NEWTON LOOP METHOD IN GAS PIPELINE NETWORK

EMMA OLGA TAIMIN GINDALAN

A thesis submitted in fulfillment
of the requirements for the award of the degree of
Bachelor of Chemical Engineering (Gas Technology)

Faculty of Chemical Engineering & Natural Resources
Universiti Malaysia Pahang

APRIL 2010

ABSTRACT

Pipeline networks are used extensively in all countries for transportation and distribution of natural gas and other light petroleum products for industrial and domestic use. However, one of the challenges of operating the pipeline network is how the operational procedures can be adjusted to meet the dynamic and future demands of customers. Thus, pipeline network simulation is an essential tool for control and operations in gas distribution systems because it can be used to stimulate and analyse networks behavior under different operating conditions. Analyses of pressures and flows are needed whenever significant changes in patterns and magnitudes of demand or supplies occur. Apart from that, there are many numerical methods that can be used in performing the gas pipeline network analysis. Thus, the hypothesis of the method states that the Newton Gauss Elimination method is faster and more accurate than the Newton Gauss Seidel method. The objectives of this research are to estimate the values of pressure drop of gas pipeline network by using Newton Loop method and to determine the accuracy and to compare the iteration number between Newton Gauss Elimination and Newton Gauss – Seidel. A case study was performed in low pressure and steady state condition. The case study covers the Gebeng Industrial Phase I and II only. FORTRAN program is developed to verify the manual calculation. Newton Gauss Elimination is more accurate than Newton Gauss – Seidel because Newton Gauss Elimination is a direct method while Newton Gauss – Seidel is an iterative method. This means Newton Gauss Elimination provides a straightforward solution while Newton Gauss – Seidel generates a sequence of successive approximation to the exact solution. The result from manual calculation and FORTRAN is approximately same.

ABSTRAK

Rangkaian paip digunakan secara menyeluruh di semua negara untuk tujuan pengangkutan dan juga pengagihan sumber gas asli dan minyak untuk keperluan industri dan domestik. Tetapi, salah satu cabaran untuk mengoperasikan rangkaian paip adalah bagaimana kaedah operasi yang boleh disesuaikan untuk memenuhi kehendak pengguna. Dengan demikian, simulasi adalah alat penting untuk kawalan dan operasi dalam system pengedaran gas kerana ia boleh digunakan dalam keadaan pengendalian yang berbeza. Analisis tekanan dan arus adalah diperlukan setiap kali ada perubahan pada pola permintaan. Dengan demikian, hipotesis telah dibuat dengan menyatakan bahawa *Newton Gauss Elimination* adalah lebih cepat dan jitu daripada kaedah *Newton Gauss – Seidel*. Objektif kajian ini adalah untuk menganggarkan nilai penurunan tekanan rangkaian dengan menggunakan kaedah *Newton Loop* dan membandingkan pengiraan diantara kaedah *Newton Gauss Elimination* dan *Newton Gauss – Seidel*. Kajian dalam kes ini dijalankan pada tekanan rendah dan dalam keadaan stabil dan meliputi kawasan industri Gebeng fasa I dan II sahaja. Program FORTRAN dibuat untuk mengesahkan pengiraan manual. *Newton Gauss Elimination* adalah lebih tepat daripada kaedah *Newton Gauss – Seidel* kerana . *Newton Gauss Elimination* adalah kaedah langsung, sedangkan *Newton Gauss - Seidel* adalah kaedah iteratif. Ini bermakna Newton Eliminasi Gauss memberikan penyelesaian mudah sedangkan Newton Gauss - Seidel menghasilkan susunan pendekatan berturut-turut untuk penyelesaian yang tepat. Hasil dari perhitungan manual dan FORTRAN sekitar sama.

TABLES OF CONTENTS

LIST OF TABLE

LIST OF FIGURES

LIST OF SYMBOLS/ABBREVIATIONS

P	-	Pressure
T	-	Temperature
K	-	Resistance Coefficient
J^k	-	Jacobi Matrix
ΔP	-	Pressure Drop
δq_1	-	Correction Loop Flow
$F(q_1)$	-	Loop Error
Z	-	Compressibility Factor
NGE	-	Newton Gauss Elimination
NGS	-	Newton Gauss-Seidel

LIST OF EQUATIONS

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Pipeline networks are used extensively in all countries for transportation and distribution of natural gas and other light petroleum products for industrial and domestic use. As well in Malaysia, many gas pipeline networks have been developed. As in January 2004, network of gas pipeline in Malaysia covering a total of 1193.9 Kilometers (831.7 kilometers completed) is constantly expanding to reach a larger population. (Gas Malaysia).

However, one of the challenges of operating the pipeline network is how the operational procedures can be adjusted to meet the dynamic and future demands of customers (Brindle et al., 1993). Thus, pipeline network simulation is an essential tool for control and operations in gas distribution systems because it can be used to simulate and analyse networks behavior under different operating conditions. Analyses of pressures and flows are needed whenever significant changes in patterns and magnitudes of demand or supplies occur. (Walski et al., 1990). In the absence of such analyses, the operational procedures may not be optimal, resulting in unnecessarily high operating cost. (Weerapong et al., 1998). Therefore, pipeline network simulation is the better way to overcome this problem.

Analysis of gas networks makes use of models of gas flow in pipes that have been developed based on the physical laws controlling the processes of flow. Analysis allows us to predict the behavior of gas network system under different condition. Such prediction can then be used to guide regarding the design and operation of the real system. And gas networks can be classified in two different types namely tree or looped. (GERG). A tree network is one where the pipes or mains are not looped and can be directly solved by using straightforward pressure drop calculations in each pipe segment. Meanwhile, a looped network is obviously a system and because of the looped nature, gas flow and direction cannot easily be calculated. The solution is only either by trial and error method or an iterative approach. Fundamental of equation for describing steady state gas flow is derived based on Bernoulli's equation, and the equations commonly used in practice. The loop and node models are formulated with the help of Kirchhoff's laws. In mathematical terms, the steady-state simulation problem of gas networks consists of solving a given system of non-linear algebraic equations. The Newton method is commonly used for this purpose.

1.2 Problem Statement

Before applying the Newton Loop method in gas pipeline network, several factors need to be considered. The factors are first, flow equations for gas flowing in pipes, the flow equations for pipeline gas describe the relation among the gas flow rate, the pressures at the two pipe ends, and related gas properties, pipe characteristics and operating conditions. Second is the pressure drop, pressure drop is a term used to describe the decrease in pressure from one point in a pipe or tube to another downstream. And the last one is numerical solution of linear algebraic equation. The commonly used numerical methods are Newton Gauss Elimination method and Newton Gauss Seidel method. In this study, it helps users to understand both numerical methods.

1.3 Objectives

The main objective of this study is to estimate the values of pressure and flowrate in gas pipeline by using Newton loop method.

In addition, the other objective of this study is to determine the accuracy and to compare the iteration number between Newton Gauss Elimination and Gauss – Seidel Method.

1.4 Scope of Study

In this study, software for gas pipeline network system will be needed, that is FORTRAN. This software can be used to simulate the gas distribution network systems. 2, 3, 4 loop gas network system will be used to perform this simulation program.

Next, studies on numerical methods, lacey equation and Kirchhoff's Laws will be done due to the different numerical methods that will be implemented into the simulator software (FORTRAN).

Then, the final step is to study the way of performing the network analysis. In performing the network analysis, Newton Loop method in steady state condition, low pressure and several equations of flow will be used. Network is in a steady state when values of the quantities characterizing the flow of gases in the system are independent of time and the system is described by a set of nonlinear algebraic equations. (Osiadacz, 1987). In steady state analysis, the pressure of the nodes and the flow rate in the pipes must satisfy the flow equation and the value of load node and source node must fulfill the Kirchhoff's Laws. (Lewandowski, 1994).

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Because of the large scale and the complexity of the network, the simulation of a natural gas pipeline network become complicated (Lewandowski). The gas pipeline network is described as a system of partial differential equation which must be solved sufficiently fast to make the solution applicable for real- time operation of the gas transmission system. Unfortunately, modeling of gas pipeline network consisting of many connected pipeline segment is more complicate. Therefore, to perform a Newton Loop method in gas pipeline network, studies on the concepts of gas network, Newton Loop method, numerical solution of linear algebraic equation and network analysis are needed.

2.2 Gas Network

Generally, the purpose to build the gas network system is for transferring natural gas in high capacity, which normal storage tanks cannot fulfill the capacity of the gas.

Gas distribution network can be classified in two different types that namely tree or loop (Steven et al., 1998). Kirchhoff was firstly proposed the concept of the tree and loop as mathematical entities in the connection with the definition of fundamental circuits used in the analysis of electric circuit.

A tree type network is much easier to calculate compare to loop type network. Because of the looped nature, gas flow and direction in each pipe cannot be easily calculated. A tree type network can be solved directly using straightforward pressure drop calculations for each pipe segment. Meanwhile, for the loop type network, the solution is only by the trial and error method with the help of some numerical solution of linear algebraic equation.

2.3 Common Flow Equations

Common flow equation can be expressed in a general form (Osiadacz, 1987). For any pipe k, the pipe flow equation from node *i* to *j* can be expressed as

$$\emptyset[(Q_n)_k] = K_k\,(Q_n^{m1})_k \qquad (1)$$

Where $\emptyset[(Q_n)_k]$ = the flow function for pipe k

K_k = the pipe constant for pipe k

$(Q_n)_k$ = the flow in pipe k

m1 = the flow exponent = 2 for low pressure networks

= 1.848 for medium pressure networks

= 1.854 for high pressure networks.

For the low pressure version of the flow equation,

$$\emptyset[(Q_n)_k] = K_k\,(Q_n^2)_k = p_i - p_j = \Delta p_k \qquad (2)$$

Where;

Δp_k = the pressure drop for pipe k,

p_i = the absolute pressure at node i, (i = the sending node of pipe k)

p_j = the absolute pressure at node j. (j = the receiving node of pipe k)

For the medium and high pressure version of the flow equation,

$$\emptyset[(Q_n)_k] = K_k\,(Q_n^{m1})_k = P_i - P_j = \Delta P_k \quad (3)$$

Where

$P_i = p_i^2$ and $P_j = p_j^2$

The equations for low pressure and for medium and high pressures can be rearranged:

$$\emptyset[\Delta p_k] = (Q_n)_k = (\Delta p_k / K_k)^{\frac{1}{2}} \quad (2.1)$$

$$\emptyset[\Delta P_k] = (Q_n)_k = (\Delta P_k / K_k)^{\frac{1}{m1}} \quad (2.2)$$

Equations (2.1) and (2.2) can be rearranged to the form as below after taking account of the fact that a change of the flow direction of the gas stream may take place in the network.

$$(Q_n)_k = S_{ij}\left(\frac{S_{ij}\,(p_i - p_j}{K_k}\right)^{\frac{1}{2}} \quad (2.3)$$

$$(Q_n)_k = S_{ij}\left(\frac{S_{ij}\,(p_i - p_j}{K_k}\right)^{\frac{1}{2}} \quad (2.4)$$

Where $S_{ij} = 1\ if\ P_i > P_j\left(p_i > p_j\right), S_{ij} = -1\ if\ P_i < P_j\left(p_i < p_j\right)$

2.3.1 Selection of Flow Equations

To calculate the pressure drop in the gas network system, flow equations are required. Recently many gas flow equations have been developed and a number have been used by the gas industry. Majorities of them are based on the result of gas flow experiments. Thus, they are only capable to limited range of flow pipe surface condition. Table 2.1 shows the guideline to the selection to selection of a gas flow equation for the distribution calculation system (Wilson, 1982).

Table 2.1: Guidelines to Selection of a flow equation for Distribution System Calculation

Type of	**Predominant Type**	**Equation Used**	**Range of Capacity**
High pressure utility supply mains	Partially turbulent	Panhandle A	Relatively good, slightly optimistic approximation for Smooth pipe Flow Law at Reynolds number > 30000
High pressure utility supply mains	Fully turbulent	Weymouth	Good approximation to Fully Turbulent Flow Law for clears rough commercial pipe of 10 to 30 inch diameter
Medium and high pressure distribution	Partially turbulent	Panhandle A	Relatively good, slightly optimistic approximation for Smooth pipe Flow Law at Reynolds number > 30000
Medium and high pressure distribution	Partially turbulent	Weymouth	Very conservative for pipe of less than 20 inch diameter
Medium and high pressure distribution	Partially turbulent	Cox's	Pressure range > 5 psi, Velocity < 20 m/s in all pipes
Low pressure distribution	Partially turbulent	Pole's	Good approximation to Smooth pipe Flow Law for pipe of 4 inch diameter or smaller

There are six equations mainly used in gas distribution system (Piggott et al., 2002). The equations are, Lacey's equation, Pole's equation, Cox's equation, Polyflo equation, Panhandle 'A' equation and Weymouth equation (Schroeder et al., 2001).

These questions are shown in the table 2.2. For the low pressure network, Lacey's and Pole's equations are used. Cox's and Polyflo equation are flow equations for medium pressure network. Panhandle 'A' and Weymouth equation are flow equations for high pressure network. For Weymouth, length L and D is in meter (m) and pressure P is in Pascal (Pa).

Table 2.2: Flow Equations

Flow Equations	**Equation**	**K value Calculation**
Lacey's/Pole's	$Q = 7.1 \times 10^{-3} \sqrt{\frac{(p_1 - p_2)D^5}{SL}}$	$K = 11.7 \times 10^3 \frac{L}{D^5}$
Cox's	$Q = 1.69 \times 10^{-3} \sqrt{\frac{(p_1^2 - p_2^2)D^5}{SL}}$	$K = 206.2252 \times 10^3 \frac{L}{D^5}$
Polyflo	$Q = 7.57 \times 10^{-4} \frac{T_n}{P_n} \sqrt{\frac{(p_1^2 - p_2^2)D^5}{fSLT}}$	$K = 27.24 \frac{L}{E^2 D^{4.848}}$
Panhandle 'A'	$Q = 7.57 \times 10^{-4} \frac{T_n}{P_n} \sqrt{\frac{(p_1^2 - p_2^2)D^5}{fSLTZ}}$	$K = 18.43 \frac{L}{E^2 D^{4.854}}$
Weymouth	$Q = 11854124.6 \frac{T_n}{P_n} D^{8/3} \sqrt{\frac{(p_1^2 - p_2^2)D^5}{SLT}}$	$K = 2590000 \frac{L}{D^{16/3}}$

Assumptions have been made for all of the six equations above. This is to avoid confusions of the users. Table 2.3 shows the limitations and assumptions made for all of the flow equations.

The fraction factor, f can be calculated using the equations given in the Table 2.3. There is no fraction factor for Cox's and Weymouth, because the value is already inserted into the flow equations. In this research, the specific of gas is assumed to be 0.589 and the compressibility factor, Z is assumed 0.95. These two values are subjected to change, depend on the type of natural gas (natural gas properties) and the type of pipe used. The temperature and pressure is assumed as standard temperature and standard pressure, normally used by international gas users in solving network analysis. The efficiency factor normally varies between 0.8 and 1 for most gas pipes (Aylmer, 1980). The actual flow in a pipe will be 80% of the flow predicted (Osiadacz, 1987). So, the efficiency factor, E in this research is assumed to be 0.8.

Table 2.3: Limitations and Assumptions Made for the Flow Equation

Flow Equations	Pressure Range (bar gauge)	Assumption Made
		Fraction factor. F Specific gravity of gas, S Temperature, T and Normal Temperature, T_n Normal Pressure, P_n Compressibility factor, Z Efficiency factor, E
Lacey's/Pole's	0 – 0.075	$f = 0.0044\left(1 + \frac{12}{0.276D}\right)$ $S = 0.589$
Cox's	0.75-7	$S = 0.589$
Polyflo	0.75-7	$\sqrt{\frac{1}{f}} = 11.98 \times E\left(\frac{SQ}{D}\right)^{0.076}$ $S = 0.589$ $T = 288K$
Panhanle'A'	>7	$\sqrt{\frac{1}{f}} = 14.94 \times E\left(\frac{SQ}{D}\right)^{0.073}$ $S = 0.589$ $T_n = T = 288K, P_n = 1.01325\ bar$

Weymouth	>7	$S = 0.589$ $T_n = T = 288K, P_n = 1.01325\ kPa$

2.3.2 Kirchhoff's First Law

Kirchhoff's first law states that the algebraic sum of the flows at any node is zero (Osiadacz, 1987). This means that the load at any node is equal to the sum of brunch flows into and out of the node.

$Q_1 + Q_2 + Q_3 \ldots + Q_n = Load$

$Load - Q = 0$

$Load = Q$

(2.5)

Where, $Q_1 + Q_2 + Q_3 \ldots + Q_n = Q$

Q = total flow in the branches, depend on the flow direction

Load = demand in the load nodes,

The nodal equation, equation (2.5) can be expressed in matrix form:

$L = A_1Q$

(2.6)

Where: L = vector of loads at the nodes,

Q = vector of flows in the branches,

A_1 = reduced branch-nodal incidence matrix.

The pressure drop in the branches can be related to the nodal pressures. The nodal pressure drop equation can be expressed in matrix form:

$\Delta P = - A^T P$

(2.7)

Where: ΔP = vector of pressure drops in the branch,

P = vector of nodal pressures,

A^T= transpose of branch-nodal incidence matrix.

From equation (2.2),

$$Q = \emptyset'(\Delta P) \tag{2.8}$$

Where $\emptyset'(\Delta P)$ = vector of pressure drop functions.

Substituting for ΔP from equation (2.7), equation (2.8) becomes

$$Q = \emptyset'(-A^T P) \tag{2.9}$$

Substituting for Q from equation (2.9), equation (2.6) becomes

$$L = A_1[\emptyset'(-A^T P)] \tag{2.10}$$

2.3.3 Kirchoff's Second Law

Kirchhoff's second law states that the pressure drop around any closed loop is zero (Osiadacz, 1987). This means that there is no pressure drop around the loop since the closed loop starts and finishes at the same node.

$$\Delta P_1 + \Delta P_2 + \Delta P_3 + \cdots + \Delta P_n = 0$$

$$\Delta P_T = 0 \tag{2.11}$$

Where, $\Delta P_1 + \Delta P_2 + \Delta P_3 + \cdots + \Delta P_n = \Delta P_T$

ΔP_T= pressure drop in the branches, depend on the loop direction

The loop equation, equation (2.11) can be expressed in matrix form:

$$B\Delta P = 0 \tag{2.12}$$

Where: ΔP = vector of pressure drops in the branches,

B = branch loop incidence matrix

Rearrange equation (2.8), give

$$\Delta P = \emptyset'(Q) \tag{2.13}$$

Where, $\emptyset'(Q)$ = vector of flow functions.

Substituting for ΔP from equation (2.13), equation (2.12) become

$$B[\emptyset(Q)] = 0 \tag{2.14}$$

2.3.4 Newton Loop Method

The set of loop equations that describes a gas network is shown as below (Hoeven, 1992).

$$B[\emptyset'(Q)] = 0 \qquad (2.15)$$

Where: B = branch loop incidence matrix,

0= zero vector, of dimension k,

ΔP = vector of pressure drop in the branches, of dimension m,

$\emptyset'(Q)$ = vector of flow functions, of dimension m.

Equation (2.15) is a mathematical representation of Kirchhoff's second Law which states that the sum of the pressure drops around any loop is zero. The loop method requires that a set of loops in the network be defined. An initial approximation is made to the branch flows ensuring that a flow balance exists at each node. Since the branch flows are approximations to their true values, a loop flow is introduced. This loop flow is the flow correction to be added to the branch flow approximations to yield the true values. In general, the branch flows are a function of the initial approximations and of all the loops flows, given like equation below.

$$Q = Q^0 + B^T q \qquad (2.16)$$

Where q = vector of loop flows of dimension k, (k is the number of loops)

Q = the branch flows,

Q^0 = initial branch flow approximations.

In the loop method, the iteration the left hand side of equation (2.14) will not be zero. The branch flows are only approximations of their true values and the pressure drops calculated from these flows will not summate to zero around each loop. This introduces a loop error into each loop which is a function of all loop flows and is denoted as f (q). There is a loop error for each loop and this set of errors is represented by:

$$F(q) = \begin{bmatrix} f_1(q_1, q_2, \dots q_k) \\ f_2(q_1, q_2, \dots q_k) \\ \dots \dots \dots \dots \dots \dots \\ f_k(q_1, q_2, \dots q_k) \end{bmatrix}$$

Where *F* denotes as a vector of functions.

2.4 Numerical Solution of Linear Algebraic Equation

Numerical methods for solving systems of linear equations fall into two general classes; they are the direct methods and the iterative methods. Direct methods lead to an exact solution in a finite number of steps if a round of error is not involved. Iterative method leads to an approximation that is acceptably close to the exact solution by performing an infinite number of arithmetic operations.

2.4.1 Gauss Elimination Method

This method is one of the earliest methods for solving simultaneous equations and it remains among the most important algorithms in use today. It is the basic for linear equation solving a many popular software packages. Its advantage is having a higher precision but the disadvantage is possible division by zero. The Gaussian elimination procedure is as follow. Assume a linear system of 3x3 equations as shown as the matrix form below.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ b_{21} & b_{22} & b_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \tag{2.17}$$

The first step is to eliminate one of the unknowns from the equations. This step is called Forward elimination. The result of this elimination step is the forming of one equation with one unknown. While the second step is solving the equation directly and the result back substituted into one of the original equations to solve the remaining unknown. The advantage of this method is it having a higher precision and disadvantage is possible division by zero (Ferziger, 1981). Assuming equations (2. 17) has initial values as below.

$$\begin{bmatrix} 3 & -0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

Separate the above equations into three equations.

$$3X_1 - 0.1X_2 - 0.2X_3 = 7.85 \tag{2.18}$$
$$0.1\ X_1 + 7\ X_2 - 0.3\ X_3 = -19.3 \tag{2.19}$$
$$0.3\ X_1 - 0.2\ X_2 + 10\ X_3 = 71.4 \tag{2.20}$$

Use the Gauss elimination method to solve the equations (2.18) to (2.20).

1) Forward elimination

The procedure is multiply equation (2. 32) by 0.1/3 and subtracts the result from equation (2.19) to get

$7.00333\ X_2 - 0.293333\ X_3 = 19.5617$

Then multiply equation (2.32) by 0.3/3 and subtracts the result form equation (2. 20) to eliminate X1. After these operations, the set of equations is

$$3X_1 - 0.1X_2 - 0.2X_3 = 7.85 \tag{2.21}$$
$$7.00333\ X_2 - 0.293333\ X_3 = -19.5617 \tag{2.22}$$
$$-0.190000X_2 + 10.0200X_3 = 70.6150 \tag{2.23}$$

Remove X_2 from equation (2.23). To accomplish this, multiply equation (2. 22) by -0.190000/7.0033 and subtract the result from equation (2. 23). An upper triangular is formed after X_2 from the equation (2.23).

$$3X_1 - 0.1X_2 - 0.2X_3 = 7.85 \tag{2.24}$$
$$7.00333X_2 - 0.293333X_3 = -19.5617 \tag{2.25}$$
$$10.0200X_3 = 70.0843 \tag{2.26}$$

2) Back substitution

Equation (2. 26) can be solved by solving the X_3.

$$X_3 = \frac{70.0843}{10.0200} = 7.00003 \tag{2.27}$$

Use equation 2.27 to solve equation 2.25

$$X_2 = \frac{-19.5617 + 0.293333(7.00003)}{7.00333} = -2.500 \tag{2.28}$$

Finally, solve X_1 by subtract equation (2. 27) and equation (2. 28) into equation (2.24).

$$X_1 = \frac{7.85 + 0.1(-2.500) + 0.2(7.00003)}{3} = 3.00$$

2.4.2 Gauss Seidel Method

This method is the most generally used iterative methods. Assume the equation (2.17) is given. If the diagonal elements are all nonzero, the first equation can be solved for X_1, the second for X_2 and the third for X_3 to yield