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IMPACT OF ALIGN MAGNETIC FIELD ON VISCOUS FLOW WITH COMBINED CONVECTIVE TRANSPORT

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Abstract

The present study focuses on the solution of the magnetic field that affected the flow of viscous fluid under combined convective transport. Flow is assumed to be moving over a stretching sheet with the Newtonian heating as the thermal condition. The mathematical model for present problem was derived from law of conservation of mass, the first law of thermodynamics and Navier Stokes equation. The formulation of the model is started by reducing the equations which were in partial differential equations to ordinary differential equation using the appropriate similarity transformations. The transformed equations were then solved by employing the finite difference scheme, known as Keller-Box method. The validation process is performed and found to be in excellent agreement with the existing results in the literature. The magnetic field that aligned 30° towards the flow boosted the value of skin friction up to 32.57% and 4.82% for Nusselt number compared to the magnetic field that acted transverse to the flow field (90°).

Introduction

The fact that the interior of the earth and the atmosphere above the earth is in a fluid state, it comes to a conclusion that a large portion of the earth is containing fluid. This leads to a high interest among researchers seeking more knowledge about fluids. Fluid flows play an important role in a large variety of natural phenomena and man-made systems. Fluid can be divided into inviscid and viscous flows. The study of viscous flow theory has gained high attention of researchers as it is widely used in various areas specifically in fluid mechanics as it concerns on the behavior of fluids in motion (dynamics mode) or at rest (stationary mode). The important aspects that always be considered in these flows are the effect of viscosity, thermal conduction as well as mass diffusion. Even though, various investigations have been made on convective fluid flows over different geometries such as stretching sheets, circular cylinder, wedges, channels and many more, the heat transfer that occurs within fluid has become one of the important topics among researchers. One of the alternative modes for heat transfer is convective which can be free or forced. Many documents have been reported in literature discussing about this topic [1-4]. When the systems involved two convections simultaneously in the heat transfer of fluid, mixed convections occur. Mixed convection flow happened when the buoyancy force developed due to the difference in temperatures at the surface and ambient fluid. There have been several studies in the literature reporting on the mixed convection flow for various physical aspects and geometries [5-10]. Motivated by the study in literature, the present study aims to develop the interest on investigating the impact of align magnetic field on viscous flow by applying the combined convection over a stretching sheet mathematically. The solutions are attained using the Keller-Box method and the analysis is concentrated on the effect of the aligned magnetic field towards the flow field.

Mathematical Formulation

The mathematical model considered in this study is limited to a steady case, two dimensional and incompressible fluid flows over a vertically stretched sheet where the x-axis is positioned in upward direction while y-axis normal to the surface. The sheet is assumed to be stretched with uniform velocity $u_w(x) = ax$ and the thermal boundary condition is taken for the case as Newtonian heating (NH). An acute angle α_1 under the range of 0°-90° towards the magnetic field is induced to the flow, as shown in Figure 1.

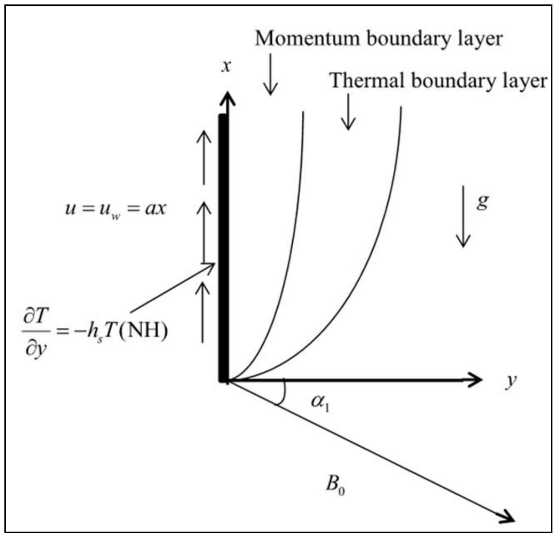

Figure 1. Flow of physical configuration.

The model is governed by continuity, momentum and energy equations where it describes the problem of flow and heat transfer involving viscous fluid embedded with aligned magnetic field and Newtonian heating as the thermal condition. The (u, v), μ, B_0, α_1, k, T, ρ and c_p represent the components of velocity in (x, y) directions, viscosity coefficient, magnetic coefficient, angle aligning to the flow field, thermal conductivity, temperature, density and specific heat at constant pressure, respectively. Now, we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}B_0^2 \sin^2 \alpha_1 u + \beta^* g(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

Equations (1) to (3) are subjected to the following boundary conditions:

$$u = u_w(x) = ax,\ v = 0,\ \frac{\partial T}{\partial y} = -h_s T \ \text{ at } \ y = 0,$$

$$u \to 0,\ T \to T_\infty \ \text{ as } \ y \to \infty, \tag{4}$$

where a is a positive constant and h_s is the heat transfer parameter, referring to the situation where the heat transfers from the surface to the surrounding fluid [11]. The governing equations (1) to (4) were simplified into ordinary differential equations, by adopting the following similarity transformations:

$$\psi = (a\upsilon)^{1/2} x f(\eta), \quad \eta = \left(\frac{a}{\upsilon}\right)^{1/2} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty}. \tag{5}$$

The resulting ordinary differential equations are as follows:

$$f'''(\eta) + f(\eta) f''(\eta) - (f'(\eta))^2 - M \sin^2 \alpha_1 f'(\eta) + \lambda\theta(\eta) = 0, \tag{6}$$

$$\theta''(\eta) + Pr\, f(\eta)\theta'(\eta) = 0 \tag{7}$$

with transformed boundary conditions (4) as

$$f(0) = 0,\ f'(0) = 1,\ \theta'(0) = -b(1 + \theta(0)) \ \text{ at } \ y = 0,$$

$$f'(\eta) \to 0,\ \theta(\eta) \to 0 \ \text{ as } \ y \to \infty. \tag{8}$$

The parameters existing in equations (6) and (7) are the magnetic field M, mixed convection λ and Prandtl number Pr. Meanwhile in equation (8), b refers to the conjugate parameter for NH. These parameters are defined as:

$$M = \frac{\sigma B_0^2}{\rho a}, \quad \lambda = \frac{Gr_x}{Re_x^2}, \quad Pr = \frac{\mu c_p}{k}, \quad b = -h_s\left(\frac{\nu}{a}\right)^{\frac{1}{2}}, \tag{9}$$

where Grashof number Gr_x and Reynolds number Re_x can be expressed as [12]:

$$Gr_x = \frac{g\beta^* T_\infty x^3}{\nu^2}, \quad Re_x = \frac{ax^2}{\nu}. \tag{10}$$

In order to make the equation more realistic, the term β^* is taken as $\beta^* = cx$ and hence change the following terms as

$$Gr = \frac{gcT_\infty x^4}{v^2}, \quad \lambda = \frac{gcT_\infty}{a^2}. \tag{11}$$

Note that the respective assumption has been proposed by [13, 14] in obtaining the similarity solution for natural convection of viscous fluid flow past a vertical plate. The exact solution for equation (6) is attained when $M = \lambda = 0$, where it can be expressed in the following form:

$$f(\eta) = 1 - e^{-\eta}. \tag{12}$$

The exact solution for temperature field is obtained by using the following expression:

$$\theta(\eta) = C_1 \int_\eta^\infty e^{-Pr\int_\eta^\infty f d\eta} d\eta, \text{ where } C_1 = \frac{b(1-\theta(0))}{e^{-Pr\int_\eta^\infty f d\eta}}. \tag{13}$$

The fluid near to the sheet surface is dragged when the fluid starts moving along it, which then actuates the friction force that behaves opposite to the flow direction. Together with that, heat begins to transfer within fluid and stretching sheet. These circumstances need to be taken into account in designing the wall of devices, so that their feature can withstand high shear stress and temperature.

Both physical quantities are respectively evaluated using the following mathematical expressions:

$$C_f = \frac{\tau_w}{\rho u_w^2(x)}, \tag{14}$$

$$Nu_x = \frac{xq_w}{k(T - T_\infty)}. \tag{15}$$

The wall shear stress τ_w and surface heat flux q_w are defined as:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{16}$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{17}$$

Here, $\mu = \rho\nu$ is dynamic viscosity. The dimensionless C_f and Nu_x are evaluated as:

$$C_f Re_x^{1/2} = f''(0), \tag{18}$$

$$Nu_x Re_x^{-1/2} = b\left(\frac{1}{\theta(0)} + 1\right). \tag{19}$$

Results and Discussion

The numerical solutions of equations (6) and (7) with respect to boundary conditions (8) were solved using the Keller-Box method where the algorithm is encoded in the Matlab software. The present solutions were carried out by selecting the finite boundary layer thickness $\eta_\infty = 10$ and step size $\Delta\eta = 0.02$ on the basis to satisfy the boundary conditions (8). Table 1 presents the comparative study between the output reported by Andersson et al. [15] and Prasad et al. [16] with the current results at $\alpha_1 = \frac{\pi}{2}$ and $\lambda = 0,$ where in this situation the equation of each problem is reduced to the similar case. The study found that there is good agreement between the present results and those given by [16] and [17].

Table 2 displays the results for exact equation (13) and presents numerical solution for surface temperature. It can be seen that the current findings are approximately approaching the exact solution, and thus the numerical algorithm developed here can be considered to be accurate.

Table 1. Comparison of skin friction coefficient $-f''(0)$ when $Pr = 7$, $b = 0.3$, $\alpha_1 = \pi/2$ and $\lambda = 0$

M	$-f''(0)$		
	[16]	[17]	Present
0	1.00000	1.000	1.000174
0.5	1.22474	1.225	1.224753
1	1.41421	1.414	1.414450
1.5	1.58114	1.581	1.581139
2	1.73205	1.732	1.732203

Table 2. Comparison of surface temperature $\theta(0)$ when $M = \alpha_1 = \lambda = 0$ and $b = 0.3$

Pr	$\theta(0)$	
	Exact equation (13)	Present
1	6.09619	6.09604
2	1.21549	1.21659
3	0.75160	0.75174
5	0.46814	0.46854
7	0.35832	0.35831
1	6.09619	6.09604
2	1.21549	1.21659

Table 3 shows the variations of skin friction coefficient and Nusselt number. It is found that, as the values of α_1 and M increase, the values of skin friction coefficient boost whilst reducing in the value of Nusselt number. This result is in contrast to the increasing value in λ. The analysis shows that for α_1 with the range of $\pi/6$ to $\pi/2$, the total increase of skin friction coefficient is 32.57% and the total decrease of Nusselt number is 4.82% while for M with the range of 0 to 4.5, the total increase of skin friction coefficient is 41.84% and the total decrease of Nusselt number is 5.30%. With λ in the range of 0.3 to 1.2, the total decrease of skin friction coefficient is 3.78% and the total of increase Nusselt number is 0.83%.

Table 3. Numerical values of skin friction coefficient $-C_f Re_x^{1/2}$ and Nusselt number $Nu_x Re_x^{-1/2}$ when $Pr = 7$, $\lambda = 1$, $b = 0.3$

α_1	M	λ	$-C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$
$\pi/6$	1.5	1	1.12433	1.86661
$\pi/4$			1.27385	1.83402
$\pi/3$			1.40817	1.80486
$\pi/2$			1.53113	1.77804
$\pi/6$	0	1	0.95288	1.90396
	1.5		1.12433	1.86661
	3.0		1.27385	1.83402
	4.5		1.40817	1.80486
		0.3	1.15795	1.85921
		0.6	1.14337	1.86140
		0.9	1.12906	1.86534
		1.2	1.11469	1.86746

Conclusion

This present paper investigates the impact of align magnetic field on viscous flow under combined convective transport. It can be concluded that the magnetic field and the align angle give significant effect to the flow characteristics which should be one of the benchmarks to be considered in the real applications.

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References

[1] S. Siddiqa, N. Begum, M. A. Hossain and R. S. R. Gorla, Natural convection flow of a two phase dusty non-Newtonian fluid along a vertical surface, International Journal of Heat and Mass Transfer 113 (2017), 482-489.

[2] N. A. Majid, N. F. Mohammad, A. R. M. Kasim and S. Shafie, Forced convective of micropolar fluid on a stretching surface of another quiescent fluid, Matematika: MJIAM 35(3) (2019), 397-413.

[3] N. R. M. Asimoni, N. F. Mohammad and A. R. M. Kasim, Unsteady MHD free convective flow past a vertical plate: an automated solution approach, Annals of Mathematical Modeling 1(2) (2019), 81-88.

[4] N. A. Majid, N. F. Mohammad, A. R. M. Kasim, M. R. Ilias and S. Shafie, Effect of constant heat flux on force convective micropolar fluid flow over a surface of another quiescent fluid, Universal Journal of Mechanical Engineering 7(4) (2019), 198-205.

[5] N. F. Mohammad, A. R. M. Kasim, A. Ali and S. Shafie, Unsteady mixed convection boundary layer flow past a sphere in a micropolar fluid, AIP Conference Proceedings, 1450, 2012, pp. 211-217.

[6] S. M. Zokri, N. S. Arifin, M. K. A. Mohamed, A. R. M. Kasim, N. F. Mohammad and M. Z. Salleh, Mathematical model of mixed convection boundary layer flow over a horizontal circular cylinder filled in a Jeffrey fluid with viscous dissipation effect, Sains Malaysiana 47(7) (2018), 1607-1615.

[7] R. Mahat, N. A. Rawi, A. R. M. Kasim and S. Shafie, Mixed convection flow of viscoelastic nanofluid past a horizontal circular cylinder with viscous dissipation, Sains Malaysiana 47(7) (2018), 1617-1623.

[8] A. R. M. Kasim and S. Shafie, Mixed convection boundary layer of a viscoelastic fluid past a circular cylinder with constant heat flux, Proceedings of the 1st Regional Conference on Applied and Engineering Mathematics 1, Eastern and Oriental Hotel, Penang, Malaysia, 2-3 June 2010, pp. 124-129.

[9] H. A. M. Al-Sharifi, A. R. M. Kasim and M. Z. Salleh, Effect of Newtonian heating on the mixed convection boundary layer flow of Eyring-Powell fluid across a nonlinearly stretching sheet, Journal of Engineering and Applied Sciences 11 (2016), 2372-2377.

[10] L. A. Aziz, A. R. M. Kasim, M. Z. Salleh and S. Shafie, Mixed convection boundary layer flow on a solid sphere in a viscoelastic micropolar fluid, Proceedings of the Third International Conference on Computing, Mathematics and Statistics (iCMS2017), Springer, 2019, pp. 111-117.

[11] N. S. Arifin, S. M. Zokri, A. R. M. Kasim, M. Z. Salleh, N. F. Mohammad and W. N. S. W. Yusoff, Aligned magnetic field of two-phase mixed convection flow in dusty Casson fluid over a stretching sheet with Newtonian heating, J. Phys.: Conf. Ser. 890 (2017), 012001.

[12] J. Merkin, R. Nazar and I. Pop, The development of forced convection heat transfer near a forward stagnation point with Newtonian heating, Journal of Engineering Mathematics 74(1) (2012), 53-60.

[13] M. Mohamed, M. Salleh, N. Noar and A. Ishak, Buoyancy effect on stagnation point flow past a stretching vertical surface with Newtonian heating, AIP Conference Proceedings, 1795, 2017, 020005.

[14] O. D. Makinde, Similarity solution of hydromagnetic heat and mass transfer over a vertical plate with a convective surface boundary condition, International Journal of Physical Sciences 5(6) (2010), 700-710.

[15] O. D. Makinde, Similarity solution for natural convection from a moving vertical plate with internal heat generation and a convective boundary condition, Journal of Thermal Science 15(1) (2011), 137-143.

[16] H. Andersson, K. Bech and B. Dandapat, Magnetohydrodynamic flow of a power-law fluid over a stretching sheet, International Journal of Non-Linear Mechanics 27(6) (1992), 929-936.

[17] K. Prasad, A. Sujatha, K. Vajravelu and I. Pop, MHD flow and heat transfer of a UCM fluid over a stretching surface with variable thermophysical properties, Meccanica 47(6) (2012), 1425-1439.