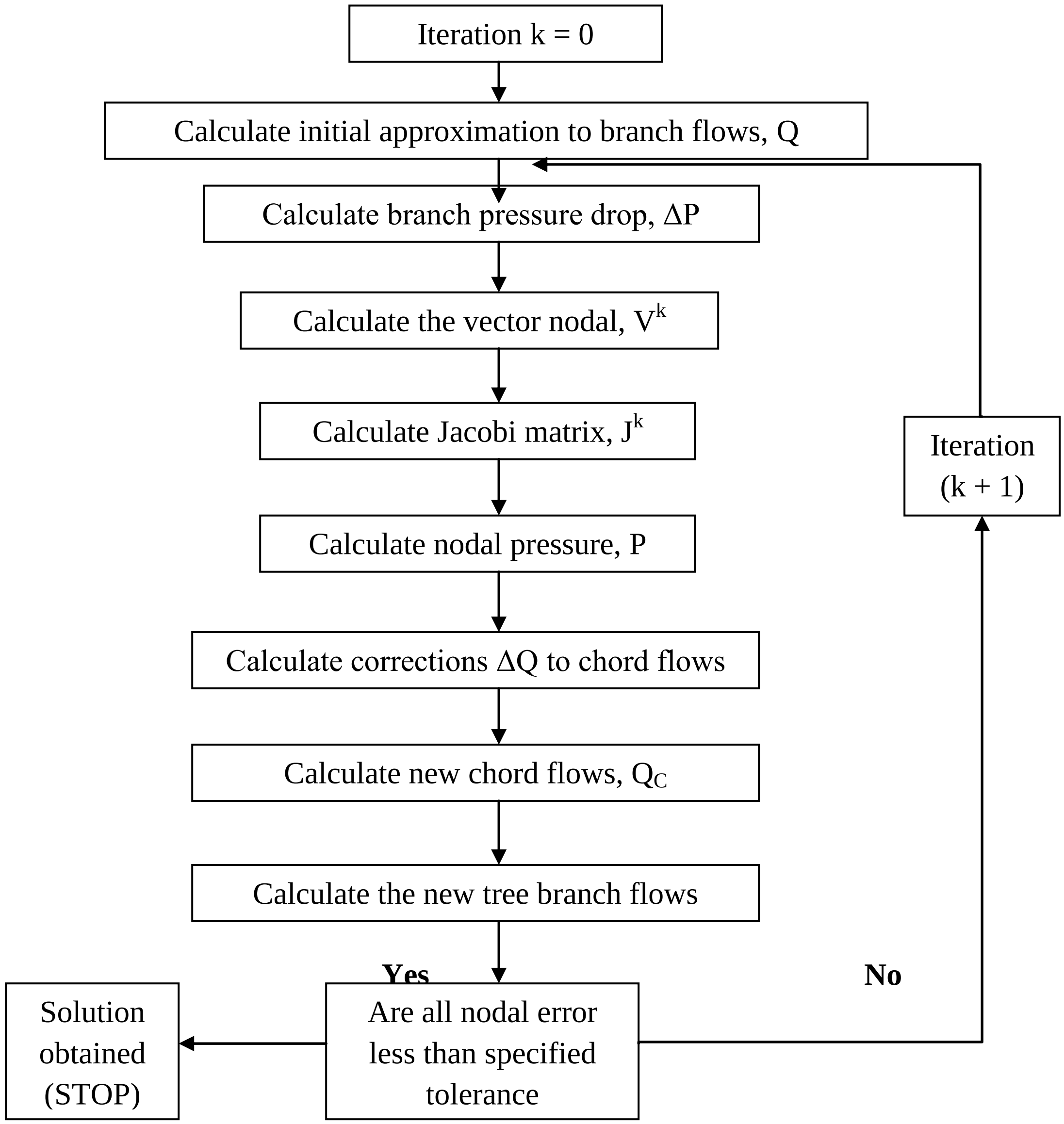

Figure 3.1: Methods for Newton Loop-Node method

3.4 Compare the Iteration Number

The numerical methods used in this research are Gauss Elimination and Gauss Seidel. These methods are used to determine the nodal pressure for the gas piping network route. Lastly, compare the iteration number between these two methods and determine the best method.

3.5 Modeling and Simulation in Fortran 90

Fortran 90 is used for simulation and analysis of gas piping network route that has been selected. Guide for Fortran 90 is referred for all steps to run this programming language software.

3.6 Compare the Result

The manual calculation is compared with programming by Fortran 90 so that can verify the result as the programming must be more accurate than manual calculation.