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Flow of Jeffrey Fluid over a Horizontal Circular Cylinder with Suspended Nanoparticles and Viscous Dissipation Effect: Buongiorno Model

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ABSTRACT

Mathematical model of Jeffrey fluid describes the property of viscoelastic that clarifies the two components of relaxation and retardation times. Nevertheless, the poor thermal performance of Jeffrey fluid has been a key issue facing the public. This issue can be accomplished by the use of nanofluid that has superior thermal performance than the conventional fluids. A better cooling rate in industry is in fact not appropriate to attain by the thermal conductivity of the conventional fluids. On that account, the present study aims to delve into the impact of viscous dissipation and suspended nanoparticles on mixed convection flow of Jeffrey fluid from a horizontal circular cylinder. A concise enlightenment on the separation of boundary layer flow is included and discussed starting from the lower stagnation point flow up to the separation point only. The non-dimensional and non-similarity transformation variables are implemented to transform the dimensional nonlinear partial differential equations (PDEs) into two nonlinear PDEs, and then tackled numerically through the Keller-box method. Representation of tabular and graphical results are executed for velocity and temperature profiles as well as the reduced skin friction coefficient, Nusselt number and Sherwood number to investigate the physical insight of emerging parameters. It was found that the incremented ratio of relaxation to retardation, Deborah number and Eckert number have delayed the boundary layer separation up to $120°$.

1. Introduction

The concept of nanofluids refers to an innovative idea of engineered heat transfer fluids by dispersing the nanometer-sized particles in the conventional fluids [1]. These particles, which are called as nanoparticles, are usually being composed of oxides $(Al_2O_3, CuO, TiO_2, SiO_2)$, metals

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(Al, Cu), nitrides (AlN, SiN), carbides (SiC), or non-metals (graphite and carbon nanotubes) with diameter between 1 and 100 nm. Some examples of conventional fluids are organic liquids such as tri-ethylene-glycols, ethylene and refrigerants, water, polymeric solution, bio-fluids, oil and lubricants, and other liquids. To accomplish the industrial cooling rate requirement, the conventional fluids are found to have limited heat transfer competency attributing to their low thermal conductivity compared to metals. In that capacity, the thermal conductivity of conventional fluids can be conceivably enhanced by suspending it with nanoparticles; nonetheless, subjected to the particles' shape, size, conductivity, amount of dispersed particles and the conventional fluid itself [2]. A number of works concerning the heat transfer in nanofluids may be found in publication by Zokri *et al.,* [3], Mohamed *et al.,* [4], Zulkifli *et al.,* [5], Azam *et al.,* [6], and Waini *et al.,* [7].

Recent studies have shown that the non-linear rheological fluids had made sizeable progression. This improvement can be tracked down through the complex nature of fluids used in various industrial applications, that a single constitutive equation is inadequate to describe such fluids. Differing to Newtonian fluid, the relationship between the stress and strain rate of non-Newtonian fluids is non-linear because of the dependency of fluid viscosity on time or deformation. The complex nature of fluids has stimulated the development of many non-Newtonian fluid models that can be mathematically recognized by its constitutive equations. Such constitutive equations are more complicated than the Navier-Stokes equations as each of the established models is fundamentally characterized by dissimilar characteristics. Most frequent highlighted non-Newtonian models in the literature comprehend the micropolar fluid model [8], viscoelastic fluid model [9], Jeffrey fluid model [10-12], Casson fluid model [13, 14], Williamson fluid model [15] and second grade fluid model [16]. Amongst all, Jeffrey fluid model has been ascertained as quite successful due to its distinct ability in explaining the dual viscoelastic properties of relaxation and retardation times, which is very much relevant with the polymer industries [17]. The important features of this fluid model include high shear viscosity, shear thinning and yield stress. At very high wall shear stress, this model degenerates to the Newtonian fluids provided that the wall shear stress is much greater than the yield stress.

Free convection flow of an incompressible fluid from a horizontal circular cylinder implicates an imperative problem in many industrial applications, for example in handling hot wire and steam pipe. Merkin [18] attempted the initial investigation on free convection boundary layer flow from a horizontal circular cylinder in a viscous fluid. He presented a complete solution of this problem from the lower stagnation point up to the upper stagnation point of circular cylinder using the Blasius and Gortler series expansion methods coupled with an integral method and finite difference scheme. Soon after, he extended the study on a horizontal cylinder of elliptic cross section when the major axis is horizontal and vertical [19]. Both the constant wall temperature and constant heat flux are incorporated. The free convection problem about a heated horizontal cylinder in a porous medium was addressed by Ingham and Pop [20], while Merkin and Pop [21] utilized a similar method as Merkin [18] to investigate the constant heat flux condition. Following the works of Merkin [18] and Merkin and Pop [21], the non-Newtonian micropolar fluid was included and thoroughly investigated by Nazar *et al.,* [22] under the constant wall temperature.

Ever since, countless investigations have been conducted from a horizontal circular cylinder in both Newtonian and non-Newtonian fluid. This takes in the published study by Molla *et al.,* [23] who utilized the free convection flow of a viscous fluid past an isothermal horizontal circular cylinder. They supposed that the fluid viscosity is proportional to an inverse linear function of the temperature. They applied the Keller-box method to solve the transformed boundary layer equations starting from the lower stagnation point of the cylinder and then proceeded round the cylinder up to the rear stagnation point. In the subsequent year, Molla *et al.,* [24] continued the investigation by incorporating the internal heat generation effect. The transformed equations were solved numerically using two methods, namely the Keller box method and series solution technique. Again, they observed that the boundary layer proceeds round the cylinder until the upper stagnation point without separating. The surface condition of Newtonian heating was studied by Salleh and Nazar [25] on free convection boundary layer flow in a viscous fluid. Here, the surface heat transfer is assumed to be proportional to the local surface temperature. They concluded that for increasing Prandtl number values, the velocity and temperature profiles were both reduced at the lower stagnation region. The combined effects of MHD, joule heating and heat generation were then presented by Azim and Chowdhury [26] on free convection flow of a viscous fluid with convective boundary conditions. With the help of Keller-box method, they noted that the skin friction along the surface of the cylinder decreases with increasing magnetic parameter and conjugate conduction parameter. Prasad *et al.,* [27] explored the flow of Jeffrey fluid past a horizontal circular cylinder with suction/injection effect. The numerical computation conducted by the Keller-box method has shown that the Deborah number has a reducing impact on the velocity and Nusselt number, but rising impact on the temperature and skin friction coefficient. Later, Makanda *et al.,* [28] deliberated the radiation effect on MHD free convection flow from a cylinder with partial slip in a non-Darcy porous medium of a Casson fluid. The cylinder surface was heated under constant surface temperature, and the partial slip factor was imposed on the surface for both velocity and temperature. The resulting system of equations was solved using the bi-variate quasilinearization method. Mohamed *et al.,* [29] solved the model of nanofluid due to a horizontal circular cylinder with viscous dissipation effect using the Keller-box method. Authors disclosed that the increase of Brownian motion parameter, thermophoresis parameter, Lewis number and Eckert number has increased the skin friction coefficient and Sherwood number, while the Nusselt number decreases. Rao *et al.,* [30] also applied the Keller-box method to scrutinize the flow of Williamson fluid with Newtonian heating. They reported that the boundary layer separation for skin friction coefficient $(x=1.5)$ is larger than the Nusselt number $(x=1.2)$. The convectively heated cylinder in MHD Tangent Hyperbolic Fluid was addressed by Gaffar *et al.,* [31]. It was identified that, for all investigated parameters, the boundary layer flow does not experience singularity. Very recently, the flow of Jeffrey nanofluid at lower stagnation point from a horizontal circular cylinder is addressed by Zokri *et al.,* [32] under the influences of suction/injection, mixed convection and convective boundary conditions.

All of the above cited works were restricted to diverse non-Newtonian fluids flow with two of them concentrated on the Jeffrey fluid. However, none of them was identified to deliberate on free convection flow of Jeffrey nanofluid. Motivated by the published works of Mohamed [29] and Dalir [33], the current investigation aims to solve the free convection flow of Jeffrey nanofluid past a horizontal circular cylinder with viscous dissipation effect.

2. Mathematical Formulation

According to Hayat and Ali [34] and Qasim [35], the constitutive equation for the model of Jeffrey fluid is

$$\boldsymbol{\tau} = -p\mathbf{I} + \mathbf{S}, \quad \mathbf{S} = \frac{\mu}{1+\lambda}\left[\mathbf{R_1} + \lambda_1\left(\frac{\partial \mathbf{R_1}}{\partial t} + \mathbf{V}.\nabla\right)\mathbf{R_1}\right]$$

where $\boldsymbol{\tau}$, $\mathbf{I}$, $\mathbf{S}$, p and μ are the Cauchy stress tensor, identity tensor, extra stress tensor, pressure and dynamic viscosity. Furthermore, the material parameters of the Jeffrey fluid are symbolized as λ and λ_1 while $\mathbf{R_1} = (\nabla\mathbf{V}) + (\nabla\mathbf{V})'$ is the Rivlin-Ericksen tensor. This model is developed with the purpose of extending the Maxwell model. The retardation time parameter which appears in Maxwell model is specifically corrected with the time derivative of the strain rate, for which it can measure the required time for the material to react to the deformation.

A steady, two-dimensional and laminar flow of the Jeffrey nanofluid model with uniform ambient temperature T_∞ and concentration C_∞ is investigated due to a horizontal circular cylinder. The cylinder is heated at the same constant temperature T_w and concentration C_w, as exhibited in the flow diagram of Figure 1.

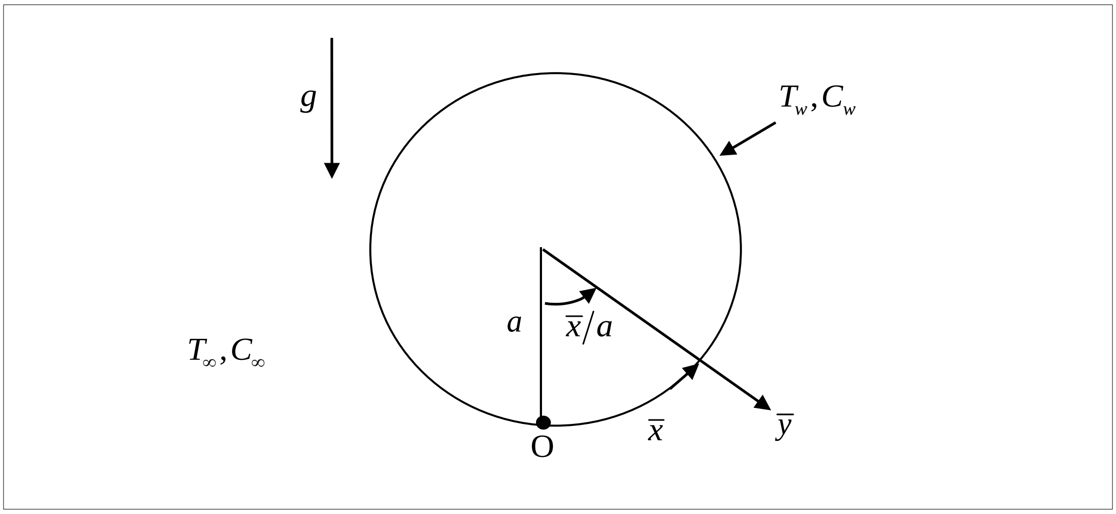

Fig. 1. Schematic diagram of free convection flow in Jeffrey fluid passing over a horizontal circular cylinder

The respective $\bar{x}-$ and $\bar{y}-$ coordinates are implicated throughout the surface of the cylinder from the lowest point, $\bar{x}=0$ and vertical to it, with a and g being the radius of the circular cylinder and gravitational acceleration, respectively. The amalgamated influences of the viscous dissipation and mixed convection are also scrutinized. The law of conservation (after applying the boundary layer approximations) is proposed as the following:

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\begin{aligned}\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\frac{\nu}{1+\lambda}\left[\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\lambda_1\left(\bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2}+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right)\right]+\\ g\beta_T(T-T_\infty)\sin\frac{\bar{x}}{a}+g\beta_C(C-C_\infty)\sin\frac{\bar{x}}{a},\end{aligned} \tag{2}$$

$$\begin{aligned}\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{\nu}{C_p(1+\lambda)}\left[\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2+\lambda_1\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)\right]+\\ \tau\left[D_B\frac{\partial C}{\partial \bar{y}}\frac{\partial T}{\partial \bar{y}}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial \bar{y}}\right)^2\right],\end{aligned} \tag{3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}}+\bar{v}\frac{\partial C}{\partial \bar{y}}=D_B\frac{\partial^2 C}{\partial \bar{y}^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial \bar{y}^2} \tag{4}$$

In the above equations, the ratio of heat capacity of the nanoparticle to the fluid and the velocity outside the boundary layer are denoted as $\tau = (\rho c)_p / (\rho c)_f$ and $\bar{u}_e(x) = U_\infty \sin(\bar{x}/a)$, respectively, whereas the velocity components along the $\bar{x}-$ and $\bar{y}-$ coordinates are symbolized as $\bar{u}$ and $\bar{v}$, respectively. Besides, the respective ratio of relaxation to retardation times, relaxation time, thermal expansion, concentration expansion, thermal diffusivity, kinematic viscosity, fluid density, local concentration, specific heat capacity at a constant pressure, local temperature, Brownian diffusion coefficient and thermophoretic diffusion coefficient are indicated as λ, λ_1, β_T, β_C, α, ν, ρ, C, C_p, T, D_B and D_T. Eqs. (1) to (4) are subjected to the following boundary conditions

$$\begin{aligned} &\bar{u}(\bar{x},0)=0,\ \bar{v}(\bar{x},0)=0,\ T(\bar{x},0)=T_w,\ C(\bar{x},0)=C_w \text{ at } \bar{y}=0 \\ &\bar{u}(\bar{x},\infty)\to 0,\ \bar{v}(\bar{x},\infty)\to 0,\ T(\bar{x},\infty)\to T_\infty,\ C(\bar{x},\infty)\to C_\infty \text{ as } \bar{y}\to\infty \end{aligned} \tag{5}$$

The above mathematical model can be furthered non-dimensionlized using the subsequent variables

$$x=\frac{\bar{x}}{a},\ y=Gr_x^{1/4}\frac{\bar{y}}{a},\ u=\frac{a}{\nu}Gr_x^{-1/2}\bar{u},\ v=\frac{a}{\nu}Gr_x^{-1/4}\bar{v},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

Using Eq. (6), Eqs. (1) to (5) yield

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{1+\lambda}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_2\left(u\frac{\partial^3 u}{\partial x\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)\right]+(\theta+N\phi)\sin x \tag{8}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+Nb\frac{\partial\phi}{\partial y}\frac{\partial\theta}{\partial y}+Nt\left(\frac{\partial\theta}{\partial y}\right)^2+\frac{Ec}{(1+\lambda)}\left[\left(\frac{\partial u}{\partial y}\right)^2+\lambda_2\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)\right] \tag{9}$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=\frac{1}{Le\Pr}\left(\frac{\partial^2\phi}{\partial y^2}+\frac{Nt}{Nb}\frac{\partial^2\theta}{\partial y^2}\right) \tag{10}$$

$$\begin{aligned} &u(x,0)=0,\quad v(x,0)=0,\quad \theta(x,0)=1,\quad \phi(x,0)=1 \quad \text{at } y=0 \\ &u(x,\infty)\to 0,\quad v(x,\infty)\to 0,\quad \theta(x,\infty)\to 0,\quad \phi(x,\infty)\to 0 \quad \text{as } y\to\infty \end{aligned} \tag{11}$$

In consequence of the above equations, we let $\Pr$, λ_2, Ec, γ, Gr_x, Re, N, Nb, Le and Nt be the Prandtl number, Deborah number, Eckert number, mixed convection parameter, Grashof number, Reynolds number, concentration buoyancy parameter, Brownian motion parameter, Lewis number and thermophoresis diffusion parameter, which can be expressed as below:

$$\Pr=\frac{\nu}{\alpha},\ \lambda_2=\frac{\lambda_1 Gr_x^{1/2}\nu}{a^2},\ Ec=\frac{\nu^2 Gr_x}{a^2 C_p(T_w-T_\infty)},\ Gr_x=\frac{g\beta_T(T_w-T_\infty)a^3}{\nu^2},\ \mathrm{Re}=\frac{U_\infty a}{\nu},$$
$$N=\frac{\beta_C(C_w-C_\infty)}{\beta_T(T_w-T_\infty)},\ Nb=\frac{\tau D_B(C_w-C_\infty)}{\nu},\ Le=\frac{\alpha}{D_B},\ Nt=\frac{\tau D_T(T_w-T_\infty)}{\nu T_\infty}$$

Next, we look for these variables to solve Eqs. (7) to (11): $\psi = xf(x,y)$, $\theta = \theta(x,y)$ and $\phi = \phi(x,y)$, in which the stream function, ψ is represented by $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Now, the satisfaction of Eq. (7) is automatically achieved and the resulting PDEs together with the related boundary conditions are

$$\begin{aligned}&\frac{1}{1+\lambda}f''' - (f')^2 + ff'' + \frac{\sin x}{x}\left[\gamma(\theta + N\phi) + \cos x\right] + \frac{\lambda_2}{1+\lambda}\left[(f'')^2 - ff^{(iv)}\right] = \\ &x\left[f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x} + \frac{\lambda_2}{1+\lambda}\left(f'''\frac{\partial f'}{\partial x} + f^{(iv)}\frac{\partial f}{\partial x} - f''\frac{\partial f''}{\partial x} - f'\frac{\partial f'''}{\partial x}\right)\right]\end{aligned} \tag{12}$$

$$\begin{aligned}&\frac{1}{\Pr}\theta'' + f\theta' + Nb\theta'\phi' + Nt(\theta')^2 = \\ &x\left[f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x} - x\frac{Ec}{(1+\lambda)}\left((f'')^2 + \lambda_2\left(xff''\frac{\partial f''}{\partial x} + f'(f'')^2 - xff'''\frac{\partial f}{\partial x} - fff'''\right)\right)\right]\end{aligned} \tag{13}$$

$$\phi'' + Le\Pr f\phi' + \frac{Nt}{Nb}\theta'' = xLe\Pr\left[f'\frac{\partial \phi}{\partial x} - \phi'\frac{\partial f}{\partial x}\right] \tag{14}$$

$$\begin{aligned}&f(x,0) = 0,\ \ f'(x,0) = 0,\ \ \theta(x,0) = 1,\ \ \phi(x,0) = 1 \ \text{ at } \ y = 0 \\ &f'(x,\infty) \to \frac{\sin x}{x},\ \ f''(x,\infty) \to 0,\ \ \theta(x,\infty) \to 0,\ \ \phi(x,\infty) \to 0 \ \text{ as } \ y \to \infty\end{aligned} \tag{15}$$

Note that primes infer the differentiation with respect to the variable y. Also, we found that Eqs. (12) to (15) can be reduced to the mixed convection Newtonian fluid as reported by Mohamed *et al.,* [36], provided the absence of the Jeffrey fluid $(\lambda = \lambda_2 = 0)$ and nanofluid $(Nt = Nb = Le = N = 0)$ parameters. At the vicinity of the lower stagnation point $(x \approx 0)$, the preceding equations (Eqs. (12) to (15)) give rise to the succeeding ordinary differential equations:

$$\frac{1}{1+\lambda}f''' + ff'' - (f')^2 + 1 + \gamma(\theta + N\phi) + \frac{\lambda_2}{1+\lambda}\left[(f'')^2 - ff^{(iv)}\right] = 0, \tag{16}$$

$$\frac{1}{\Pr}\theta'' + f\theta' + Nb\theta'\phi' + Nt(\theta')^2 = 0 \tag{17}$$

$$\phi'' + Le\Pr f\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{18}$$

$$\begin{aligned}&f(0) = 0,\ \ f'(0) = 0,\ \ \theta(0) = 1,\ \ \phi(0) = 1 \\ &f'(\infty) \to 1,\ \ f''(\infty) \to 0,\ \ \theta(\infty) \to 0,\ \ \phi(\infty) \to 0\end{aligned} \tag{19}$$

The non-appearance of parameter Ec in Eq. (17) clearly signifies that the profiles of velocity, temperature and concentration are no longer being influenced by Ec at the stagnation point of the cylinder. Further, the local Nusselt and Sherwood numbers are exemplified as follows

$$\begin{aligned}&C_{fr} = \frac{S_w}{\rho_f U_\infty^2},\ S_w = \frac{\mu}{1+\lambda}\left[\frac{\partial \bar{u}}{\partial \bar{y}} + \lambda_1\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \bar{v}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)\right]_{\bar{y}=0},\ Nu_x = \frac{aq_w}{k(T_w - T_\infty)},\ q_w = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} \text{ and} \\ &Sh_x = \frac{aj_w}{D_B(C_w - C_\infty)},\ j_w = -D_B\left(\frac{\partial C}{\partial \bar{y}}\right)_{\bar{y}=0}\end{aligned} \tag{20}$$

The reduced Nusselt and Sherwood numbers are now given by

$$C_{fr}Gr_x^{1/4} = \frac{x}{1+\lambda} f''(x,0),\ Nu_x Gr_x^{-1/4} = -\theta'(x,0) \text{ and } Sh_x Gr_x^{-1/4} = -\phi'(x,0) \tag{21}$$

3. Results and Discussion

The non-linear PDEs (Eqs. (12) to (14)) with the respective boundary conditions (Eq. (15)) are treated through the Keller-box method. The numerical solutions start at the lower stagnation point, $x = 0°$ with initial profiles being given by Eqs. (16) to (18) accompanied by boundary conditions (19) and then preceded round the circular cylinder up to the separation point $x = 120°$. The step size of $\Delta x = \Delta y = 0.01$ and the boundary layer thickness, $y_\infty = 4$ to 6 are implemented to obtain the numerical results. The results of this study are comprehensively explored and discussed for diverse values of dimensionless governing equations λ, λ_2 and Ec, as illustrated in Figures 1 to 9.

In order to authenticate the engaged numerical method, the comparative benchmark of the $C_{fr}Gr_x^{1/4}$ and $Nu_x Gr_x^{-1/4}$ values against position of x are presented through Tables 1 and 2. The limiting results of the current study are matched with the tabulated values of Merkin [18], Nazar [22], Molla [24], Azim and Chowdhury [26] and Mohamed [29], who applied the Keller-box method in solving the free convection flow of viscous, micropolar and nanofluid. A proper match among the comparative values of both tables has manifestly validated the present results. Furthermore, it can be concluded from the comparative values that the $C_{fr}Gr_x^{1/4}$ rises to a maximum value before declining to a finite value, while the $Nu_x Gr_x^{-1/4}$ decelerates with increasing position of x.

The graph for velocity $f'(y)$, temperature $\theta(y)$ and concentration $\phi(y)$ profiles are portrayed in Figures 2 to 4 for different values of λ and λ_2. Initially, a rise in λ is noticed to boost the velocity profile; however, the velocity profile starts to deteriorate as the momentum boundary layer thickness increases. Physically, λ is dependent on the retardation time. An increase in λ signifies weaker retardation time while a decrease in λ indicates stronger retardation time. Such change in retardation time leads to the increment and decrement in the momentum boundary layer thickness. Instead, a reversal graph trend is observed for increasing λ_2 values. It is perceived that λ_2 displays a trivial effect at the cylinder surface, but the effect comes to be highly substantial as the thickness of boundary layer increases up to the freestream. Moreover, with increasing value of λ, the decrease in temperature profile is found to be slightly significant than the decrease in concentration profile. This outcome goes in the same way as for rising λ_2 values, where a slight significant increase in temperature rather than the concentration profile is spotted. These profiles also decline continuously towards the freestream following the escalation of the boundary layer thickness. The incremented temperature and concentration profiles can be directly related with the behaviour of Deborah number which liable to the changes in retardation time. An increase in retardation time increases the λ_2, which eventually reduces the resistance of fluid motion within the boundary layer. This has subsequently resulted in high impact of fluid motion, which does not only thicken the momentum boundary layer, but also the thermal and concentration boundary layers.

Salient features of skin friction coefficient $C_{fr}Gr_x^{1/4}$, Nusselt number $Nu_x Gr_x^{-1/4}$ and Sherwood number $Sh_x Gr_x^{-1/4}$ are portrayed in Figures 5 to 10 for various values of λ, λ_2 and Ec. These figures have demonstrated that the boundary layer separation had occurred at $x = 120°$, regardless of the varied parameter values. Figures 5 to 7 demonstrate that the $C_{fr}Gr_x^{1/4}$ is a lessening function of λ and a rising function of λ_2, while both the $Nu_xGr_x^{-1/4}$ and $Sh_xGr_x^{-1/4}$ perform reversely. It is observed that the heat and nanoparticle concentration transfer rates reduce sequentially as the tangential coordinate value, x increases. Figures 8 to 10 exhibit that, Ec enunciates a rising impact over the $C_{fr}Gr_x^{1/4}$ and the $Sh_xGr_x^{-1/4}$, but a lessening impact over the $Nu_xGr_x^{-1/4}$. Here, the reversal behaviour of heat transfer transpires as the $Nu_xGr_x^{-1/4}$ values become negative by virtue of escalating Ec from 0 to 2. Such behaviour transpires as a result of dissipative heat effect, thus can be explained as a reversal of the heat flow. One would also expect that the $Nu_xGr_x^{-1/4}$ always gives positive value when $Ec < 0$ and tends to result in negative value when $Ec > 0$. Besides, the impact of Ec for each profile is not plotted here because the graph generates a unique solution. Mathematically, this can also be connected with discontinuation of Ec in the energy equation (Eq. (17)), which subsequently leads to a unique solution of $C_{fr}Gr_x^{1/4}$, $Nu_xGr_x^{-1/4}$ and $Sh_xGr_x^{-1/4}$ at $x = 0°$.

Table 1
Comparative values of $C_{fr}Gr_x^{1/4}$ for different values of x when $\lambda = 0$, $\lambda_2 \to 0$ (very small), $N = Ec = Nb = Nt = Le = 0$ and $\Pr = 1$

			$C_{fr}Gr_x^{1/4}$			
x	Merkin [18]	Nazar [22]	Molla [24]	Azim and Chowdhury [26]	Mohamed [29]	Present
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\pi/6$	0.4151	0.4148	0.4145	0.4139	0.4121	0.4120
$\pi/3$	0.7558	0.7542	0.7539	0.7528	0.7538	0.7507
$\pi/2$	0.9579	0.9545	0.9541	0.9526	0.9563	0.9554
$2\pi/3$	0.9756	0.9698	0.9696	0.9678	0.9743	0.9728
$5\pi/6$	0.7822	0.7740	0.7739	0.7718	0.7813	0.7761
π	0.3391	0.3265	0.3264	0.3239	0.3371	0.3302

Table 2
Comparative values of $Nu_xGr_x^{-1/4}$ for different values of x when $\lambda = 0$, $\lambda_2 \to 0$ (very small), $N = Ec = Nb = Nt = Le = 0$ and $\Pr = 1$

			$Nu_xGr_x^{-1/4}$			
x	Merkin [18]	Nazar [22]	Molla [24]	Azim and Chowdhury [26]	Mohamed [29]	Present
0	0.4214	0.4214	0.4214	0.4216	0.4214	0.4214
$\pi/6$	0.4161	0.4161	0.4161	0.4163	0.4163	0.4162
$\pi/3$	0.4007	0.4005	0.4005	0.4006	0.4008	0.4009
$\pi/2$	0.3745	0.3741	0.3740	0.3742	0.3744	0.3743
$2\pi/3$	0.3364	0.3355	0.3355	0.3356	0.3364	0.3363
$5\pi/6$	0.2825	0.2811	0.2812	0.2811	0.2824	0.2814
π	0.1945	0.1916	0.1917	0.1912	0.1939	0.1932

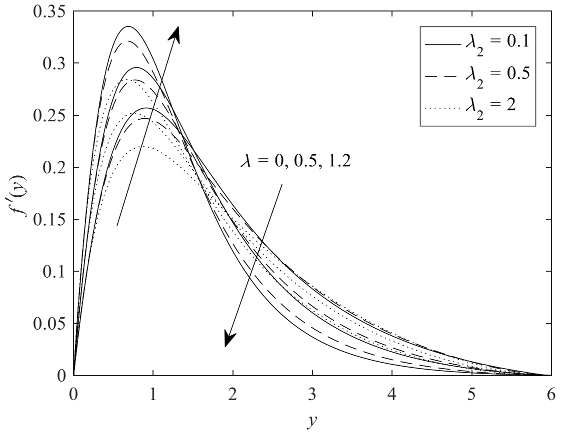

Fig. 2. Variation of $f'(y)$ for several values of λ and λ_2 when $N = Nb = Nt = Ec = 0.1,\ Le = 10$ and $\Pr = 7$

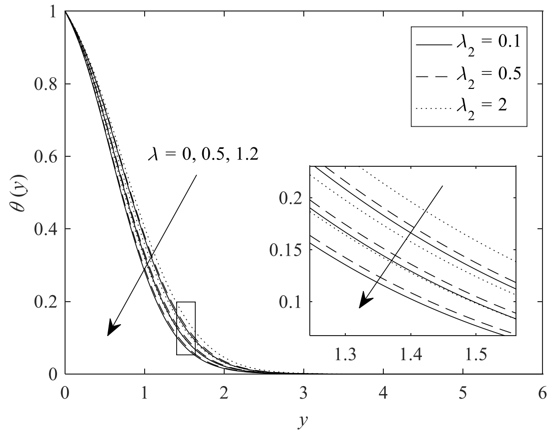

Fig. 3. Variation of $\theta(y)$ for several values of λ and λ_2 when $N = Nb = Nt = Ec = 0.1,\ Le = 10$ and $\Pr = 7$

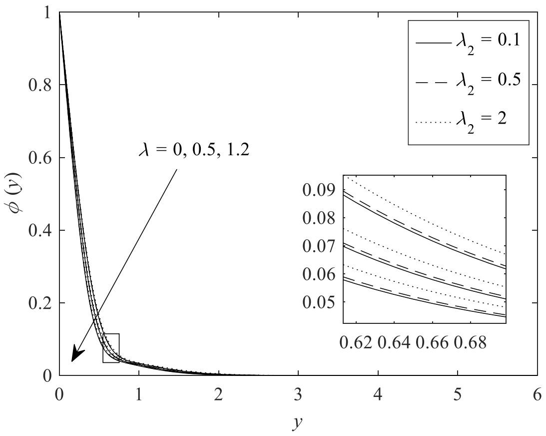

Fig. 4. Variation of $\phi(y)$ for several values of λ and λ_2 when $N = Nb = Nt = Ec = 0.1,\ Le = 10$ and $\Pr = 7$

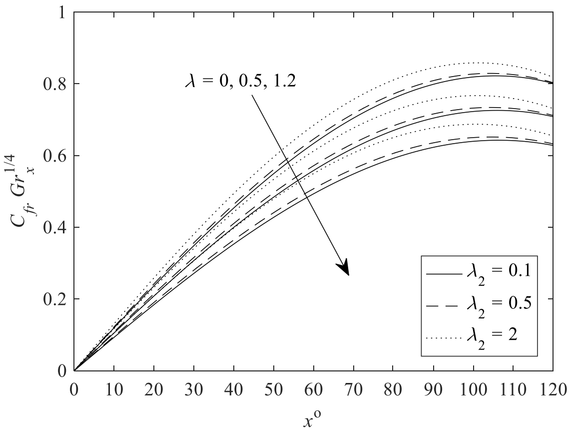

Fig. 5. Variation of $C_{fr} Gr_x^{1/4}$ for several values of λ and λ_2 when $N = Nb = Nt = Ec = 0.1,\ Le = 10$ and $\Pr = 7$

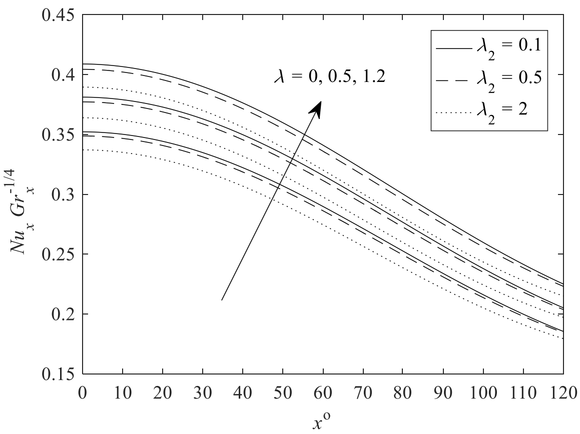

Fig. 6. Variation of $Nu_x Gr_x^{-1/4}$ for several values of λ and λ_2 when $N = Nb = Nt = Ec = 0.1,\ Le = 10$ and $\Pr = 7$

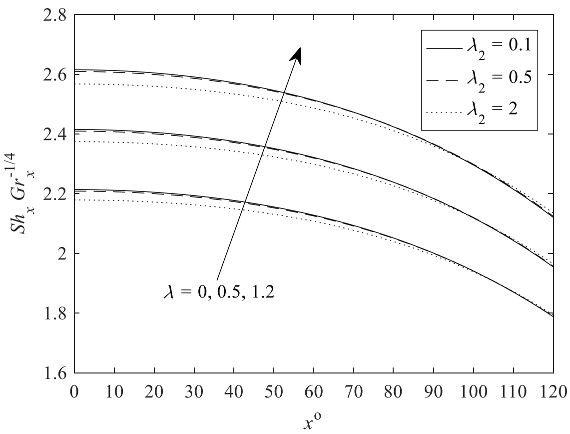

Fig. 7. Variation of $Sh_x Gr_x^{-1/4}$ for several values of λ and λ_2 when $N = Nb = Nt = Ec = 0.1,\ Le = 10$ and $\Pr = 7$

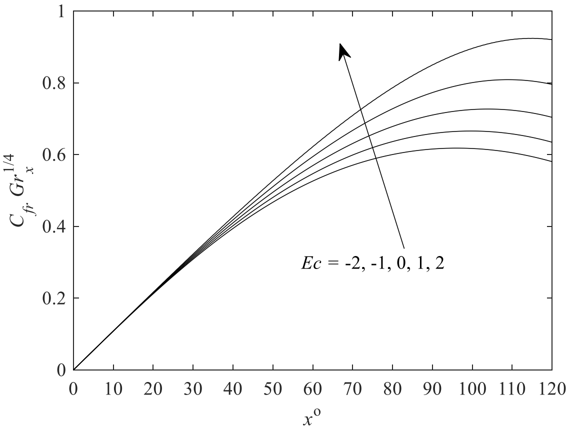

Fig. 8. Variation of $C_{fr}Gr_x^{1/4}$ for several values of Ec when $\lambda = \lambda_2 = 0.5,\ N = Nb = Nt = 0.1,\ Le = 10$ and $\Pr = 7$

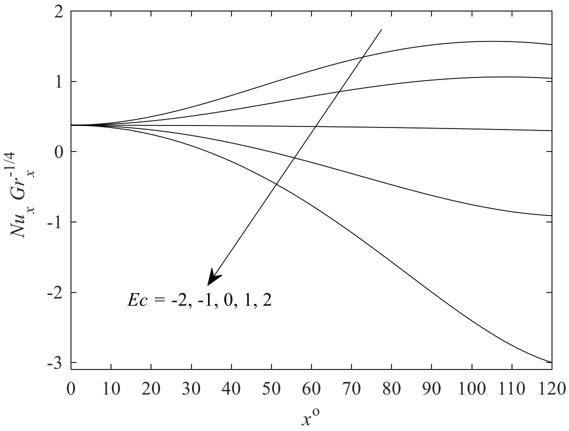

Fig. 9. Variation of $Nu_xGr_x^{-1/4}$ for several values of Ec when $\lambda = \lambda_2 = 0.5,\ N = Nb = Nt = 0.1,\ Le = 10$ and $\Pr = 7$

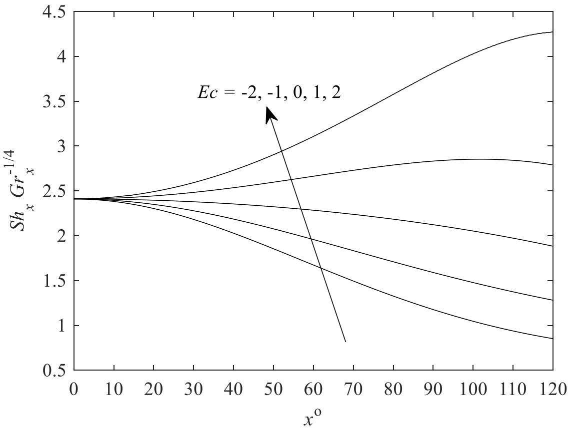

Fig. 10. Variation of $Sh_xGr_x^{-1/4}$ for several values of Ec when $\lambda = \lambda_2 = 0.5,\ N = Nb = Nt = 0.1,\ Le = 10$ and $\Pr = 7$

4. Conclusions

The free convection boundary layer flow problem of Jeffrey nanofluid on a horizontal circular cylinder with viscous dissipation effect was deliberated. The effects of Jeffrey fluid parameter and viscous dissipation on the velocity, temperature and concentration profiles as well as the reduced skin friction coefficient, Nusselt number and Sherwood number have been discussed and explained. On the whole, the concise outcome of this investigation is provided as follows:

I. The similar distribution shows the opposite behaviour for both Jeffrey fluid parameters.
II. An increase in Ec shows no effects on the velocity, temperature and concentration profiles at the lower stagnation point. Augmenting Ec has enlarged the skin friction coefficient and Sherwood number, but reduced the Nusselt number.
III. The increase of λ, λ_2 and Ec has delayed the boundary layer separation up to $x = 120°$.

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