Stagnation Point Flow over a Stretching Sheet with Convective Boundary Conditions

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Abstract. In this study, the numerical investigation of stagnation point flow over a stretching surface with Newtonian heating and convective boundary conditions are considered. The transformed boundary layer equations are solved numerically using the Keller-box method. Numerical solutions are obtained for the local heat transfer coefficient, the surface temperature as well as the temperature profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number, stretching parameter and conjugate parameter are analyzed and discussed. As a conclusion, the thermal boundary layer thickness depends strongly on these three parameters. It is found that, as Prandtl number and stretching parameter increase, the temperature profiles decrease. While, as conjugate parameter decrease, the temperature profiles also decrease.

Keywords: Convective boundary conditions, numerical solution, stagnation point flow, stretching sheet
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INTRODUCTION

Problems related to convection boundary layer flows are important in engineering and industrial activities. Such flows are applied to manage thermal effects in many industrial outputs for example in electronic devices, computer power supply and also in engine cooling system such as heatsink in car radiator. Sakiadis [1] was the first to study the boundary layer flow on a continuous solid surface moving at constant speed. Due to entrainment of the ambient fluid, this boundary layer flow is quite different from Blasius flow past a flat plate. Sakiadis's theoretical predictions for Newtonian fluids were later corroborated experimentally by Tsou et al. [2]. The flow of a viscous fluid past a stretching sheet is a classical problem in fluid dynamics. Crane [3] was the first to study the convection boundary layer flow over a stretching sheet. The heat and mass transfer on a stretching sheet with suction or blowing were investigated by Gupta and Gupta [4]**.** They considered on isothermal moving plate and obtained the temperature and concentration distributions. Chen and Char [5] studied the laminar boundary layer flow and heat transfer from a linearly stretching, continuous sheet subjected to suction or blowing with prescribed wall temperature and heat flux.

Ishak et al. [6, 7, 8] studied the MHD stagnation point flow towards a stretching sheet, mixed convection towards vertical and continuously stretching sheet, and post stagnation-point towards vertical and linearly stretching sheet. This type of problem was then extended to viscous fluids, viscoelastic fluids or micropolar fluids by many investigators by considering the usually applied boundary conditions, either prescribed wall temperature or prescribed wall heat flux. Recently, Mohamed et al. [9] studied on the stagnation point flow over a stretching sheet and Hayat et al. [10] investigated flow of a second grade fluid over a stretching surface with Newtonian heating.

On the other hand, Merkin [11] has shown that, in general, there are four common heating processes specifying the wall-to-ambient temperature distributions, namely, (i) constant or prescribed wall temperature; (ii) constant or prescribed surface heat flux; (iii) Newtonian heating (NH); and (iv) convective/conjugate boundary conditions (CBC), where heat is supplied through a bounding surface of finite thickness and finite heat capacity. The interface temperature is not known a priori but depends on the intrinsic properties of the system, specifically the thermal conductivity of the fluid or solid.

Since the early paper by Luikov et al. [12], many contributions to the topic of conjugate heat transfer have been studied. The conjugate/convective boundary condition, has been used only quite recently by Aziz [13] who studied the laminar thermal boundary layer over a flat plate. This Blasius flow with conjugate boundary condition then have been revisited by Rashidi and Erfani [14] and Magyari [15]. Makinde and Aziz [16] considered the MHD mixed convection over a vertical plate in porous medium. Ishak [17, 18] have studied the thermal boundary layer flow on a moving plate (Sakiadis flow) with radiation effects. Recently, Merkin and Pop [19], Yao et al. [20], Yacob et al. [21] and Yacob and Ishak [22] investigated the boundary layer flow past a shrinking/stretching sheet with convective boundary conditions in a viscous fluid, nanofluid or micropolar fluid, respectively. Excellent reviews of the topics of convective heat transfer problems can be found in the books by Kimura et al. [23] and Martynenko and Khramtsov [24].

The aim of this study is to investigate the problem of stagnation point flow over a stretching sheet with convective boundary conditions. The governing nonlinear partial differential equations are first transformed into a system of ordinary differential equations by the similarity transformation, before being solved numerically using the Keller-box method. To the best of our knowledge this problem has not been considered before, so the reported results are new.

MATHEMATICAL FORMULATION

A steady two-dimensional stagnation-point flow over a stretching/shrinking plate immersed in an incompressible viscous fluid of ambient temperature, T_∞ is considered. It is assumed that the external velocity $u_e(x)$ and the stretching velocity $u_w(x)$ are of the forms $u_e(x) = ax$ and $u_w(x) = bx$ where *a* and *b* are constants. It is further assumed that the plate is subjected to a conjugate boundary condition. The boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions (Salleh [25] and Aziz [13])

$$\begin{aligned} &u = u_w(x), \quad v = 0, \quad \frac{\partial T}{\partial y} = -h_f T \text{ (NH)}, \quad -k\frac{\partial T}{\partial y}(x,0) = h_f(T_f - T(x,0)) \text{ (CBC) at } y = 0 \\ &u = u_e(x), \quad T \to T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{4}$$

where *u* and *v* are the velocity components along the *x* and *y* directions, respectively. Furthermore, *T* is temperature, T_f is the fluid at temperature *T*, ν is the kinematic viscosity, k is the thermal conductivity, α is the thermal diffusivity and h_f is the heat transfer coefficient.

We now introduce the following similarity variables (see Salleh et al. [26] and Aziz [13]):

$$\eta = \left(\frac{u_e}{\nu x}\right)^{1/2} y, \quad \psi = \left(\nu x u_e\right)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty} \; (NH), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty} (CBC), \tag{5}$$

where ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, which identically satisfies Equation (1). Thus, we have

$$u = axf'(\eta), \quad v = -(a\nu)^{1/2} f(\eta) \tag{6}$$

where prime denotes differentiation with respect to η. Substituting (5) and (6) into Equations (2) and (3), we obtain the following nonlinear ordinary differential equations:

$$f''' + ff'' + 1 - f'^2 = 0 \tag{7}$$

$$\frac{1}{\Pr}\theta'' + f\theta' = 0 \tag{8}$$

where $\Pr = \dfrac{\nu}{\alpha}$ is the Prandtl number. The boundary conditions (4) become

$$f(0) = 0, \quad f'(0) = \varepsilon, \quad \theta'(0) = -\gamma(1+\theta(0)) \text{ (NH)}, \quad \theta'(0) = -\gamma(1-\theta(0)) \text{ (CBC)} \tag{9}$$

$$f'(\eta) \to 1, \; \theta(\eta) \to 0 \quad \text{as } \eta \to \infty \tag{10}$$

where $\varepsilon = \dfrac{b}{a}$ is the stretching parameter. Besides, $\gamma = h_s \left(\dfrac{\nu}{a}\right)^{1/2}$ (NH) or $\gamma = h_f \left(\dfrac{\nu}{a}\right)^{1/2} k^{-1}$ (CBC) is the conjugate parameter for the convective boundary conditions. It is noticed that $\gamma = 0$ is for the insulated plate and $\gamma \to \infty$ is when the surface temperature is prescribed.

SOLUTION PROCEDURE

Equations (7) and (8) subject to boundary conditions (9) and (10) are solved numerically using the Keller-box method as described in the books by Na [27] and Cebeci and Bradshaw [28]. The solution is obtained in the following four steps:

1. reduce Equations (7) and (8) to a first-order system,
2. write the difference equations using central differences,
3. linearize the resulting algebraic equations by Newton's method, and write them in the matrix-vector form,
4. solve the linear system by the block triadiagonal elimination technique.

RESULTS AND DISCUSSION

The Equations (7) and (8) subject to the boundary conditions (9) were solved numerically using the Keller-box method with three parameters considered, namely the Prandtl number $\Pr$, the conjugate parameter γ and the stretching parameter ε. Due to the decoupled boundary layer equations (7) and (8), for $\varepsilon = 0$, it is found that there is a unique value of the skin friction coefficient, $f''(0) = 1.23258766$, which is in very good comparison with the classical value $f''(0) = 1.232588$ by Hiemenz [29]. Table 1 presents the comparison between the present results with the previously reported results by Salleh [25] and Mohamed et al. [9] for various values of the Prandtl number Pr when $\gamma = 1$ and $\varepsilon = 0$, considering the case of Newtonian heating (NH). It is found that they are in good agreement. We can conclude that this method works efficiently for the present problem and we are also confident that the results presented here are accurate.

TABLE 1. Comparison between the present solution of Equations (7) and (8) with previously published results when $\gamma = 1$ and $\varepsilon = 0$ (NH)

Pr	**Salleh [25]**		**Mohamed et al. [9]**		**Present**	
	$\theta(0)$	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$
5	23.0042	24.0042	23.0239	24.0239	23.0042	24.0042
7	5.6872	6.6872	5.6062	6.6062	5.6873	6.6873
10	2.9226	3.9226	2.9516	3.9516	2.9227	3.9227
100	0.6866	1.6866	0.5034	1.5034	0.6866	1.6866
1000	0.2593	1.2593	0.1809	1.1809	0.2577	1.2577

Figure 1 presents the temperature profiles for various values of Pr. It is found that as Pr increases, the temperature in the boundary layer decreases, and the thermal boundary layer thickness also decreases. This is because for small values of the Prandtl number, the fluid is highly thermal conductive. Physically, if Pr increases, the thermal diffusivity decreases and these phenomena lead to the decreasing of energy ability that reduces the thermal boundary layer.

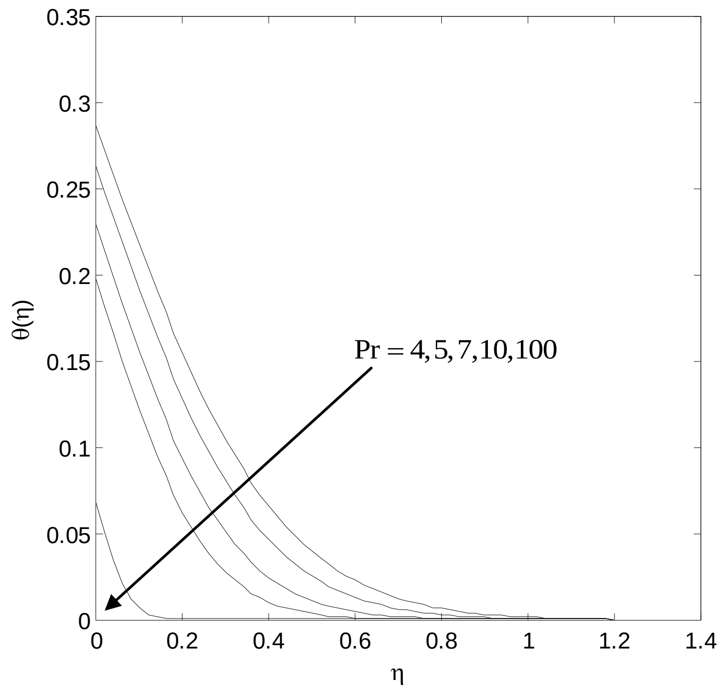

FIGURE 1. Temperature profiles $\theta(\eta)$ for various values of Pr when $\gamma = 1$ and $\varepsilon = 3$ (CBC)

The temperature profiles with various values of ε are presented in Figure 2, and it is found again that as ε increases, the temperature decreases, and the thermal boundary layer thickness also decreases, similar to Figure 1.

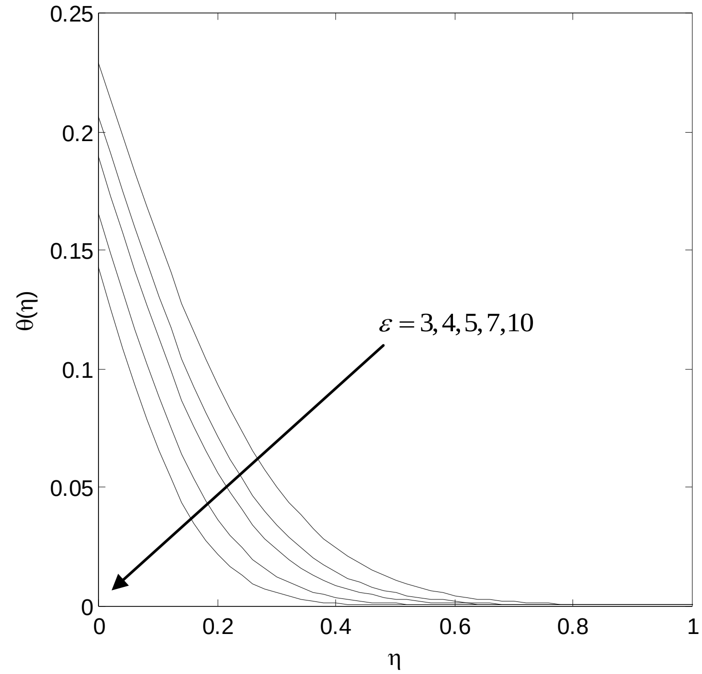

FIGURE 2. Temperature profiles $\theta(\eta)$ for various values of ε when $\gamma = 1$ and $\Pr = 7$ (CBC)

The temperature profiles presented in Figure 3 show that when the value of the conjugate parameter γ increases, it is found that the temperature also increases, contrary to the temperature profiles with various values of $\Pr$ and ε in Figures 1 and 2.

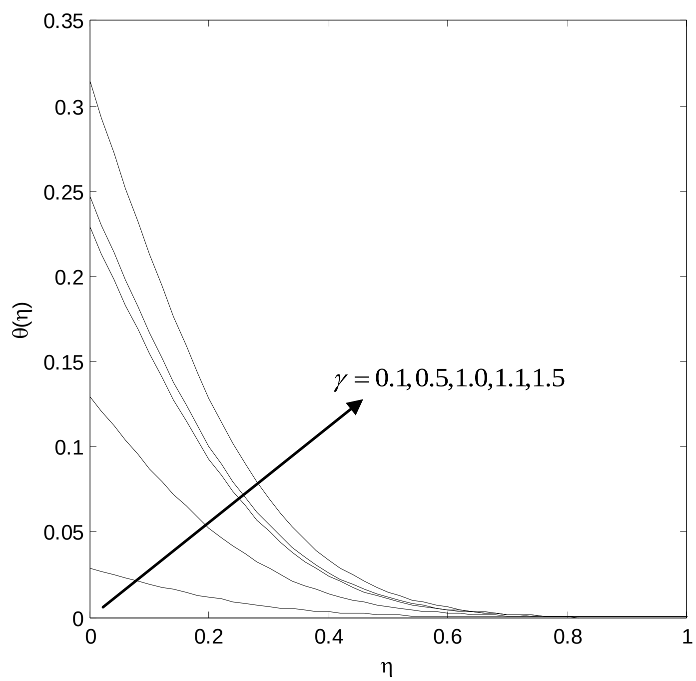

FIGURE 3. Temperature profiles $\theta(\eta)$ for various values of γ when $\varepsilon = 3$ and $\Pr = 7$ (CBC)

CONCLUSION

In this paper we have numerically studied the problem of stagnation point flow over a stretching sheet with convective boundary conditions. It is shown how the Prandtl number $\Pr$, stretching parameter ε and conjugate parameter γ affect the values of the temperature profiles $\theta(0)$ and heat transfer coefficient $-\theta'(0)$.

We can conclude that the thermal boundary layer thickness depends strongly on these three parameters. Furthermore, it is seen that an increase in the Prandtl number $\Pr$ and stretching parameter ε result in the decrease of the temperature. The reason is that smaller values of $\Pr$ are equivalent to increasing thermal conductivity, and, therefore, heat is capable of diffusing away from the heated wall more rapidly than at higher values of $\Pr$. However, the increase of conjugate parameter γ lead to the increase of temperature profiles $\theta(0)$.

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