NUMERICAL METHOD APPROACH FOR MAGNETOHYDRODYNAMIC RADIATIVE FERROFLUID FLOWS OVER A SOLID SPHERE SURFACE

by

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In this paper, the theoretical study on the laminar boundary-layer flow of ferrofluid with influences of magnetic field and thermal radiation is investigated. The viscosity of ferrofluid flow over a solid sphere surface is examined theoretically for magnetite volume fraction by using boundary-layer equations. The governing equations are derived by applied the non-similarity transformation then solved numerically by utilizing the Keller-box method. It is found that the increments in ferroparticles (Fe_3O_4) volume fraction declines the fluid velocity but elevates the fluid temperature at a sphere surface. Consequently, the results showed viscosity is enhanced with the increase of the ferroparticles volume fraction and acts as a pivotal role in the distribution of velocity, temperature, reduced skin friction coefficient, and reduced Nusselt number of ferrofluid.

Key words: *ferrofluid, magnetohydrodynamic, thermal radiation, solid sphere, magnetite, free convection*

Introduction

The magnetic nanofluid or better known as ferrofluid contains magnetic nanoparticles (ferroparticles) such as iron oxide compounds with the general formula of ferrite, $MO \cdot Fe_2O_3$ where M is denoted by Cu, Fe, Mg, Mn, and Ni and suspended in a liquid carrier, Rosensweig *et al.* [1]. The black iron oxide or called magnetite, Fe_3O_4, is classified in non-metallic nanofluids has been discovered most satisfactory in practice, Papell [2]. In addition, ferrofluid permanently magnetized in the presence of magnetic field and exhibit superparamagnetism. There are several methods to synthesize the ferrofluid because it is not found in nature such as decomposition of the iron carbonyl, ball milling and co-precipitation. Nowadays, electronic devices such as a loudspeaker use ferrofluid to prevent the voice coil from overheating and to improve the sound quality, Tsuda *et al.* [3]. The ferrofluid flow behaviour in the existence of the magnetic field shows the MHD effect that influences the ferrofluid flow and its thermophysical properties [1]. In this study, the MHD effect is taking into account for the mathematical model formulation where the effects of polarization and electric conductivity can be neglected in ferrofluid, Blums [4].

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The fluid behaviour will change according to the fluid-flow at different geometries and assumptions, Darus [5]. Many researchers [6-11] have done extensive theoretical studies about the MHD flow of ferrofluid on various surfaces. From this literature discovered that the magnetite nanoparticles volume fraction (ferroparticles volume fraction) with water-based, thermal radiation and magnetic parameters are the factors to enhance the heat transfer of ferrofluid. Nevertheless, a few papers had merely investigated the ferrofluid flow and heat transfer over bluff bodies surfaces. Consequently, the present work is focused on the laminar boundary-layer flow and heat transfer over a solid sphere. Besides, the experiments by [12-15] investigated the heat transfer, viscosity and thermophysical properties of water-based magnetite that provided important references for explaining the potential mechanism in enhancing the thermal conductivity of ferrofluid and show remarkable agreement between theory and experiment.

It should be emphasized that the mathematical model is extended from Alwawi [16] with consideration thermal radiation and MHD flow. The governing equations are extended by implementing the Tiwari and Das model [17] and numerically solved by Keller-box method. Hence, the effect of the magnetic field with the presence of Lorentz force as a body force and buoyancy force is reported graphically.

Mathematical model

The ferrofluid flow starts from a lower stagnation point, $\bar{x} = 0,$ then slowly embedded in a ferrofluid as illustrated in fig. 1. This physical phenomenon is under the same assumptions with the references to the mathematical model proposed by [16, 18]. Further, $\bar{r}(\bar{x}) = a\sin(\bar{x}/a)$ denotes the radial distance from the symmetrical axis to the surface of the sphere.

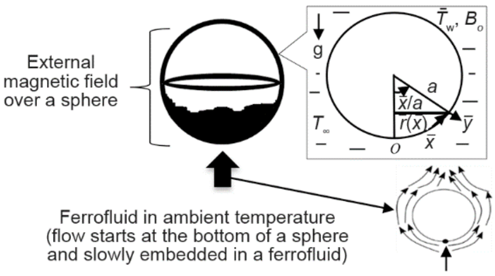

Figure 1. Physical model and coordinate system

Here, the ferrofluid with ferroparticles (magnetite, Fe_3O_4) dispersant and suspended in a base fluid (water) are assumed as Newtonian fluid and behave as single-phase fluid. By implementing the Boussinesq approximation with the source term previously mentioned, the governing equations are [16]:

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\bar{v}) = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu_{\mathrm{ff}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{(\rho\beta)_{\mathrm{ff}}}{\rho_{\mathrm{ff}}}\mathrm{g}(T - T_\infty)\sin\frac{\bar{x}}{a} - \frac{\sigma_{\mathrm{ff}} B_o^2}{\rho_{\mathrm{ff}}}(\bar{u}) \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha_{\mathrm{ff}}\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{(\rho C_p)_{\mathrm{ff}}}\frac{\partial q_{\mathrm{r}}}{\partial \bar{y}} \tag{3}$$

subject to the boundary conditions as considered by [16] where $\bar{u}$ and $\bar{v}$ indicate the velocity components along the $\bar{x}$ and $\bar{y}$ axes, respectively. Meanwhile, the effective thermophysical properties that determine the calculation of fluid-flow and heat transfer behaviour of ferrofluid, subscript, ff, can be expressed in terms of base fluid, subscript, f, ferroparticles, subscript, s, and ferroparticles volume fraction, ϕ, as defined in [9, 15, 18]. Next, the Rosseland approximation is employed when thermal radiation is considered where the radiative heat flux, q_r, is defined:

$$q_{\mathrm{r}} = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial \bar{y}} \tag{4}$$

The term T^4 represented a linear function of the temperature difference within the flow is assumed to be sufficiently small expanding in a Taylor series about T_∞ and considering up to the second term where the higher-order terms will be neglected in the eq. (3), which becomes:

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha_{\mathrm{ff}}\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{1}{(\rho C_p)_{\mathrm{ff}}}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial \bar{y}^2} \tag{5}$$

Now, employing the transformation variables as suggested by [16] and the effective thermophysical properties definition in eqs. (1), (2), and (5) becomes:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\nu_{\mathrm{ff}}}{\nu_{\mathrm{f}}}\frac{\partial^2 u}{\partial y^2} + \frac{(1-\phi)\rho_{\mathrm{f}} + \dfrac{\phi(\rho\beta)_s}{\beta_{\mathrm{f}}}}{(1-\phi)\rho_{\mathrm{f}} + \phi\rho_s}\theta\sin x - \frac{\sigma_{\mathrm{ff}}}{\sigma_f\left[(1-\phi) + \phi\dfrac{\rho_s}{\rho_{\mathrm{f}}}\right]}Mu \tag{7}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{(\rho C_p)_{\mathrm{f}}}{(\rho C_p)_{\mathrm{ff}}}\left(\frac{k_{\mathrm{ff}}}{k_{\mathrm{f}}} + \frac{4}{3}Nr\right)\frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

subject to the boundary conditions as [16] where the magnetic parameter, the Prandtl number and radiation parameter are expressed:

$$M = \frac{\sigma_{\mathrm{f}} a^2 B_o{}^2(x)}{\nu_{\mathrm{f}}\rho_{\mathrm{f}}\mathrm{Gr}^{1/2}}, \quad \Pr = \frac{\nu_{\mathrm{f}}(\rho C_p)_{\mathrm{f}}}{k_{\mathrm{f}}}, \quad Nr = \frac{4\sigma^* T_\infty^3}{k^* k_{\mathrm{f}}} \tag{9}$$

The following non-similarity functions are used to solve eqs. (6)-(8) by reducing the number of dependent variables:

$$\psi = xr(x)f(x,y), \quad \theta = \theta(x,y) \tag{10}$$

where ψ is the stream function defined:

$$u = \frac{1}{r}\left(\frac{\partial \psi}{\partial y}\right) \quad \text{and} \quad v = \frac{1}{r}\left(\frac{\partial \psi}{\partial x}\right)$$

which satisfies eq. (6). Hence, by utilizing eq. (10) into eqs. (7) and (8), the following partial differential equations turn into:

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\dfrac{\phi\rho_s}{\rho_f}\right)}\left(\frac{\partial^3 f}{\partial y^3}\right)+\left(1+\frac{x}{\sin x}\cos x\right)\left(f\frac{\partial^2 f}{\partial y^2}\right)-\left(\frac{\partial f}{\partial y}\right)^2+$$

$$+\frac{(1-\phi)\rho_\mathrm{f}+\dfrac{\phi(\rho\beta)_\mathrm{s}}{\beta_\mathrm{f}}}{(1-\phi)\rho_\mathrm{f}+\phi\rho_\mathrm{s}}\left(\frac{\sin x}{x}\theta\right)-\frac{\dfrac{\sigma_\mathrm{ff}}{\sigma_\mathrm{f}}}{(1-\phi)+\phi\dfrac{\rho_s}{\rho_\mathrm{f}}}M\left(\frac{\partial f}{\partial y}\right)=x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \quad (11)$$

$$\frac{1}{\mathrm{Pr}}\frac{(\rho C_p)_\mathrm{f}}{(\rho C_p)_\mathrm{ff}}\left(\frac{k_\mathrm{ff}}{k_\mathrm{f}}+\frac{4}{3}Nr\right)\frac{\partial^2\theta}{\partial y^2}+\left(1+\frac{x}{\sin x}\cos x\right)f\frac{\partial\theta}{\partial y}=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right) \quad (12)$$

Then, the boundary conditions becomes as [16]. The following the local skin friction coefficient and the local Nusselt number are defined as [16] where the wall shear stress, τ_w, and the surface heat flux, q_w, in the presence of thermal radiation are given by:

$$\tau_\mathrm{w}=\mu_\mathrm{ff}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0}, \quad q_\mathrm{w}=-k_\mathrm{ff}\left(\frac{\partial T}{\partial\bar{y}}\right)_{\bar{y}=0}+q_\mathrm{r} \quad (13)$$

The transformation variables by [16] and eq. (10) with the effective thermophysical properties expressions are applied to get the dimensionless expression for reduced skin friction and the reduced Nusselt number:

$$C_\mathrm{f}\mathrm{Gr}^{1/4}=\frac{x}{(1-\phi)^{2.5}}\frac{\partial^2 f}{\partial y^2}(x,0), \quad \mathrm{Nu}_x\mathrm{Gr}^{-1/4}=-\left(\frac{k_\mathrm{ff}}{k_\mathrm{f}}+\frac{4}{3}Nr\right)\frac{\partial\theta}{\partial y}(x,0) \quad (14)$$

Results and discussion

The implicit finite difference method or known as Keller-box method is applied in this study for solving the differential equation. Afterwards, the derivation of this method with the appropriate initial profiles that satisfied boundary conditions is programmed in MATLAB software. The precision of the numerical method utilized is verified by comparing it with the outcome from previously numerical results. Table 1 shows the present results are found in good agreement with previously published results. Therefore, the present numerical method and MATLAB programme codes work accurately, effectively and efficiently.

The increase of ferroparticles volume fraction prompt a decrease of the $C_\mathrm{f}\mathrm{Gr}^{1/4}$ and $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$ as depicted in figs. 2 and 3 when the MHD principles and thermal radiation emerge in ferrofluid flow. The viscosity of Fe_3O_4-water based increases with an increase of Fe_3O_4 volume fraction as experiment results by [13, 14]. In line with the experiment and theoretical result, viscosity has a fundamental role in determining the $C_\mathrm{f}\mathrm{Gr}^{1/4}$. The skin friction of ferrofluid against the sphere surface is directly proportional to the viscosity of ferrofluid where the increment in viscosity elevates the skin friction because of the exertion of the drag force on the surface that influences the velocity of ferrofluid. However, a different phenomenon takes place to the reduced Nusselt number. Although the temperature increases (fig. 5) when the Fe_3O_4 volume fraction is increased, it does not enhance the $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$ that measures the convection

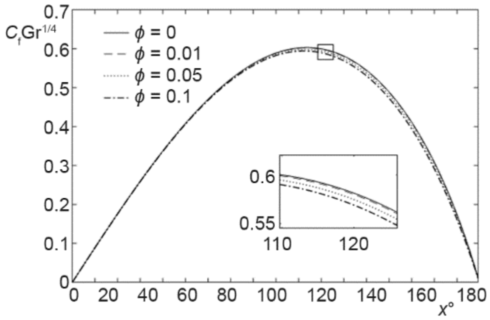

Figure 2. The $C_f Gr^{1/4}$ for several values of ϕ when M = 1 and Nr = 1

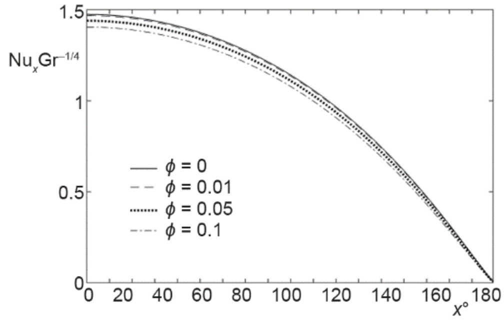

Figure 3. The $NuGr^{-1/4}$ for several values of ϕ when M = 1 and Nr = 1

heat transfer on the sphere surface. These results are related to the buoyancy force and Lorentz force that dominate over a sphere surface. The thermophysical properties and the superparamagnetism magnetic behaviour exhibit in ferrofluid gives significant results to alter the ferrofluid flow and heat transfer in order to enhance the thermal conductivity.

Figure 4 portrays the velocity of ferrofluid at stagnation region $(\bar{x} = 0)$ is diminishing but then increases which cause it to enhance the momentum boundary-layer thickness with the increment in ferroparticles volume fraction. Initially, this phenomenon is related to skin friction and viscosity as stated above but as mentioned by [19] the temperature is also one of the elements that affect the variation of dynamic viscosity. Figure 5 observed the increment in ferroparticles volume fraction elevates the temperature and thermal boundary-layer thickness. Consequently, the thermal conductivity also increases as the temperature increased as reported by Haiza *et al*. [12]. Obviously, the velocity of ferrofluid correlates with the temperature of ferrofluid where the viscosity increases in parallel with the increase of Fe_3O_4 volume fraction but decreases when the temperature is increased and caused the velocity of ferrofluid to decline as discovered in [13-15]. Physically, the intermolecular force of attraction is less effective and weak when the temperature increases which then implies more vigorous random motion appears. Therefore, it will reduce the viscosity and elevates the velocity of the fluid.

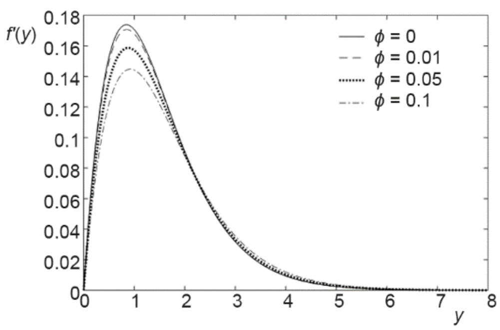

Figure 4. Velocity profile for several values of ϕ when M = 1 and Nr = 1

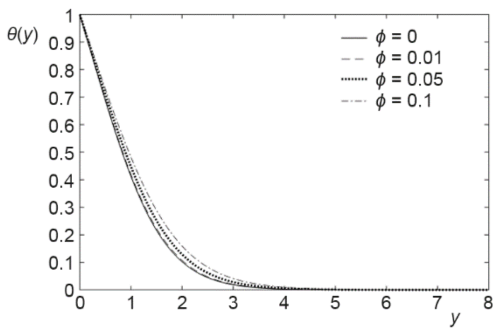

Figure 5. Temperature profile for several values of ϕ when M = 1 and Nr = 1

Table 1. Comparison values of $Nu_x Gr^{-1/4}$ with previously published results when $\phi = M = Nr = 0$ and Pr = 0.7

x	[20]	[21]	[16]	Present
0°	0.4574	0.4576	0.4576	0.4576
40°	0.4407	0.4406	0.4406	0.4406
90°	0.3694	0.3692	0.3693	0.3693
120°		0.2925	0.2927	0.2927
170°		0.0712		0.0714
180°				0.0045

Conclusion

The MHD flow in free convection heat transfer of ferrofluid with the presence of thermal radiation over a sphere surface has been investigated. The influence of the viscosity, Lorentz force and buoyancy force is scrutinized to determine the ferrofluid behaviour and heat transfer when the ferroparticles volume fraction parameter increases or decreases. The crucial discovery of the study is the reduced skin friction and velocity of ferrofluid depends on the viscosity and temperature of ferrofluid when the Fe_3O_4 volume fraction increase. It is observed that the viscosity of ferrofluid is the determining factor in influencing the ferrofluid behaviour on the sphere surface even the buoyancy force and Lorentz force exist.

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