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# Investigation on linear and nonlinear dynamic equation for vehicle model in numerical simulation

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# Investigation on linear and nonlinear dynamic equation for vehicle model in numerical simulation

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**Abstract.** With the increase in vehicles in the world today, more accidents have occurred. Therefore, more safety systems are needed for conventional vehicles. However, physical testing is dangerous and costly compared with numerical simulation. So, a good numerical simulation is especially important. In general, the vehicle dynamics equations were used in the numerical simulation. There are two types of vehicle dynamics equations that are linear and nonlinear. This paper researched which one is more practical between linear and nonlinear equations in the simulations versus real-time experiments. In this paper, the equation of motion was used to simulate the state of vehicle motion during cornering conditions in the MatLab Simulink software. The results show that the value of the linear and non-linear yaw is close to the yaw from experimental at lower speed conditions. With the speed increase, the linear yaw value will decrease and farther and farther away from experimental yaw results. Although the linear equation is relatively simple to calculate, the results of the nonlinear equations are closer to the results of real experiments. In summary, the nonlinear equation is more applicable, while the linear equation is not applicable to simulate the motion of the vehicle.

**Keyword.** Numerical Simulation; Steering Performance; Passive Control; Vehicle Dynamics.

## 1. Introduction

Most of the traffic accidents that happened were due to the vehicle motion of cornering [1]. These accidents can be summaries as two factors which are a human factor and vehicle factor [2][3]. If the accident happened is because of the people wrong operating, then it can be solved most accidents by strengthen driver safety awareness, increase road signs, etc. If the vehicle factor makes the accident occur, more vehicle safety systems should be developed. Usually, the vehicle will have an understeering and oversteering phenomenon happen when the vehicle turns [4]. Understeer (US) and oversteer (OS) are important criteria for measuring vehicle handling steering balance [5][6]. It can be understood professionally as the true steering angle of the vehicle is greater or less than the steering angle of the wheels when the vehicle doing cornering, and the vehicle will easily lose control at this time [7][8]. The OS phenomenon occurs because when the vehicle is turning, the vehicle speed will be slightly lower than the normal driving speed. And the center of gravity will be shifted forward from the position [9]. The yaw of the vehicle increases at this time, so the lateral force of the vehicle will be greater than the tire and the ground of friction [10]. Therefore, the vehicle will skid. And the vehicle OS is mostly because of the fact that the driver fails to turn to the appropriate steering angle in time to make the front wheels of the car skid [11][12]. And the rear wheels have sufficient grip during this time, which makes the vehicle uncontrollably deviate to the outside of the road then leads to accidents occur. We cannot avoid many traffic accidents, but the safety performance of the vehicle itself can be developed as much as possible[13]. However, physical vehicle testing is both dangerous and very costly [14]. Therefore, it is very important to have a suitable numerical simulation system [15]. This paper is mainly to analyze which of the linear equation and the nonlinear equation is more suitable for analyzing the movement of the vehicle when the vehicle dynamics equation is used for numerical simulation. In order to accurately determine which equation is more suitable, the author used an experimental vehicle to conduct vehicle steering experiments before the simulation started. The process of the test is that the test vehicle drives around a roundabout to get the parameters of the vehicle in the process of turning. Then compare with the results of the simulation results. The equations of motion (EOM) use to simulate the trajectory of the vehicle [16]. And the EOM has two types which the linear vehicle dynamics (VD) equation and the nonlinear VD equation [17]. in order to study which of the linear and nonlinear equations is more suitable for the numerical simulation of vehicles, this paper uses the parameters from the testing car to do the numerical simulation for the linear and nonlinear equations respectively [18][19]. Although the linear equation is transformed from the nonlinear equation, the linear equation simplifies the calculation of most details. And it is still very different from the result obtained by the nonlinear equation, and a conclusion can be drawn from the analysis of the experimental results: When the vehicle speed is low, the result of the linear equation is very close to the true value. However, as the speed of the car gradually increases, the result obtained by the linear equation will be more and more offset from the true value.

## 2. Experimental vehicle

The Figure 1 is Malaysian national car and this car using for doing experimental, this test car from University Malaysia Pahang, under the Automotive Engineering Center. The car is a sedan type with 1.6-liter engine capacity with a manual transmission. In the research, this test car will be used for the experimental of the 2WS. This experimental use to validate the linear equation and non-linear that which one is more suitable for using in the simulation.


**Figure 1.** Testing car form automotive engineering center.

### *2.1. Place to doing the experimental*

This experiment was done at Universiti Malaysia Pahang campus Pekan. Due to the vehicle need to doing cornering to see the steering performance, the roundabout was selected as the experimental place. The testing roundabout located in the middle of the campus. There are three exits at this roundabout,

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and the diameter of it is about 55 meters. So, during the experiment, when there are some other vehicles drive in or some danger happens the testing vehicle will quickly exit from this roundabout.

## 3. Vehicle dynamics

The vehicle dynamics equation is used for the numerical simulation to simulate the movement of the vehicle. The results of the linear and nonlinear equations are used to compare with the results of the experiment. The tire characteristics are used to calculate the vehicle state of motion when the vehicle doing cornering. The vehicle's state of motion is affected by many factors, and the tire is the only vehicle in contact with the ground during movement. The steering of the vehicle will also be intuitively expressed on the tires. Therefore, the characteristics of tires are essential when simulating the movement of a vehicle.

### *3.1. Non-linear Equation*

Vehicle dynamics is a part of engineering and also refers to vehicle dynamics, mainly derived from classical mechanics. The dynamic equation of motion is derived from the application of Newton's law in the inertial reference frame. The dynamic equations are used in this article to simulate the longitudinal velocity, lateral velocity, and yaw rotation velocity of the model vehicle during its movement.

$$m\left(\frac{du}{dt}-v\gamma\right)=(X_{FR}+X_{FL})\,cos\,\theta_F+(X_{RR}+X_{RL})\,cos\,\theta_R-(Y_{FR}+Y_{FL})\,sin\,\theta_F-(Y_{RR}+Y_{RL})\,sin\,\theta_R \tag{1}$$

$$m\left(\frac{dv}{dt}+u\gamma\right)=(X_{FR}+X_{FL})\,sin\,\theta_F+(X_{RR}+X_{RL})\,sin\,\theta_R+(Y_{FR}+Y_{FL})\,cos\,\theta_F+(Y_{RR}+Y_{RL})\,cos\,\theta_R \tag{2}$$

$$\begin{aligned}I\frac{d\gamma}{dt}=\;&l_F[(X_{FR}+X_{FL})\,sin\,\theta_F+(Y_{FR}+Y_{FL})cos\theta_F]\\&+l_R[(X_{RR}+X_{RL})sin\theta_R+(Y_{RR}+Y_{RL})cos\theta_R]\\&+\frac{d_F}{2}[(X_{FR}+X_{FL})\,cos\,\theta_R+(Y_{FR}+Y_{FL})\,sin\,\theta_F]\\&+\frac{d_R}{2}[(X_{RR}+X_{RL})\,cos\,\theta_R+(Y_{RR}+Y_{RL})\,sin\,\theta_R]\end{aligned} \tag{3}$$

### *3.2. Tire Characteristics*

The wheel is an important factor in the simulation. Therefore, during the modelling process, slip ratio, tire side slip angle, and weight distribution is all take into consideration in calculating the friction force and the side lateral force. The deformation of the tire tread rubber is also used to derive these shown below equations

When $\xi_s > 0$, then the longitudinal force and lateral force can be written as

$$\begin{aligned}F_x&=-K_s s\xi_s^2-6\mu F_z\,cos\,\theta\left(\frac{1}{6}-\frac{1}{2}\xi_s^2+\frac{1}{3}\xi_s^3\right)\\F_y&=-K_\beta(1+s)\,tan\,\beta\,\xi_s^2-6\mu F_z\,sin\,\theta\left(\frac{1}{6}-\frac{1}{2}\xi_s^2+\frac{1}{3}\xi_s^3\right)\end{aligned} \tag{4}$$

And when $\xi_s \leq 0$, then

$$\begin{aligned}F_x&=-\mu F_z\cos\theta\\F_y&=-\,\mu F_z\sin\theta\end{aligned} \tag{5}$$

Where:

$$\tan\theta=\frac{K_\beta\tan\beta\,(1+s)}{K_s s}\qquad\qquad \cos\theta=\frac{s}{\lambda}$$

$$sin\,\theta = \frac{K_\beta\, tan\,\beta\,(1+s)}{K_s\lambda}$$

During the simulation, the point at which the contact surface moves from the adhesive area to the slip area is defined as the following equations:

$$\xi_s = 1 - \frac{K_s}{3\mu F_z}\lambda \tag{6}$$

Where:

$$K_s = \frac{bl^2}{2}K_x \qquad K_\beta = \frac{bl^2}{2}K_y$$

$$\lambda = \sqrt{s^2 + \left(\frac{K_\beta}{K_s}\right)^2 (1+s)^2\, tan^2\,\beta}$$

The side-slip angle equation for each tire is shown below:

$$\begin{aligned} \beta_{FL} &= tan^{-1}\left(\frac{v + l_F\gamma}{u + d_F\frac{\gamma}{2}}\right) - \theta_F \;; \quad \beta_{RL} = tan^{-1}\left(\frac{v - l_R\gamma}{u + d_R\frac{\gamma}{2}}\right) - \theta_R \\ \beta_{FR} &= tan^{-1}\left(\frac{v + l_F\gamma}{u - d_F\frac{\gamma}{2}}\right) - \theta_F \;; \quad \beta_{RR} = tan^{-1}\left(\frac{v - l_R\gamma}{u - d_R\frac{\gamma}{2}}\right) - \theta_R \end{aligned} \tag{7}$$

Tire slip ratio $s$ is used to measure and express the slipping behavior of the vehicle tire. The equation as below:

$$s = \frac{u - r\omega}{r\omega} \tag{8}$$

The coefficient of tire friction μ can be approximated by the following equation:

$$\mu = -1.10k \times (e^{35\rho} - e^{0.35\rho}) \tag{9}$$

*3.3. Linear equations*

The linear equation is assumed the sideslip angle is small. When the sideslip angle is small, it can be assumed that the direction of the vehicle almost coincides with the lateral direction. And the fundamental equations of motion for the linear is:

$$mV\frac{d\beta}{dt} + 2\left(K_f + K_r\right)\beta + \left\{mV + \frac{2}{V}\left(l_fK_f - l_rK_r\right)\right\}\gamma = 2K_f\theta \tag{10}$$

$$2\left(l_fK_f - l_rK_r\right)\beta + I\frac{d\gamma}{dt} + \frac{2(l_f^2K_f + l_r^2K_r)}{V}\gamma = 2l_fK_f\theta \tag{11}$$

## 4. Passive control system

This article aims to verify which of the nonlinear equations and linear equations in the vehicle dynamic equation is more suitable for numerical simulation of vehicles. So, this study uses a passive control system. At first, the values of lateral speed, longitudinal speed, and yaw angle when the vehicle is turning at different speeds are obtained by turning tests on the vehicle. Then the vehicle is simulated and compared with the experimental results. The vehicle speed is controlled by changing the tire rotation speed to make it consistent with the experimental speed. In this research, the author wants to test the phenomenon that the vehicle during cornering when the vehicle speed is 25km/h, 30km/h, and 35km/h. However, since the vehicle cannot reach the exact integer value when driving, the average value of each speed when turning is finally taken, namely 26.367km/h, 30.688km/h, and 34.699km/h. In order to get more accurate simulation results to compare with the experimental, three significant figures are reserved.

## 5. Simulation procedure

This research was completed by using the MatLab software to do the simulation. First, complete the modelling in the MatLab Simulink by using the EOM. The modelling has two main parts which are linear VD and nonlinear VD, the independent variable is the tire rotational speed. Then, change the speed of the vehicle by changing the tire rotation speed so that the speed of the vehicle is the same as the speed reached by the vehicle in the experiment. When the simulation results are consistent with the experimental results, analyse the linear equation and the nonlinear equation by comparing the yaw angle values output from the experiment and the simulation, which one is more suitable for modelling the vehicle

## 6. Results and discussion

### *6.1. Experimental results*

The experimental results come from the test car, and the export of the results is using a software called DEWEsoft. The testing car itself is equipped with an additional computer, which is only used to receive the data sensed by various sensors on the vehicle. Then it is analyzed and processed by DEWEsoft software, and the changes of the data can be seen intuitively in the software also can export to an excel file. The 25km/h, 30km/h, and 35km/h appearing in the results below are not the final speeds of the vehicles in the test results. These three speeds are the target speeds, but because the driver cannot artificially control the speed exactly to each target speed when the vehicle is turning, these three speeds are only used as a reference, and the real vehicle speed will be used in the simulation and finally compared with the simulation results. From Figure 2 to Figure 7, it can be clearly seen that the data obtained by the experiment is uneven and it seems that the data is still very large. This is because the sensitivity of the sensor was adjusted to collect data every millisecond during the experiment. And although the driver tries to control the speed of the vehicle to keep the vehicle at a constant speed during the test, it still cannot maintain a perfectly constant speed. So, this has led to the fact that the result does not look ideal, but it is not the case.

Each of the figures from Figure 2 to Figure 7 has a dotted line and an equation. This line is the trend line of the set of data, and the equation is the expression formula of the trend line. Because the experimental results are uneven, the author uses the trend line of each set of data to analyse. It is not difficult to see from each equation that the independent variable X (time) has a very small effect on the value of the corresponding dependent variable Y (actual vehicle speed or yaw moment). Comparing Figure 3, Figure 5, and Figure 7, the yaw moment of the vehicle is gradually increasing when the vehicle turning with the vehicle speed increases. The amount of the increased yaw moment is basically the same. This proves that the results of these data can be used for comparison with the results of the simulation.

*25km/h*

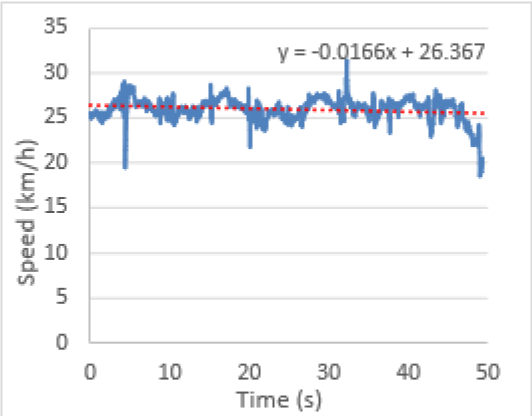


**Figure 2.** The actual vehicle speed for the experimental at 25km/h

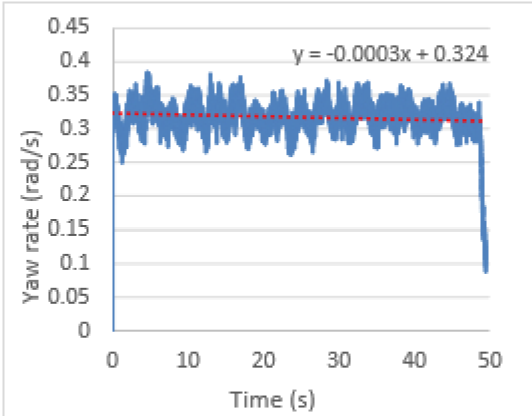


**Figure 3.** The yaw moment for the vehicle speed at 25km/h

*30km/h*

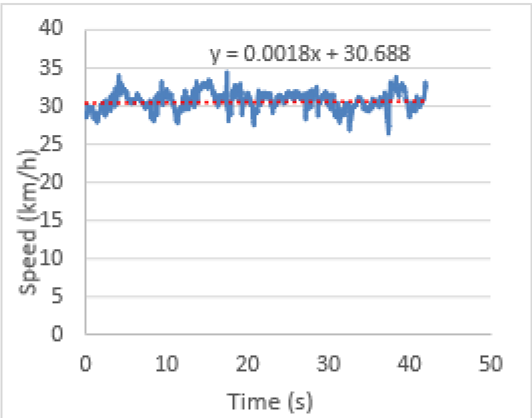


**Figure 4.** The actual vehicle speed for the experimental at 30km/h

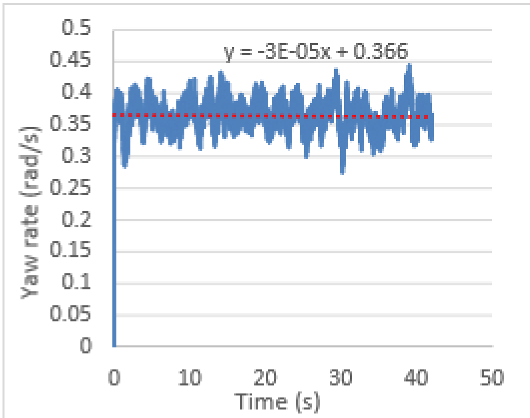


**Figure 5.** The yaw moment for the vehicle speed at 30km/

*35km/h*

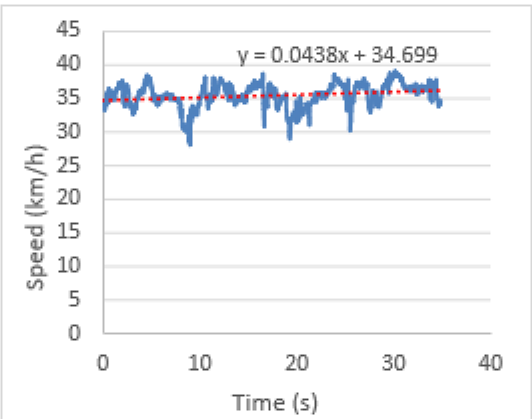


**Figure 6.** The actual vehicle speed for the experimental at 35km/h

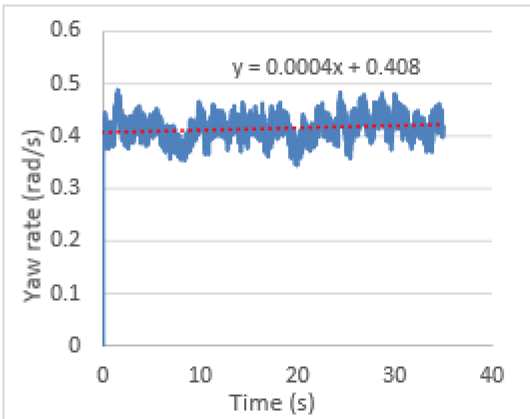


**Figure 7.** The yaw moment for the vehicle speed at 35km/h

### *6.2. Simulation results*

Figure 8 and Figure 9 are the results of the linear VD simulated by simulation, respectively corresponding to the vehicle speed and yaw moment derived from the linear equation. "LV1", "LV2", and "LV3" in Figure 8 and Figure 9 refer to linear speeds 1, 2, and 3. Corresponding to this are the target speeds of 25km/h, 30km/h, and 35km/h mentioned above. The straight-up part in the figure represents that the vehicle is accelerating until it reaches a constant speed. The declining part of each line represents that the vehicle starts to turn after reaching a constant speed and driving in a straight line, so the speed of the vehicle drops rapidly, but in the end, all reach a constant value. This also means that the vehicle can steer in a steady state at this moment. Comparing Figures 8 & 10, the simulated speed results are basically the same whether it is a linear or a nonlinear equation. However, in Figures 9 & 11, the yaw moment simulated by the linear equation decreases with the increase of speed, and the result of the nonlinear equation is that the yaw moment increases with the increase of speed. As the speed increases, the time required for the vehicle to reach the average speed gradually becomes longer in Figure 8, and "LV3" is obviously longer than the time required for the "LV1" to reach the average speed. And as the speed increases, the difference between the speed of the vehicle after turning and the speed of the vehicle before turning becomes larger at a different speeds condition. It can be speculated that when the speed is 40km/h. It takes longer for the vehicle to reach the average speed.

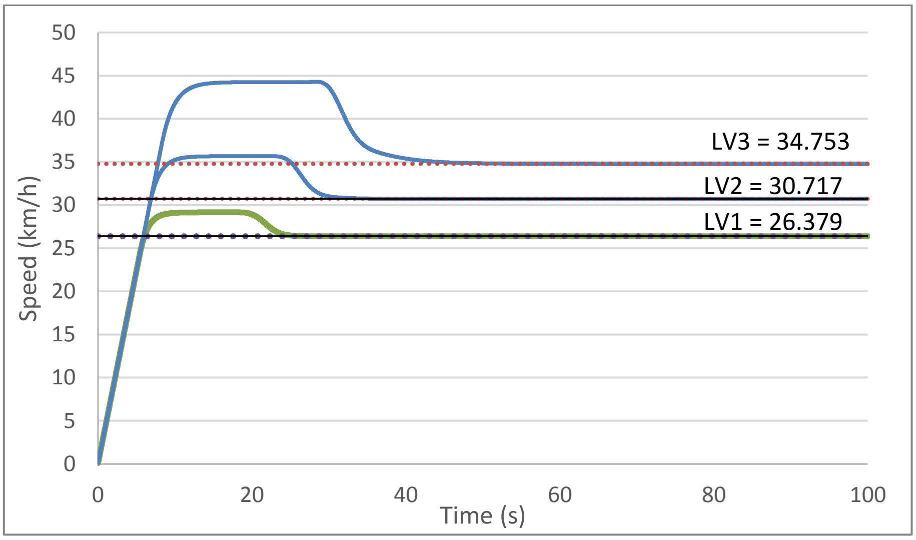


**Figure 8.** The linear VD simulation results for the vehicle speed

In Figure 9, "r" represents the yaw moment. And "Lr1", "Lr2" and "Lr3" are the yaw moment generated by the vehicle at the corresponding speed. Combining Figure 8 shows that the yaw moment of the vehicle is always zero before the vehicle turns, which also indirectly proves that the calculation of the equation is correct. When the vehicle is not turning, there is no deviation of the vehicle, so there is no yaw moment. Because zero yaw moment usually means that the vehicle is not moving or traveling in a straight line, and when the vehicle has a yaw moment and still maintains a constant value, then the vehicle is in a state of constant turning. After the vehicle starts to turn, the yaw moment increased extremely rapidly, and when it reaching the peak value, the speed at which the yaw moment increased slowly until it reaches a constant value. The Figure 9 shows that as the speed increases, the fluctuation before the yaw moment reaches a constant value will become larger and larger. This also means that when turning at high speeds, the vehicle shakes more severely. And as the speed increases, the shaking will increase. Therefore, it is not recommended that the driver turn the steering wheel drastically during high-speed driving because it is very easy to cause the vehicle to lose control.

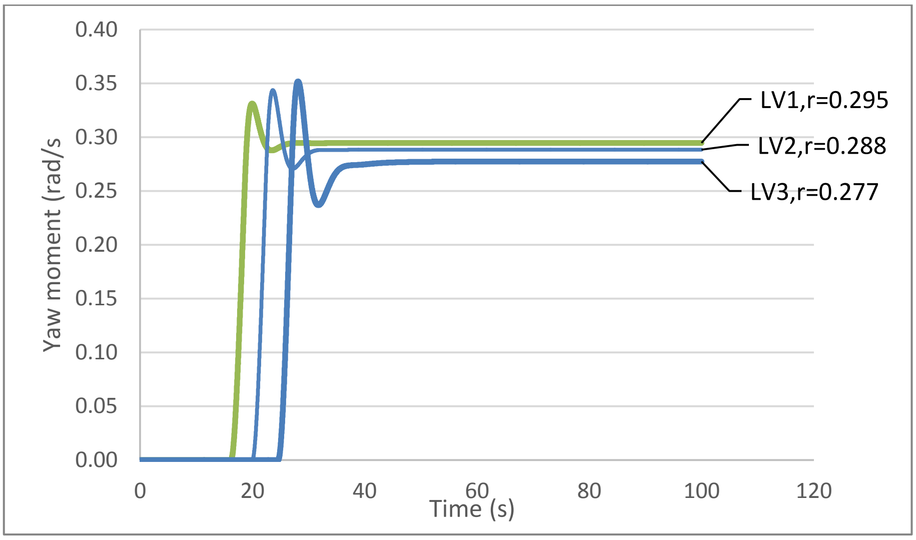


**Figure 9.** The linear VD simulation results for the yaw moment

Figures 10 and 11 show the simulation results of the nonlinear VD equation, Figure 10 shows the speed of the vehicle in the simulation results, and Figure 11 shows the yaw moment of the vehicle. Among them, "NLV1", "NLV2" and "NLV3" in the two figures correspond to the three speeds of the vehicle in the simulation. "NLr1", "NLr2" and "NLr3" are the yaw moment of the vehicle at this speed.

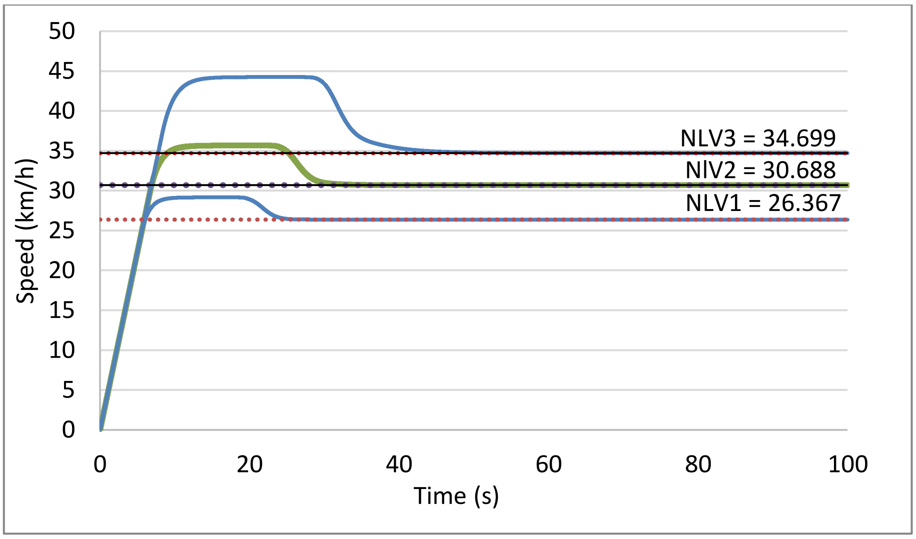


**Figure 10.** The non-linear VD simulation results for the vehicle speed

In Figure 12, the blue line represents the result of the test using the testing car. With the speed increases, the yaw moment increased gradually, and the ascent rate increases after 30km/h. The orange line represents the result of the linear equation simulation. As the speed increases, the yaw moment is slowly reduced. The yaw is lower than the experimental value at the speed of around 25km/h, and the yaw value differs as the speed increases. Is also getting bigger. However, compared with the non-linear result, when the speed is small, the result of the linear equation is closer to the result of the experiment. The gray line represents the results of the non-linear yaw moment simulation, and the yaw gradually increases with the increase of speed. It can be seen from the figure that when the speed is high, nonlinear equations are more suitable for vehicles to do the numerical simulation than linear equations.

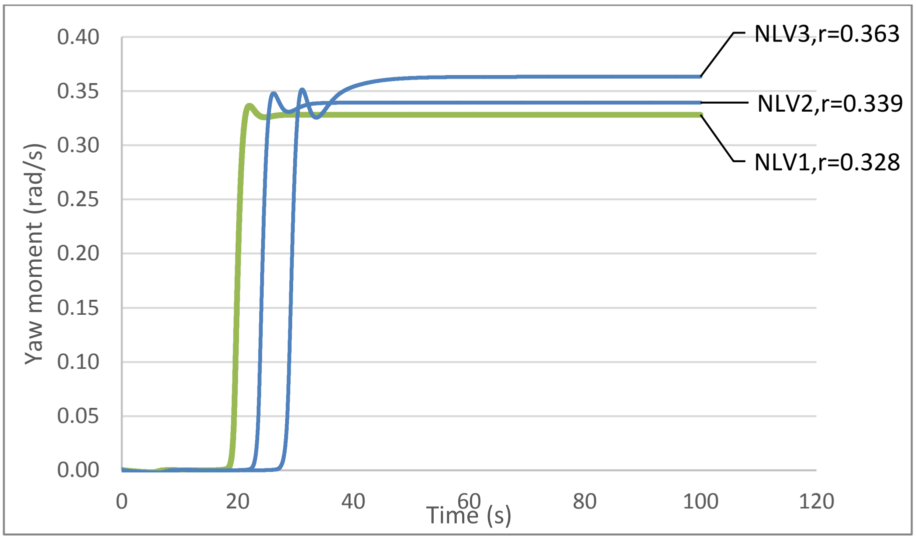


**Figure 11.** The non-linear VD simulation results for the yaw moment

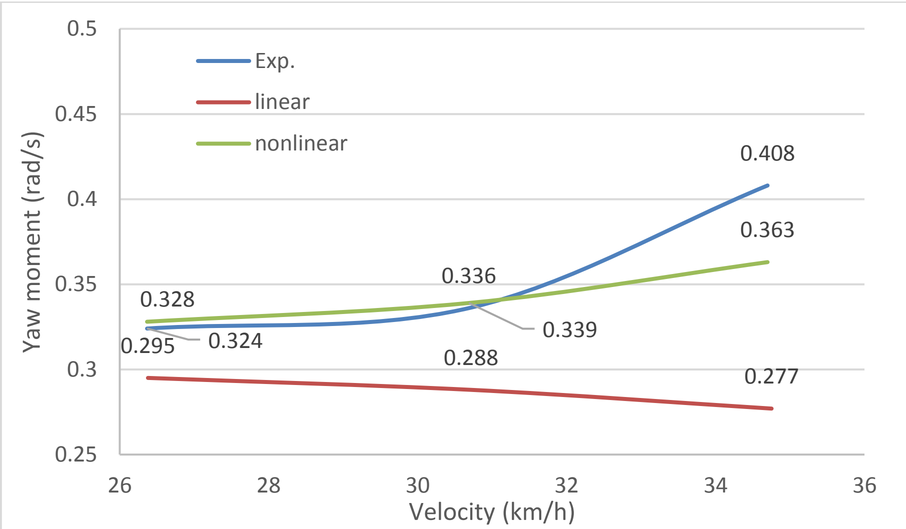


**Figure 12**. The change in yaw moment with increasing speed

## 7. Conclusion

This article mainly studies which equation is more suitable to do the numerical simulation between either linear equations or nonlinear equations. The whole study is divided into three stages. the first stage is to do the road testing using the testing car; the second stage is using the parameter of the testing car to build the simulation model and to do the simulation test; the third stage is to collect all the data and doing the analysis and compare the results between simulation and experimental. By comparing the results of the experiment and simulation, when the vehicle is running at a slower speed, the yaw moment simulated by the linear equation differs from the experimental yaw moment by 0.029 rad/s, while the yaw moment simulated by the nonlinear equation differs from the experimental result by 0.004 rad / s. As the speed increases, the results of the linear equation simulation become smaller and deviate more and more from the experimental results. The nonlinear simulation results are gradually rising with the speed increase and compared with the linear equation, it is closer to the experimental results, and when the speed is about 31km/h, the results of the nonlinear equation simulation are consistent with the experimental results. So, it can be said that the nonlinear equations are more suitable as equations for simulating vehicle dynamics.

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