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TWO-PHASE FLOW OF NON-NEWTONIAN EYRING FLUID OVER A VERTICAL STRETCHED SURFACE WITH TEMPERATURE DEPENDENT VISCOSITY

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Abstract

The investigation of the fluid flow problem via mathematical approach for non-Newtonian fluid is challenging due to the rise in complexity in its model. However, the study still attracted researchers since the model is able to capture properties of the existing fluid involved in industrial applications. There are several models representing the non-Newtonian fluid. In this paper, the model of Eyring-Powell fluid with dust particle under influence of temperature dependent viscosity is discussed. The model is formulated using the law of conservation of mass, the first law of thermodynamics and Navier-Stokes equation. The complexity of the model is reduced to a set of ordinary differential equations and the computation is done by using the finite difference method. The validation of the present results is attained by direct comparison with those existing in literature which is found to be in excellent agreement. The investigation revealed the viscosity of the fluid affecting the flow characteristics in both the phases.

Introduction

During the last decades, a considerable amount of effort has been made in the study of non-Newtonian fluid flow in industrial applications due to the limitation analysis of Newtonian fluid flow. Such studies have practical applications, for instance, in manufacture of plastic film, in extrusion of polymer sheet from a die and in fibre industries. This type of fluid also can be found in many daily materials such as coal oil slurries, shampoos, paints, clay coatings and suspensions, cosmetic products, grease, custard, animal blood and body fluids. The principle of Navier-Stokes equations has failed to accurately portray the properties of non-Newtonian fluids. A number of constitutive equations have been introduced for non-Newtonian fluids by considering the natural density of such fluids. Unlike in viscous fluids, shear stresses of non-Newtonian fluids are highly complex which result in multifaceted equations. The study on non-Newtonian fluid flow can be found in documents reported in [1-7].

Recently, a subfamily of non-Newtonian fluid, namely Eyring-Powell fluid has gained worldwide attention owing to its prominent applications in industry and engineering, for example, in the thermal oil recovery, food and slurry transportation and polymer and food processing. Keeping views of its rheological features, numerous researchers have concentrated on the flows dealing with this versatile nature fluid. A whole mathematical model for non-Newtonian fluids had been proposed by Eyring and Powell in 1944 called as the Eyring-Powell model (Powell and Eyring [8]). [9, 10] studied the problem of non-Newtonian Eyring-Powell fluid over a moving of a stretching surface. The authors are motivated to study this problem by the fact that the Eyring-Powell fluid type has a clear characteristic under other non-Newtonian fluids and, it is derived from the kinetic theory of gases instead of from empirical relations. Most significantly, it behaves like a viscous fluid at increased shear rates. Hayat et al. [11] studied an analytical solution for the effects of Newtonian heating (NH) boundary conditions over a moving surface in the presence of mixed convection stagnation flow of Eyring-Powell fluid. The advancement in fluid mechanics has led to the development of innovative way in investigating the suspension of dust particles in fluid flow known as the two-phase flow model, which describes the behavior of fluid-dust system. Again, Eyring-Powell fluid is one of the non-Newtonian fluids in which it possesses the interactive behaviour of dust and liquid phases such as transportation of petroleum, treatment of waste-water, emission of smoke from vehicles, piping of power plants, corrosive particles, honey, soup, jelly, tomato sauce and concentrated fruit juices in mining generally involve the activities of fluid-solid movement, where the interaction of these phases is significant. In conjunction with these implementations, a number of research activities in the respective flow can be found in the literature under various circumstances, for instance, in numerous geometries, boundary conditions, and type of fluid based. Aljabali et al. [12] presented the progress on the development of mathematical model on non-Newtonian two-phase model where the fluid flow is studied together with the dust particles.

Motivated by the above mentioned works, the current study attempts to explore the flow problem embedded in the two-phase boundary layer flow of Eyring-Powell fluid with dusty effect past a vertical stretching sheet by focusing temperature dependent viscosity. The modified mixed convection influences with Newtonian heating (NH) are also implanted in the investigation. Numerical solutions are obtained by using finite difference method and expressed through graphs and tables.

Mathematical Formulation

The steady, incompressible two dimensional boundary layer flows of Eyring-Powell fluid with dusty effect and Newtonian heating included with temperature-dependent viscosity are considered. The sketch of a physical model is displayed in Figure 1.

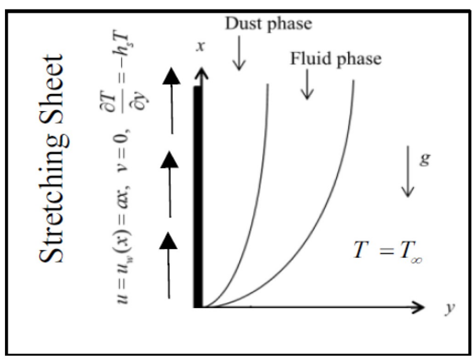

Figure 1. Physical model of two-phase flow.

Summing up from the aforementioned assumptions and adopting the boundary layer approximation, the governing equations of dusty Eyring fluid can be expressed as

Fluid phase:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{1}{\rho\tilde{\beta}c}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{2\rho\tilde{\beta}c^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}$$

$$+ g\beta_T(T - T_\infty) + \frac{\rho_p}{\rho\tau}(u_p - u), \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{\rho_p c_s}{\gamma_T}(T_p - T). \tag{3}$$

For dust phase:

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0, \tag{4}$$

$$\rho_p\left(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \frac{\rho_p}{\tau_v}(u - u_p), \tag{5}$$

$$\rho_p c_s\left(u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right) = -\frac{\rho_p c_s}{\gamma_T}(T_p - T). \tag{6}$$

The symbols (u, v), T, ρ, c_p, μ, k, g, β_T, τ, $\tilde{\beta}$ and c^* represent the components of velocity in (x, y) directions, temperature, density, specific heat at constant pressure, viscosity coefficient, Stokes' resistance, gravity acceleration, thermal expansion coefficient, relaxation time of particles $\tau = 1/k$ and fluid parameters of Erying-Powell model, respectively. Further, (u_p, v_p), T_p, ρ_p, c_s, τ_v and γ_T denote the velocity components in (x, y) directions, temperature, density, specific heat, velocity and thermal relaxation time for dust phase, respectively. The corresponding fluid and particle phase boundary conditions are given as

$$u = u_w(x) = ax,\ v = 0,\ \frac{\partial T}{\partial y} = -h_s T \text{ at } y = 0,$$

$$u \to 0,\ u_p \to 0,\ v_p \to v,\ T \to T_\infty,\ T_p \to T_\infty \text{ as } y \to \infty. \tag{7}$$

In (7), the parameters correspond to the velocity of the stretching surface with a being a positive constant $u_w(x)$, heat transfer parameter h_s and ambient temperature T_∞. To obtain the set of similarity equations in the form of ordinary differential equations, the similarity transformations (8) are adopted and applied to the governing equations (1)-(6):

$$u = axf'(\eta), \quad v = -(a\upsilon)^{1/2} f(\eta), \quad \eta = \left(\frac{a}{\upsilon}\right)^{1/2} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty}(\mathrm{NH}),$$

$$u_p = axF'(\eta), \quad v_p = -(a\upsilon)^{1/2} F(\eta), \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_\infty}. \tag{8}$$

The Reynolds exponential viscosity model is needed to predict temperature-dependent variability in viscosity that gives a detailed approach as

$$\mu(\theta) = \mu_0 e^{-(\beta_1\theta)} = \mu_0[1 - (\beta_1\theta) + O(\beta_1^2)], \tag{9}$$

where the subscript 0 denotes the reference state.

After making equations as nonlinear differential equations governing the problem (1-6), we have

$$(1 + M)f'''(\eta) - (f'(\eta))^2 + f(\eta)f''(\eta) + \beta N(F'(\eta) - f'(\eta))$$

$$- BM(f''(\eta))^2 f'''(\eta)$$

$$- \alpha f''(\eta)\theta'(\eta) - \alpha\theta(\eta)f'''(\eta) + \lambda\theta = 0, \tag{10}$$

$$\theta''(\eta) + Prf(\eta)\theta'(\eta) + \frac{2}{3}\beta N(\theta_p(\eta) - \theta(\eta)) = 0, \tag{11}$$

$$(F'(\eta))^2 - F(\eta)F''(\eta) + \beta(F'(\eta) - f'(\eta)) = 0, \tag{12}$$

$$\theta'_p(\eta)F(\eta) + \frac{2}{3}\frac{\beta}{Pr\gamma}(\theta(\eta) - \theta_p(\eta)) = 0. \tag{13}$$

The above are subjected to the following converted (fluid and dust) particle phase with boundary conditions:

$$f(0) = 0,\ f'(0) = 1,\ \theta'(0) = -b(1 + \theta(0)) \text{ at } \eta = 0,$$

$$f'(\eta) \to 0, F'(\eta) \to 0, F(\eta) \to f(\eta), \theta(\eta) \to 0, \theta_p(\eta) \to 0 \text{ as } \eta \to \infty. \quad (14)$$

In equations (10)-(14), a notation prime $(')$ corresponds to the differentiation with respect to η. The dimensionless numbers and parameters: fluid parameters M and B, Prandtl number Pr, viscosity parameter α, conjugate parameter b, parameter of mass concentration of particle phase N, specific heat ratio of mixture parameter γ, fluid-particle interaction parameter β, mixed convection parameter λ, Grashof number Gr_x and Reynolds number Re_x can be defined as

$$M = \frac{1}{\mu_0 \tilde{\beta} c},\ B = \frac{a^3 x^2}{2c^2 \nu_f},\ Pr = \frac{\nu_f}{\alpha},\ \alpha = \frac{k}{\rho c_p},\ b = -h_s\left(\frac{\upsilon}{a}\right)^{1/2},\ N = \frac{\rho_p}{\rho},$$

$$\gamma = \frac{c_s}{c_p},\ \beta = \frac{1}{a\tau_v},\ \lambda = \frac{Gr_x}{Re_x^{1/2}},\ Gr_x = \frac{g\beta^* T_\infty x^3}{\upsilon^2},\ Re_x = \frac{u_x(x)x}{\upsilon}. \quad (15)$$

Validation Procedure

The numerical solutions for this current investigation have been obtained by finite difference method where equations (10) to (14) are computed using MATLAB software. It is well known that this method is undeniably one of suitable approaches for solving the two-phase flow problem. By having the finite boundary layer thickness, $\eta_\infty = \infty$, the boundary conditions of this study are fully satisfied on the basis of both velocity and temperature profiles attaining the asymptotic behavior, as displayed in Figures 2 to 5.

A limiting case noticed under the absence of effect of dust particles and parameters representing Eyring fluid, viscosity with negligible buoyancy force is obtained by using the following expression presented by Imtiaz et al. [13]:

$$f(\eta) = 1 - \exp(-\eta), \quad \theta(\eta) = \frac{b}{1-b}\exp(-\eta). \tag{16}$$

The non-dimensional quantities of physical and practical interest such as skin friction coefficient C_{fx} and the local Nusselt number Nu_x are defined as

$$C_{fx} = \frac{\tau_w}{\rho u_w^2(x)}, \quad Nu_x = \frac{xq_w}{k(T - T_\infty)},$$

where the shear stress τ_w and surface heat q_w are given as

$$\tau_w = \left(\mu_0 + \frac{1}{\tilde{\beta}c^*}\right)\frac{\partial u}{\partial y} - \frac{1}{6\tilde{\beta}}\left(\frac{1}{c^*}\frac{\partial u}{\partial y}\right)^3 \text{ and } q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

and the coefficient of skin friction and heat transfer coefficient become:

$$C_f Re_x^{1/2} = ((1 - \alpha\theta(0)) + M)f''(0) - \frac{B}{3}Mf''^3(0),$$

$$Nu_x Re_x^{1/2} = b\left(1 + \frac{1}{\theta(0)}\right). \tag{17}$$

Results and Discussion

In this section, numerical solutions obtained for the convective flow of dusty Eyring fluid have been analyzed in details. This present paper concentrated on the discussion of the solutions for various values of N and α by assigning a set of fixed values of $Pr = 10$, $M = B = 0.5$, $b = 0.6$, $\lambda = 1$, $\beta = 0.9$. The computation is done by assigning a set of fixed values of the parameter. Later, the values of $C_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$ are computed. The direct comparison is made with the exact expression (16). From Table 1, an excellent agreement is achieved which indicates that the current model and its findings are acceptable. Table 2 depicts the variations of $C_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$ with regard to parameters of N and α, respectively. From this table, the similar influences of N and α on the variation of magnitude values of $C_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$ have been observed. It is found that, both parameters improve the values of $C_f Re_x^{1/2}$, but the opposite trend is observed in the magnitude of the value of $Nu_x Re_x^{-1/2}$ for higher values *N* of both fluid and dust particles.

Table 1. Comparative study on $f''(0)$

Exact solution	Present study
−1.0000	−1.0015

Table 2. Numerical results of $C_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$ for various values of *N* and α

α	N	$C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$
0.3	0.9	−1.076987	0.104349
0.5		−1.074772	0.104350
0.8		−1.070934	0.104353
0.1	0.2	−1.174082	0.104271
	0.7	−1.067285	0.104069
	1.2	−1.012968	0.103920

Further, the analyses of parameters of *N* and α on the velocity and temperature profiles of fluid and dust phases, respectively, are depicted in Figures 2 to 5. Figures 2 and 3 are plotted to evaluate all fluid and particle-phase velocity and temperature components for the variance of *N* at which a constant value of $\alpha = 0.1$ is considered. Within the boundary layer, temperature profiles are reduced with improvement within *N*. On the other hand, the velocity profile is improved by increasing the parameter *N*. That is because the fluid has a tendency to raise the intensity of drag between the phases, with the rise in mass content of dust particles. The fluid movement is thus slowed down, resulting into the reduced surface-phase energy, because the surface layer is pulled together with the liquid. By continuing to increase the mass content of the dust particles, on the other hand, more fluid-phase energy is converted into a larger number of particles, but less energy from the fluid phase is supplied to the individual particles. Therefore, one can infer that varying N will greatly affect the flow characteristics. Furthermore, the boundary momentum layer for ordinary Eyring fluid is observed to be thinner than that of the dusty Eyring fluid.

Figures 4 and 5 indicate that the momentum of both the processes decrease owing to a change in the amount of viscosity parameter and temperature profiles of both phases enhance for increasing values of α, when $N = 0.9$ is fixed. Physically, more heat energy is released to the fluid flow resulting from enhancement of the effect of α which subsequently decreases the fluid viscosity and raises the temperature profile.

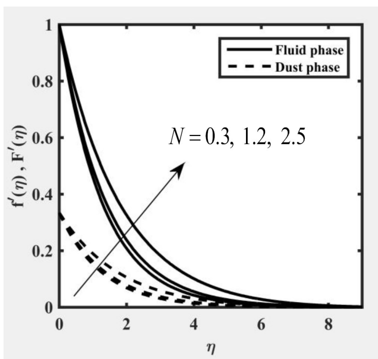

Figure 2. Variation of $f'(\eta)$ and $F'(\eta)$ for various values of N.

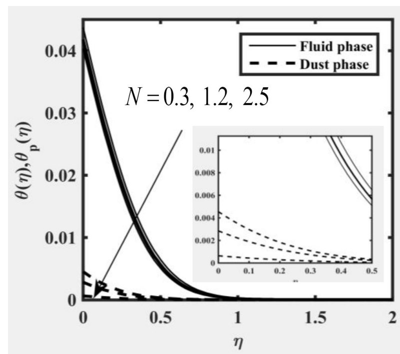

Figure 3. Variation of $\theta(\eta)$ and $\theta_p(\eta)$ for various values of N.

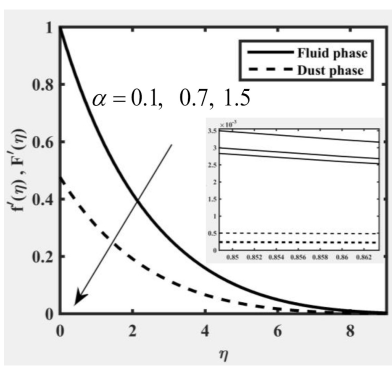

Figure 4. Variation of $f'(\eta)$ and $F'(\eta)$ for various values of α.

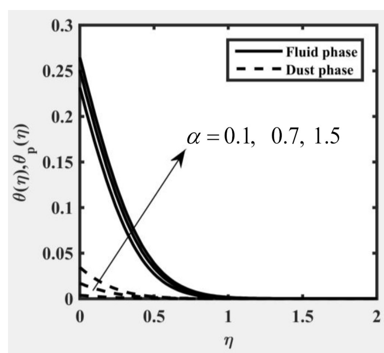

Figure 5. Variation of $\theta(\eta)$ and $\theta_p(\eta)$ for various values of α.

Conclusion

The present investigation reviewed the two-phase convective flow of dusty Eyring-Powell fluid over a vertical stretching sheet by highlighting the effect of temperature-dependent viscosity which involves the parameters N and α. From the mathematical analysis, a similar trend can be noticed in the motion and temperature distributions of fluid and dust phases, respectively, when both the parameters are increased. Nevertheless, the rising values of α exhibit significant influence on temperature profile as compared to velocity profile. In the same manner, the Nusselt number is enhanced significantly when the values of N and α are changed, while the magnitude of the value of skin friction coefficient increases gradually.

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