Simple Pole Placement Controller for Elastic Joint Manipulator

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Abstract - **This paper presents investigations into the development of simple pole placement controller for tip angular position tracking and deflection reduction of an elastic joint manipulator system. A Quanser elastic joint manipulator is considered and the dynamic model of the system is derived using the Euler-Lagrange formulation. The pole placement controller is designed based on integral state feedback structure and the feedback gain is computed based on the desired time response specifications of tip angular position. The proposed control scheme is also compared with a hybrid Linear Quadratic Regulator (LQR) with input shaper control scheme. The performances of the control schemes are assessed in terms of tip angular tracking capability, level of deflection angle reduction and time response specifications. Finally, a comparative assessment of the control techniques is presented and discussed.**

Keywords – **Elastic joint, vibration control, pole placement, LQR and input shaper.**

I. INTRODUCTION

Recently, elastic joint manipulators have received a growth number of attention from many researchers due to its light weight, high manoeuvrability, flexibility, high power efficiency, and large number of applications. Nevertheless, controlling such systems still faces numerous degree of difficulty that need to be addressed before they can be used in abundance in everyday real-life applications. The control issue of the elastic joint is to design the controller so that link of robot can reach a desired position or track a prescribed trajectory precisely with minimum deflection to the link. In order to achieve these objectives, various methods using different technique have been proposed. Such as adaptive output-feedback controller based on a backstepping design [1],[2],[3], non linear control approach using namely feedback linearization technique and the integral manifold technique [4],[5], robust control design [6],[7], LQR with input shaping scheme [8], LQR with non-collocated PID [9], and intelligent schemes based on fuzzy logic controller [10,11,12] with the combination of proportional and derivatives gain. In general, the above mentioned control strategies is highly complicated and need to consider various parameter in the design requirement. In particular, some of the parameters need to be determined in a heuristic manner which is a very exhausted work.

This paper addresses a simple pole placement controller based on integral state feedback structure for elastic joint manipulator. This study shows that by only setting the desired settling time and overshoot, the dominant poles of the closed loop system can be easily determined. The designed dominant poles and other poles which are located ten times from the dominant poles are capable to control the tip angular position with minimal deflection angle. To examine the effectiveness of the proposed controller, it is compared with hybrid LQR with input shaper control scheme. The performances of both controllers are investigated in terms of tip angular tracking capability, level of deflection angle reduction and time response specifications. The implementation results show that the simple pole placement controller provide a fast input tracking response with very minimal deflection angle as compared to hybrid control schemes.

This paper is organized as follows: The next section provides a description of the linear model of elastic joint manipulator system in a state-space form. Section III is devoted to develop a tip angular tracking and deflection angle reduction control schemes for elastic joint manipulator system. Implementation results are shown in section IV and conclusions are drawn in section V.

II. MODELING OF ELASTIC JOINT MANIPULATOR

The elastic joint manipulator system considered in this work is shown in Figure 1, where θ, is the tip angular position and α is the deflection angle of the elastic joint. The base of the elastic joint manipulator which determines the tip angular position of the flexible link is driven by servomotor, while the flexible link will response based on base movement. The deflection of link will be determined by the flexibility of the spring as their intrinsic physical characteristics.

This section provides a brief description on the modelling of the elastic joint manipulator system, as a basis of a simulation environment for development and assessment of the pole placement control technique. The Euler-Lagrange formulation is considered in characterizing the dynamic behaviour of the system.

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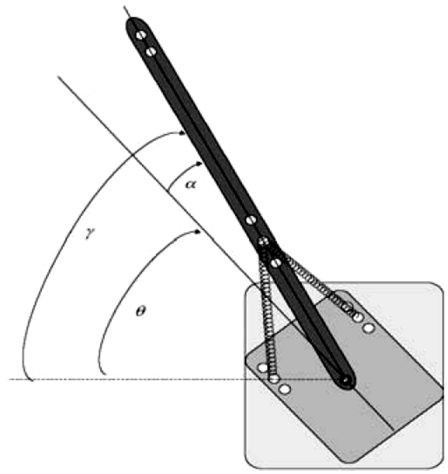

Figure 1. Elastic joint manipulator system.

The linear model of the uncontrolled system can be represented in a state-space form [13] as shown in equation (1), that is

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

with the vector $x = \begin{bmatrix}\theta & \alpha & \dot{\theta} & \dot{\alpha}\end{bmatrix}^T$ and the matrices *A*, *B* and *C* are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{K_{stiff}}{J_{eq}} & \dfrac{-\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \\ 0 & \dfrac{-K_{stiff}(J_{eq} + J_{arm})}{J_{eq} J_{arm}} & \dfrac{\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & \dfrac{\eta_m \eta_g K_t K_g}{J_{eq} R_m} & \dfrac{-\eta_m \eta_g K_t K_g}{J_{eq} R_m} \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

In equation (1), the input *u* is the input voltage of the servomotor, V_m which determines the elastic joint manipulator base movement. In this study, the values of the parameters are defined in Table 1.

TABLE I.
SYSTEM PARAMETERS

Symbol	Quantity	Value
R_m	Armature Resistance (Ohm)	2.6
K_m	Motor Back-EMF Constant (V.s/rad)	0.00767
K_t	Motor Torque Constant (N.m/A)	0.00767
J_{link}	Total Arm Inertia (kg.m^2)	0.0035
J_{eq}	Equivalent Inertia (kg.m^2)	0.0026
K_g	High gear ratio	14:5
K_{stiff}	Joint Stiffness	1.2485
B_{eq}	Equivalent Viscous Damping (N.m.s/rad)	0.004
$_g$	Gearbox Efficiency	0.9
$_m$	Motor Efficiency	0.69

III. CONTROLLER DESIGN

This section provides a description on the pole placement and hybrid LQR and input shaper control design for elastic joint manipulator system. The main objective of both controllers is to achieve good performance in input tracking of tip angular position with minimal deflection angle.

A. Pole placement controller

In this study, an integral state feedback control is used as a platform to design the proposed controller. The block diagram of integral state feedback control is shown in Figure 2.

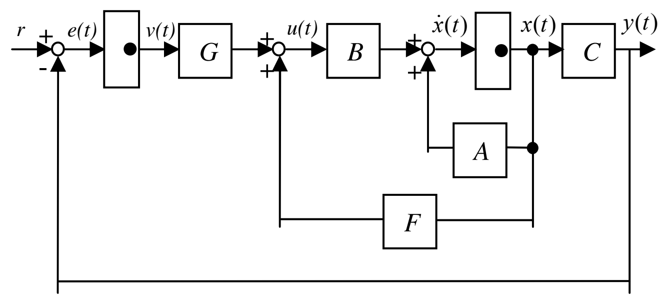

Figure 2. Block diagram of integral state feedback control.

The main objective of the proposed controller is to find the gain parameter matrix, *F* and *G* such that it fulfils the design requirement. From the block diagram of Figure 2, the control input of the system is derived as follow

$$u(t) = Fx(t) + Gv(t) \tag{3}$$

where $v(t) = \int_0^t e(\tau)d\tau$ and $e(t) = r - y(t)$

Using new state variable $x_e = \begin{bmatrix}x^T & v\end{bmatrix}^T$ and equation (3) the representation of state space equation can be rewrite as

$$\begin{aligned} \begin{bmatrix}\dot{x}(t) \\ \dot{v}(t)\end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}\begin{bmatrix}x(t) \\ v(t)\end{bmatrix} + \begin{bmatrix}B \\ 0\end{bmatrix}u(t) + \begin{bmatrix}0 \\ 1\end{bmatrix}r \\ e(t) &= r - Cx(t) \end{aligned} \tag{4}$$

Next, at the steady state condition as $t \to \infty$, the state space equation can be written in the following form

$$\begin{aligned} \begin{bmatrix}0 \\ 0\end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}\begin{bmatrix}x(\infty) \\ v(\infty)\end{bmatrix} + \begin{bmatrix}B \\ 0\end{bmatrix}u(\infty) + \begin{bmatrix}0 \\ 1\end{bmatrix}r \\ 0 &= r - Cx(\infty) \end{aligned} \tag{5}$$

By subtracting (4) to (5), the state space form is converted to

$$\dot{\tilde{x}}_e(t) = \tilde{A}\tilde{x}_e(t) + \tilde{B}_2\tilde{u}(t)$$
$$\tilde{e}(t) = \tilde{C}_1\tilde{x}_e(t) \tag{6}$$

where

$$\tilde{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix},\ \tilde{B}_2 = \begin{bmatrix} B \\ 0 \end{bmatrix},\ \tilde{x}_e = \begin{bmatrix} \tilde{x} \\ \tilde{v} \end{bmatrix} = \begin{bmatrix} x - x(\infty) \\ v - v(\infty) \end{bmatrix}$$
$$\tilde{C}_1 = \begin{bmatrix} -C & 0 \end{bmatrix},\ \tilde{e}(t) = e - e(\infty)$$

Then, the new control input function is described as follow

$$\tilde{u}(t) = F\tilde{x}(t) + G\tilde{v}(t) = K_{PP}\tilde{x}_e(t) \tag{7}$$

Finally, a closed loop state space equation with controller gain, K_{PP} can be obtained below

$$\dot{\tilde{x}}_e(t) = \tilde{A}_{cl}\tilde{x}_e(t) + \tilde{B}_1 w$$
$$\tilde{y}(t) = \tilde{C}_1\tilde{x}_e(t) + \tilde{D}_{11}w + \tilde{D}_{12}u \tag{8}$$

where

$$\tilde{A}_{cl} = (\tilde{A} + \tilde{B}_2 K),\ \tilde{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \end{bmatrix},\ \tilde{D}_{11} = 1,\ \tilde{D}_{12} = 0$$

and w is exogenous input disturbance or reference input to the system. Let $G_{yw}(s)$ denote the closed loop transfer function from w to y under state feedback control $u = K_{PP}x$. The controller gain, K_{PP} can be determined from the location of poles that form the characteristics equation. In particular, the dominant poles, $\sigma \pm j\omega$ can be determined from the desired settling time (T_s) and percentage of overshoot (OS) of the system as shown in (9) and (10).

$$\sigma = \zeta\omega_n \tag{9}$$

$$\omega = \omega_n\sqrt{1-\zeta^2} \tag{10}$$

where damping ratio and natural frequency are given as $\zeta = -\dfrac{\ln\left(\frac{OS}{100}\right)}{\sqrt{\pi^2 + \ln\left(\frac{OS}{100}\right)^2}}$ and $\omega_n = \dfrac{4}{\zeta T_s}$, respectively. Then, the other poles are located at a distance 10 times the value of the dominant poles location on the left hand side of s-plane.

B. Hybrid LQR with input shaper controller

In this hybrid control technique, initially an LQR controller is developed for tip angular position control of elastic joint manipulator [9]. Then, this is extended to incorporate input shaper control schemes for deflection angle suppression. Figure 3 shows the block diagram of LQR with input shaper control structure. The tracking performance of the LQR control applied to the elastic joint manipulator system was investigated by obtaining the value of vector K_{LQR} and $\bar{N}$ which determines the feedback control law and for elimination of steady state error respectively. The natural frequency was obtained by exciting the elastic joint manipulator system with an unshaped reference input under LQR controller. The input shapers were designed for pre-processing the trajectory reference input and applied to the system in a closed-loop configuration.

For an linear time invariant (LTI) system in (1), technique involves choosing a control law $u = \psi(x)$ which stabilizes the origin (i.e., regulates x to zero) while minimizing the quadratic cost function

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t)dt \tag{11}$$

where $Q = Q^T \geq 0$ and $R = R^T > 0$. The term "linear-quadratic" refers to the linear system dynamics and the quadratic cost function.

The matrices Q and R are called the state and control penalty matrices, respectively. If the components of Q are chosen large relative to those of R, then deviations of x from zero will be penalized heavily relative to deviations of u from zero. On the other hand, if the components of R are large relative to those of Q, then control effort will be more costly and the state will not converge to zero as quickly.

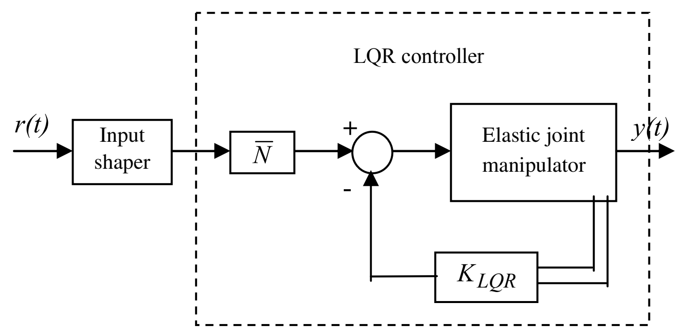

Figure 3. Block diagram of the hybrid LQR with input shaper control schemes configuration.

A famous and somewhat surprising result due to Kalman is that the control law which minimizes J always takes the form $u = \psi(x) = -K_{LQR}x$. The optimal regulator for a LTI system with respect to the quadratic cost function above is always a linear control law. With this observation in mind, the closed-loop system takes the form

$$\dot{x} = (A - BK_{LQR})x \tag{12}$$

and the cost function J takes the form

$$J = \int_0^\infty x(t)^T Qx(t) + (-K_{LQR}x(t))^T R(-K_{LQR}x(t))dt \tag{13}$$

$$J = \int_0^{\infty} x(t)^T (Q + K_{LQR}{}^T R K_{LQR}) x(t) dt \quad (14)$$

Next, the input shaping techniques are designed based on the amplitude and time locations of the impulses in order to reduce the detrimental effects of the system flexibility. These parameters are obtained from the natural frequency and damping ratio of the closed-loop system under LQR control schemes.

The requirement of positive amplitudes for the input shapers has been used in most input shaping schemes. The requirement of positive amplitude for the impulses is to avoid the problem of large amplitude impulses. For the case of positive amplitudes, each individual impulse must be less than one to satisfy the unity magnitude constraint. In order to increase the robustness of the input shaper to errors in natural frequency, the positive Zero-Vibration-Derivative-Derivative (ZVDD) input shaper, is designed by solving the derivatives of the system vibration equation. This yields a four-impulse sequence with parameter as

$$t_1 = 0,\ t_2 = \frac{\pi}{\omega_d},\ t_3 = \frac{2\pi}{\omega_d},\ t_4 = \frac{3\pi}{\omega_d}$$

$$A_1 = \frac{1}{1+3K+3K^2+K^3},\ A_2 = \frac{3K}{1+3K+3K^2+K^3}$$

$$A_3 = \frac{3K^2}{1+3K+3K^2+K^3},\ A_4 = \frac{K^3}{1+3K+3K^2+K^3} \quad (15)$$

where

$$K = e^{-\zeta\pi / \sqrt{1-\zeta^2}},\quad \omega_d = \omega_n \sqrt{1-\zeta^2}$$

(ω_n and ζ representing the natural frequency and damping ratio respectively) and t_j and A_j are the time location and amplitude of impulse j respectively. Please refer [9] for the detail of the design scheme.

In this work, the design of the pole placement and hybrid LQR with input shaper controllers must fulfil the following specifications:

- Settling time of less than 1.5 s with overshoot less than 5% and zero steady state error for the tip angular position
- Deflection angle is less than ±1.5 degree.

V. Implementation and Results

In this section, the proposed control schemes are simulated and tested to the elastic joint manipulator model and the corresponding results are presented. The tip angular position is required to follow a trajectory motion of 50 degree. System responses namely the tip angular position, deflection angle and power spectral density of deflection angle are observed. The performances of the control schemes are assessed in terms of input tracking, deflection angle reduction and time response specifications. Finally, a comparative assessment of the performance of the control schemes is presented and discussed.

Applying equation (9) and (10) based on the design requirement; the dominant poles are obtained as $-4.444 \pm j1.516$ while the other poles are setting at -30, -35, -40. Hence, the state feedback gain for pole placement controller, K_{PP} is obtained as follows:

$$K_{PP} = [5055.7 \quad -8494.2 \quad 922.04 \quad 463.35 \quad -10318].$$

On the other hand, using the *lqr* function in the Matlab, both vector K_{LQR} and $\overline{N}$ were set as

$$K_{LQR} = [28.2843 \quad 23.0634 \quad 11.3478 \quad 5.5665] \text{ and}$$
$$\overline{N} = [28.2843].$$

Then, the positive ZVDD input shaper is designed based on the single mode of vibration frequency (6 Hz) from the analysis of closed-loop configuration with LQR control [9].

The response of tip angular position, deflection angle and power spectral density (PSD) of deflection angle of the elastic joint manipulator is depicted in Figs. 4-6 for both pole placement and hybrid LQR with input shaper controller. It shows that both controller can track the desired trajectory input with zero steady state error and achieve zero vibration from the response of deflection angle. Hence, in overall both controllers successfully fulfil the design requirement. Table 1 summarises the time response specifications of tip angular position. It is noted that the pole placement controller produces a fast settling time with zero overshoot as compared to hybrid LQR with input shaper controller. In addition, the pole placement controller also shows a very minimal oscillation (low frequency) at the deflection angle response as compared to hybrid LQR with input shaper controller. In terms of magnitude of oscillation, the deflection angle response of the pole placement controller was found to oscillate between ±1.22 degree which is lower than hybrid control schemes with ±1.36 degree. In addition, in frequency domain response, the pole placement controller also achieved smaller level of vibration magnitude as compared to hybrid LQR with input shaper.

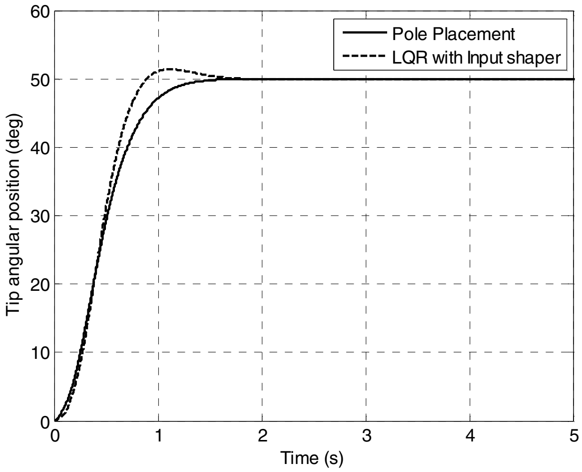

Figure 4. Tip angular position response with pole placement and hybrid LQR-input shaper control.

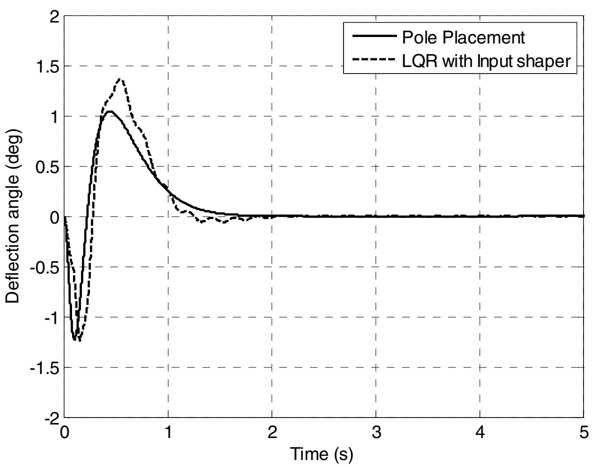

Figure 5. Deflection angle response with Pole placement and hybrid LQR-input shaper control.

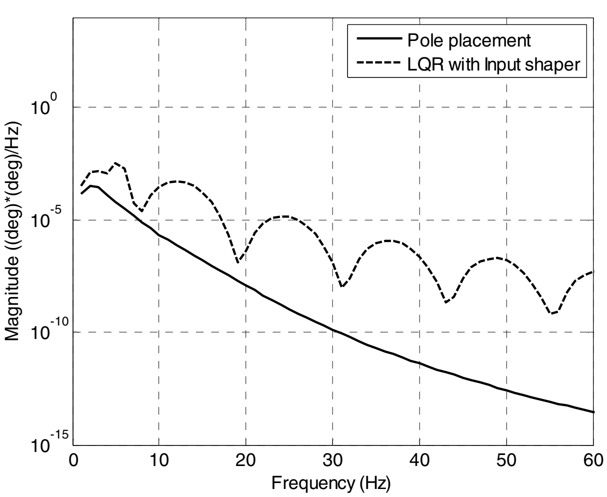

Figure 6. PSD response with Pole placement and hybrid LQR-input shaper control.

TABLE II. TIME RESPONSE SPECIFICATIONS OF TIP ANGULAR POSITION

Controller		Pole placement	LQR-input shaper
Specifications of tip angular position response	Settling time (s)	1.219	1.329
	Rise time (s)	0.701	0.515
	Overshoot (%)	0.00	2.88

VI. CONCLUSIONS

The control schemes development of simple pole placement controller based on integral state feedback has been presented. The proposed controller can easily achieved the design requirement based on the exact location of poles. The performances of the proposed control schemes are compared with LQR with input shaper controller and have been evaluated in terms of tip angular tracking capability and deflection angle reduction. The results show that the pole placement controller provide a faster tip angular tracking response with zero overshoot and minimal level of deflection angle as compared to hybrid LQR and input shaper control schemes. The work thus developed and reported in this paper forms the basis of design and development of experimental work for trajectory tracking and vibration suppression of others flexible structure system.

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REFERENCES

[1] W. Yim, "Adaptive Control of a Flexible Joint Manipulator," Proc. 2001 IEEE, International Robotics & Automation, Seoul, Korea, pp. 3441–3446, 2001. doi: 10.1109/ROBOT.2001.933150.

[2] J.H. Oh, and J.S. Lee, "Control of Flexible Joint Robot System by Backstepping Design Approach," Proc. of IEEE International Conference on Robotics & Automation, pp. 3435–3440, 1997. doi: 10.1109/ROBOT.1997.606867.

[3] F. Ghorbel, J.Y. Hung, and M.W. Spong, M.W., "Adaptive Control of Flexible Joint Manipulators," Control Systems Magazine, Vol. 9, pp. 9-13, 1989.

[4] L.C. Lin, and K. Yuan, "Control of Flexible Joint Robots via External Linearization Approach," Journal of Robotic Systems, Vol. 1 (1), pp. 1-22, 2007. doi: 10.1002/rob.4620070102.

[5] M.W. Spong, K. Khorasani, and P.V. Kokotovic, "An Integral Manifold Approach to the Feedback Control of Flexible Joint Robots," IEEE Journal of Robotics and Automation, Vol. RA-3, No.4, pp. 291-300, 1987. doi: 10.1109/JRA.1987.1087102.

[6] P. Tomei, "A Simple PD Controller for Robots with Elastic Joints," IEEE Trans. on Automatic Control, Vol. 36, No. 10, pp. 1208-1213, 1991. doi: 10.1109/9.90238.

[7] J.S. Yeon, and J.H. Park, "Practical Robust Control for Flexible Joint Robot Manipulators," Proc. 2008 IEEE International Conference on

Robotic and Automation, Pasadena, CA, USA, pp. 3377–3382, 2008. doi: 10.1109/ROBOT.2008.4543726.

[8] M.A. Ahmad, "Vibration and Input Tracking Control of Flexible Manipulator using LQR with Non-collocated PID controller," Proc. of 2nd UKSIM European Symposium on Computer Modelling and Simulation, UK, pp. 40-45, 2008. doi: 10.1109/EMS.2008.75.

[9] M.A. Ahmad, M.S. Ramli, R.M.T. Raja Ismail, N. Hambali, M.A. Zawawi, "The investigations of input shaping with optimal state feedback for vibration control of a flexible joint manipulator" Proc. of Conference on Innovative Technologies in Intelligent Systems and Industrial Applications, Kuala Lumpur, pp. 446-451, 2009. doi: 10.1109/CITISIA.2009.5224165

[10] M.A. Ahmad, R.M.T. Raja Ismail, M.S. Ramli, M.A. Zawawi, N. Hambali, and N.M. Abd Ghani, "Vibration Control of Flexible Joint Manipulator using Input Shaping with PD-type Fuzzy Logic Control," Proceedings of IEEE International Symposium on Industrial Electronics, Seoul, pp. 1184-1189, 2009. doi: 10.1109/ISIE.2009.5215717.

[11] M.A. Ahmad, A.N.K. Nasir, N. Hambali, and H. Ishak, "Vibration and Input Tracking Control of Flexible Manipulator using Hybrid Fuzzy Logic Controller," Proceedings of the IEEE International Conference of Mechatronics and Automation, Kagawa, pp. 593-598, 2008. doi: 10.1109/ICMA.2008.4798823.

[12] M.A. Ahmad, M.H. Suid, M.S. Ramli, M.A. Zawawi, and R.M.T. Raja Ismail, "PD fuzzy logic with non-collocated PID approach for vibration control of flexible joint manipulator," Proc. of 6th International Colloquium on Signal Processing and Its Applications, Mallaca city, pp. 1-5, 2010. doi: 10.1109/CSPA.2010.5545243.

[13] Quanser Student Handout, Rotary Flexible Joint Module. http://www.quanser.com.