

**EXACT SOLUTIONS ON UNSTEADY  
CONVECTIVE FLOW OF VISCOUS, CASSON,  
SECOND GRADE AND MAXWELL  
NANOFLUIDS**

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We hereby declare that we have checked this thesis, and, in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Doctor of Philosophy.

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I hereby declare that the work in this thesis is based on my original work except for quotations and citation which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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**EXACT SOLUTIONS ON UNSTEADY CONVECTIVE FLOW OF VISCOUS,  
CASSON, SECOND GRADE AND MAXWELL NANOFUIDS**

SIDRA AMAN

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for the award of the degree of  
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*TO MY BELOVED FAMILY ESPECIALLY MY LATE PARENTS, SIBLINGS AND  
BROTHER IN LAW*

*THANK YOU FOR EVERYTHING*

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## **ABSTRAK**

Aliran pemindahan haba dan jisim bagi bendalir Newtonan dan tak-Newtonan disebabkan oleh olakan mempunyai banyak kegunaan yang penting, seperti di dalam industri, kimia, kosmetik, farmasi dan kejuruteraan. Dalam tesis ini, aliran olakan tak mantap dengan pelbagai bendalir Newtonan tak-Newtonan dan bendalir hybrid nano tak Newtonan seperti hibrid Casson, gred kedua dan bendalir nano Maxwell dalam saluran menegak atau melepas plat menegak dikaji. Tiub nano karbon, zarah nano graphene, kobalt, tembaga dan alumina telah digunakan untuk meningkatkan kadar pemindahan haba bendalir di dalam kerja penyelidikan ini. Bendalir nano mempunyai pelbagai kegunaan dalam automotif seperti bahan penyejuk, mikroelektronik, mikrocip dalam komputer, sel bahan api dan bioperubatan. Permasalahan aliran olakan bebas dan bendalir nano campuran dikaji dalam media berliang dan tak berliang, dengan/tanpa pengaruh magnetohidrodinamik (MHD). Syarat-syarat yang lain seperti plat menegak berayun, kesan radiasi dan penjanaan haba dipertimbangkan. Idea derivatif pecahan masa Caputo telah digunakan dalam beberapa permasalahan adalah topik yang novel pada masa kini. Kelebihan derivatif pecahan adalah julat peningkatan derivatif dalam kes ini dan derivatif pembolehubah telah digunakan untuk julat nombor. Pembolehubah tak berdimensi digunakan untuk menurunkan persamaan menakluk berdimensi bersama dengan syarat awal dan syarat sempadan kepada bentuk tak berdimensi. Penyelesaian tepat untuk halaju, suhu dan kepekatan diperoleh dengan menggunakan teknik jelmaan Laplace, dan beberapa permasalahan diselesaikan dengan menggunakan teknik pertubasi bersama dengan jelmaan Laplace songsang, seperti teknik Zakian. Ungkapan sepadan untuk geseran kulit, nombor Nusselt dan nombor Sherwood juga dihitung. Hasil dapatkan diplot menggunakan perisian MathCAD-15 dengan sifat termofizik zarah nano dan bendalir asas. Keputusan bergrafik membincangkan dengan terperinci kesan pelbagai parameter seperti radiasi, nombor Peclet, nombor Grashof, parameter pecahan dan pecahan isipadu zarah nano. Melalui objektif kajian didapati halaju bendalir nano meningkat dengan peningkatan nombor haba/larutan nombor Grashof, parameter radiasi sementara berkurangan dengan pecahan isipadu zarah nano. Profil suhu meningkat dengan parameter radiasi, penjanaan haba dan pecahan isipadu. Kekonduksian termal dan nombor Nusselt bendalir nano memaparkan peningkatan yang ketara dengan peningkatan pecahan isipadu.

## **ABSTRACT**

The heat and mass transfer flow of Newtonian and non-Newtonian nanofluids caused by convection has much practical significance, such as in industries, chemicals, cosmetics, pharmaceuticals and engineering. In this thesis, the unsteady convection flows of Newtonian, non-Newtonian and non-Newtonian hybrid nanofluids such as Casson hybrid, second grade and Maxwell nanofluids in a vertical channel or past a vertical plate will be studied. Carbon nanotubes (CNTs), graphene, cobalt, copper and alumina nanoparticles are used for the enhancement of heat transfer rate of fluids in this research work. Nanofluids have a range of applications in automobiles as coolants, microelectronics, microchips in computer, fuel cells and biomedicine. The problem of free and mixed convection flow of nanofluids is studied in a porous as well as non-porous media, with or without magnetohydrodynamics (MHD) influence. Other conditions like oscillating vertical plate, radiation effect and heat generation have been considered. The idea of Caputo time fractional derivative is used in some problems which is a novel topic nowadays. The advantage of fractional derivative is that the range of derivative increases in this case and the derivative of variable are used for a range of numbers. Appropriate non-dimensional variables are used to reduce the dimensional governing equations along with imposed initial and boundary conditions into dimensionless forms. The exact solutions for velocity, temperature and concentration are acquired via Laplace Transform technique and, in some places, regular perturbation technique along with inverse Laplace transform i.e. Zakian technique. The corresponding expressions for skin friction, Nusselt number and Sherwood's number have been calculated. The outcomes acquired are plotted via computational software MathCAD-15 using the specific thermophysical properties of nanoparticles and base fluids. The graphical outcomes have been discussed to delineate the impact of various embedded parameters such as radiation parameter, Peclet number, Grashof number, fractional parameter and volume fraction of nanoparticles. Throughout the objectives, velocity of the nanofluid is found to be increasing with increasing thermal/solutal Grashof number, radiation parameter while decreasing with volume fraction of nanoparticles. Temperature profile increases with radiation parameter, heat generation and volume fraction. Thermal conductivity and Nusselt number of the nanofluids exhibit significant increment with increasing volume fraction.

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## LIST OF SYMBOLS

$\mathbf{A}_1, \mathbf{A}_2$	Rivlin-Ericksen tensors
$Al_2O_3$	Alumina
$Ag$	Silver
$b$	Body force per unit mass vector
$\mathbf{b}$	Induced magnetic field
$B_0$	Magnitude of magnetic field
$Br$	Brinkman number
$CuO$	Copper oxide
$Cu$	Copper
$C_0$	Concentration on the left wall
$C_d$	Concentration on the right wall
$Cf$	Skin friction
$C_\infty$	Ambient concentration
<b>D</b>	Rate of strain
$D_m$	Mass diffusion
$\epsilon_{ij}$	$(i,j) - th$ component of the deformation rate
<b>E</b>	Electric field
<b>g</b>	Gravitational acceleration
$Gm$	Solutal Grashof number
$Gr$	Grashof number
$h_f$	Heat flux
<b>I</b>	Unit vector
$J$	Current density
$k_l$	Variable permeability,
$M$	Magnetic parameter
$N$	Radiation parameter
$Nu$	Nusselt number

$p$	Pressure
$Pe$	Peclet number
$p_h$	Hydrostatic pressure
$p_d$	Dynamic pressure
$p_y$	Yield stress of the fluid
$q$	Laplace transform parameter
$\mathbf{q}''$	Heat flux
$q_x, q_y, q_z$	Heat fluxes in $x, y, z$ directions
$Q$	Total Heat
$r$	Real number
$Re$	Reynolds number
$\mathbf{S}$	Extra stress tensor
$Sc$	Schmidt number
$Sh$	Sherwood number
$t$	Time
$\mathbf{T}$	Cauchy stress tensor
$T$	Temperature of the fluid
$T_0$	Temperature on the left wall
$T_d$	Temperature on the left wall
$T_\infty$	Ambient temperature
$TiO_2$	Titanium oxide formula
$u, v, w$	Velocity components in $x, y$ and $z$ direction
$U$	Internal energy
$\mathbf{V}$	Velocity vector field
$V_w$	Suction/Injection parameter
$W$	Work done
$x, y, z$	Cartesian coordinates

## Greek Letters

$\alpha$	Fractional parameter
$\alpha_0$	Mean radiation absorption coefficient
$\alpha_1, \alpha_2$	Normal stress moduli
$\alpha_s$	Second grade fluid parameter
$\beta$	Casson fluid parameter
$\beta_c$	Volumetric solutal expansion coefficient
$\beta_{CNT}$	Volumetric coefficient of thermal expansion of CNTs
$\beta_T$	Volumetric thermal expansion coefficient
$\beta_f$	Volumetric coefficient of thermal expansion of base fluid
$\beta_{nf}$	Volumetric coefficient of thermal expansion of nanofluid
$\rho_{nf}$	Density of nanofluid
$\rho$	Fluid density
$\rho_\infty$	Density of the ambient fluid
$\rho_0$	Density of ambient fluid
$(\rho\beta)_{nf}$	Thermal expansion coefficient of nanofluid
$(\rho c_p)_{nf}$	Specific heat capacitance of nanofluid
$\tau$	Dimensionless shear stress
$\mu$	Dynamic viscosity
$\nu$	Kinematic viscosity
$\mu_B$	Plastic dynamic viscosity of non-Newtonian fluid
$\mu_{nf}$	Viscosity of the nanofluid
$\sigma$	Electrical conductivity of fluid
$\sigma_{nf}$	Electrical conductivity of nanofluid
$\omega$	Frequency of the pressure gradient.
$\lambda$	Relaxation time
$\lambda_s$	Steady part of amplitude

$\lambda_p$	Pulsating part of amplitude
$\pi_1$	Product of component of deformation rate with itself
$\pi_c$	Critical value of the product of component of deformation rate with itself
$k_i, \gamma_i$	Constants for Zakian technique
$\phi$	Volume fraction of nanoparticles
$\phi_1$	Volume fraction of copper nanoparticles
$\phi_2$	Volume fraction of Alumina nanoparticles
$\varphi$	Porosity of porous medium
$\nabla$	Vector operator

## SUBSCRIPTS

$\infty$	Ambient condition
$*$	Dimensional
$f$	Fluid
$nf$	Base fluid
$nf$	Nanofluid
$hbnf$	Hybrid nanofluid
$s1$	Copper nanoparticles
$s2$	Alumina nanoparticles
$w$	Condition on the wall

## **LIST OF ABBREVIATION**

CNTs	Carbon Nanotubes
KKL	Koo-Kleinstreuer-Li
MathCAD	Mathematical Computer-Aided Design
MHD	Magneto Hydrodynamic
MWCNTs	Multiple Walls Carbon Nanotubes
PDE	Partial Differential Equations
ODE	Ordinary Differential Equations
PPF	Pulsatile Poiseuille Flow
SWCNTs	Single Walls Carbon Nanotubes

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