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An overview: on path planning optimization criteria and mobile robot navigation

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Abstract. Mobile robots are growing more significant from time to time and have been applied to many fields such as agriculture, space, and even human life. It could improve mobile robot navigation efficiency, ensure path planning safety and smoothness, minimize time execution, etc. The main focus of mobile robots is to have the most optimal functions. An intelligent mobile robot is required to travel autonomously in various environments, static and dynamic. This paper article presents the optimization criteria for mobile robot path planning to figure out the most optimal mobile robot criteria to fulfill, including modeling analysis, path planning and implementation. Path length and path smoothness are the most parameters used in optimization in mobile robot path planning. Based on path planning, the mobile robot navigation is divided into three categories: global navigation, local navigation and personal navigation. Then, we review each category and finally summarize the categories in a map and discuss the future research strategies.

1. Introduction

During the early stage of establishment, mobile robot applications were restricted to industries only. Nowadays, mobile robots have been grown their demand in various fields, including mining equipment, nuclear applications, aerospace systems [1], education, entertainment, rescuing, medicine, military, and agriculture. Besides, mobile robots are commonly practical used in human life [2], such as in [3] the sweeping robot is introduced for daily household cleaning, and some airports have deployed cleaning robots to replace human cleaners [4].

A mobile robot's real challenge is to move from one place to another place set as a start point to the target point correctly as smooth as possible with obstacle avoidance. Thus, the navigation of a mobile robot is essential to move the robot and essential for the robot that is entitled to be mobile [5]. Mobile robot navigation confronts three problems such as motion control, path planning and localization. Motion control is a decisive process on what action to be taken next concerning the current situation. Path planning generates a smooth path from starting to goal location as fast as possible and avoiding obstacles in the environment [6]. Sarif and Buniyamin state that path planning is the main problem in mobile robots and acts as an essential part of navigation [7]. Localization is a determining process of the current location of a mobile robot. Knowing the mobile robot's location is needed to decide for future action [8]. To conclude, mobile robot navigation problems are more understood by answering these three questions [9]:

i. Where am I?
ii. How do I get to the other places from here?
iii. Where are the other places related to me?

Since the development in technologies and methods approaches become more profound and broader, the mobile robot navigation thought would answer these two additional questions:

i. How is the structure of the environment I am in?
ii. How is this place like?

According to Nourbakhsh and Siegwart [10], path planning and localization are the essential factors in mobile robot navigation; besides, they state that perception is considered a mobile robot navigation problem. Path planning is a central problem in robotics since its intricacy increases tremendously with the configuration area's dimension. Figure 1 shows the part of the robot navigation problem in a real-world environment.

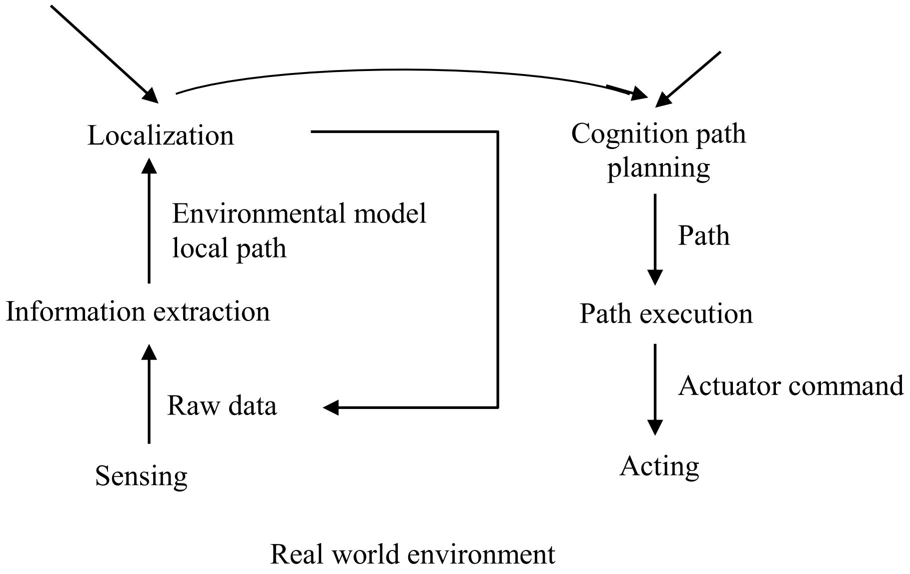

Figure 1. Part of robot navigation problem [10]

Mobile robot path planning is developing a path from the starting point to the working area's goal point. The focus of mobile robot path planning is to find the safe shortest path for the robot to move from a recent location to the destination point without obstacle-collision. Then, the path should be optimal to reduce energy consumption, timely execution and communication delay. Path planning also helps in saving a lot of time and the capital investment and the wear of a mobile robot; besides provides the free obstacle path with respect to distance, time, safety and smoothness [11].

Most research presenting the mobile robot navigation algorithms and optimization criteria [12]; however, the necessity of versatile research emerged from classic approach to heuristic approach to meta-heuristic approach and recently hybridization. This paper review the categories of mobile robot navigations and optimization criteria to figure out the scope of innovation in the particular area for suitable environments. This paper highlights mobile robot navigation efficiency and the effectiveness of a mobile robot's optimization criteria to be fulfilled. It is necessary to determine whether the developed path achieved optimality conditions for extreme trajectories.

2. Mobile robot navigation

Mobile robot path planning navigation can be divided into three categories according to the data received from the environment [13]; global navigation, local navigation, and personal navigation. The three of them differ in terms of definition and methods, but they focus on answering the same question. Earlier studies combined the previously mentioned two types of algorithms; however, the combination must be depending upon the robot's autonomy and environmental factors [14].

Global navigation can see the environment's position elements regarding the reference axis and stir towards the pre-selected goal [12]. The mobile robots need advanced information on the atmosphere, obstacle status and goal location. Global navigation strategies deal with a wholly known environment. The path planning algorithm for a known environment is based on a classical approach such as cell decomposition (CD), roadmap approach (RA), and artificial potential field (APF). These algorithms are traditional and have limited intelligence due to the less robustness in terrain uncertainty [15]. According to authors in [16], a global navigation algorithm solves collisions by building a global environment map. When planning a path on the map, graph-based search methods' accuracy and speed depend on the search space's granularity. Hence, those approaches are not suitable for real-time application. As a result, some studies introduced local navigation algorithm.

Local navigation identifies the environment's dynamic conditions and establishes positional relationships among various elements. Unlike global navigation, local navigation does not involve advanced information of the atmosphere, obstacle status and goal location. Local navigation strategies deal with the unknown and partially known environment, while global navigation deals with a completely known environment [12]. Local navigational algorithms are known as reactive approaches as they are more intelligent and able to control and execute a plan autonomously, such as genetic algorithm (GA), fuzzy logic (FL), neural network (NN), firefly algorithm (FA), particle swarm algorithm (PSO), ant colony optimization (ACO), cuckoo search algorithm (CSA) and many more. Local algorithms show more elasticity in the environment and provide an optimized path [15]. However, local navigation's main weakness is the local minima. To avoid the problem, select the local target of the robot's surroundings, and calculate the next step's direction [17].

Lastly, personal navigation handles the various elements relative to each other by considering their position [12]. Personal navigation systems for localizing a user indoors; runs a map, user's location and direction of some kind—the summarized categories of mobile robot navigation algorithms as shown in Figure 3. Path planning can also be categorized as offline (static) path planning or online (dynamic) path planning depending on the surrounding features. Offline (static) path planning is the condition where the path choice is made in the case of static obstacles, while online (dynamic) path planning is when the path is made when the obstacles are moving [14], as shown in figure 2.

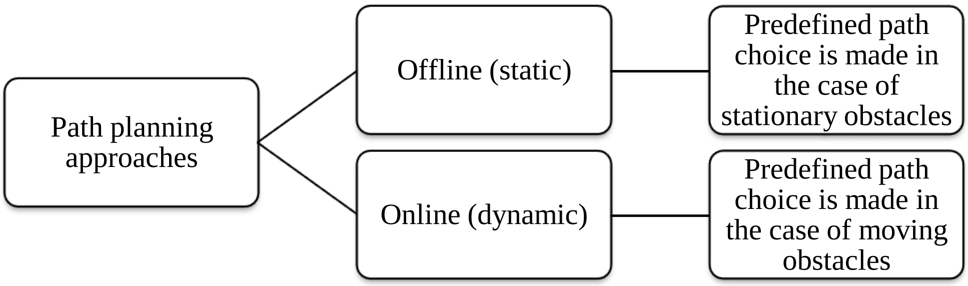

Figure 2. A path planning approach

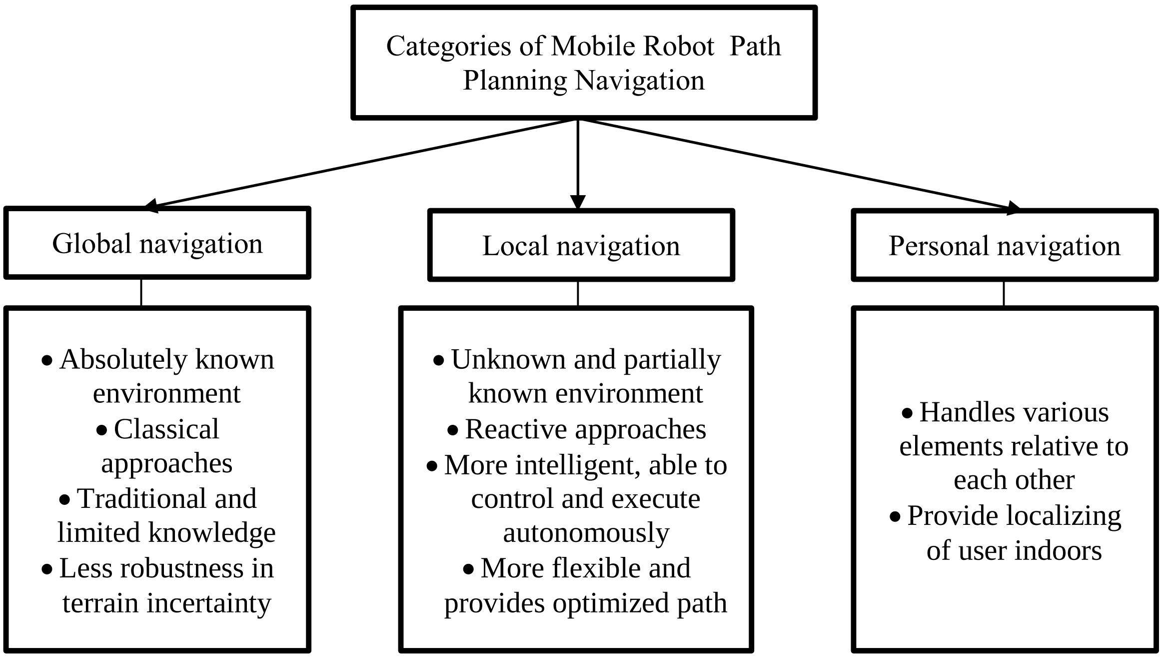

Figure 3. Mobile robot path planning categories

3. Optimization criteria for path planning

Standard multi-objective path planning functions are path length, path smoothness, path safety, total execution time and total energy consumption. The main goal in path planning is to create a set of route points for the robot from the start point to the goal point in an environment crowded with barriers while achieving specific optimization criteria such as shortest route, minimum energy use, path smoothness, minimum time execution, and maximum safety. At least two functions must be satisfied and classified as an NP-hard problem [18].

Given the workspace $W(S,O,G)$ where S is a starting point, G is a goal point, and O_i $i = 1,2,3,...,N$ denotes the set of obstacles, the objective is to find a collision-free path from S to G while optimizing the path. Notably, a path P is represented as a sequence of nodes it goes through, i.e. $P = \{p_1, p_2,..., p_n\}$, where $p_1 = S$, $p_n = G$ and $p_i (1 < i < n)$ could be any vertex of any obstacle. A path is considered valid if it does not cut through any obstacles, for example, $P_1 = \{S, O_{12}, O_{21}, O_{25}, O_{33}, O_{29}, G\}$ and $P_2 = \{S, O_{19}, O_{20}, O_{13}, G\}$ as shown in figure 4 with brown and blue lines, respectively.

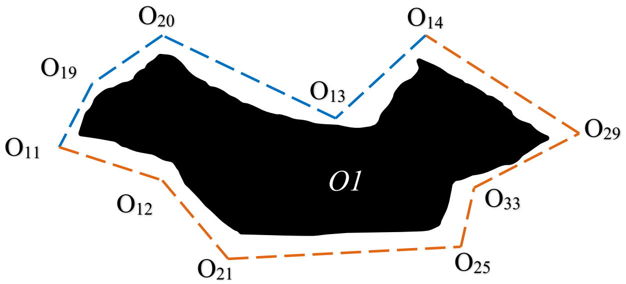

Figure 4. Representation of an obstacle in configuration space [19]

3.1. Path length

Path length in a mobile robot field refers to the robot's entire route from the starting point to the goal point. The length of a path P is defined as follows:

$$L(P)=\sum_{i=1}^{n-1}\left|P_iP_{i+1}\right| \tag{1}$$

where $\left|P_iP_{i+1}\right|$ indicates the Euclidean distance between two nodes P_i and P_{i+1}. The shortest path length involves reducing the overall total path distance between the start point S and goal point G. Uniformly distributed random points in a plane are generated using the following function [20]:

$$F_1(x,y)=d(wp_j,wp(K)) \tag{2}$$

The shortest distance (SD) such that the total length is the sum of all waypoints $wp_j\,(j=1,2,...,K)$ produced by the path planning algorithm from the start point $S(wp_1)$ and the goal point $G(wp_K)$.

$$SD=\sum\nolimits_{t=1}^{K-1}d(wp_i(t),wp_i(t+1))=\sum\nolimits_{t=1}^{K-1}d_t \tag{3}$$

where *i* is the index of the best solution obtained by the optimization-based path planning algorithm.

$$d_t=\sqrt{(x_{wp_{i+1}(t+1)}-x_{wp_i(t)})^2+(y_{wp_{i+1}(t+1)}-y_{wp_i(t)})^2}$$

Kayraki et al. [20] presented an algorithm using roadmap approach to getting the optimal path length. The approach has a failure to do so. Sanchez et al. [21] come up with minor modification using probabilistic roadmap approach to improve the process of getting the shortest path length. Xiao et al. [22] adopted genetic algorithm in order to achieve the optimization of path length, path smoothness and obstacle avoidance. A firefly algorithm have been presented in [23] and achieved three optimization criteria such as path smoothness, path length and path safety. In [24] adopted also firefly algorithm for shortest path length and obstacle avoidance. Li and Chow [25] produced self-adaptive learning particle swarm optimization (SLPSO) approach to solve path planning problem and generate shortest path length, path smoothness and obstacle avoidance.

3.2. Path smoothness

Smoothness involves minimizing the angle between the current goal position and the produced-current position. The angle between the two lines connecting G to the robot initial two successive positions in every iteration number. Thereby, the smoothness is reflected as follows [26]:

$$F_2(x,y)=\sum_{j=0}^{K-1}\left|\theta\left(_{j,j+1}\right)-\theta\left(_{j,K+1}\right)\right| \tag{4}$$

where

$$\theta\left(_{j,j+1}\right)=\tan^{-1}\frac{y_{j+1}-y_j}{x_{j+1}-x_j} \tag{5}$$

$$\theta\left(_{j,K+1}\right)=\tan^{-1}\frac{y_{K+1}-y_j}{x_{K+1}-x_j} \tag{6}$$

The overall objective function for multi-objective optimization is a weighted sum of the above two objectives, which can also be summarized as,

$$F(x,y)=c_1F_1(x,y)+c_2F_2(x,y) \tag{7}$$

where c_1 and c_2 are considered a degree of importance, their value must satisfy the following constraint $c_2 = 1 - c_1$.

Other than the length of the path, the smoothness is considered to be another significant criteria [27, 28]. Castillo [27] prove that path smoothness has attract researchers' attention recently. An approach is proposed in [29-31] by using particle swarm optimization and other methods. An idea is introduced in [32] to generate collision-free path under curvature constraints. In [33], another idea is produces for mobile robot path planning by using a Bezier-curve-based approach in a multi-agent soccer system. A method developed in [34] has been used to drive the mobile robot. On and Yazici [35] introduced A* algorithm to generate optimal smooth path using arc-line approach. PSO algorithm is proposed in [36] to plan a path and using cubic B-spline curve to generate path smoothness. In [37], generic algorithm has been used to generate a path and piecewise Hermite interpolating polynomials is used to have a smooth path. Xu [38] proposed a new approach to generate smooth optimal path using Bezier curve combine with improved PSO algorithm. An idea in [39] produced optimal curvature smoothing path for flying robots.

3.3. Time execution

Time execution in a mobile robot field refers to the time taken for a mobile robot to complete the entire route from the starting point S to the goal point G. Short execution times are related to the environment's high productivity [40]. According to [41], trajectory planning must be aware of changes in the robot's dynamic characteristics since conventional robot trajectory planning calculates the robot's trajectory parameters under robot kinematic and geometric constraints. The time-optimal trajectory planning (TOTP) algorithm on finding the minimum time to complete a task by improving the robot speed is first presented by [42-43]. However, TOTP leads to the robot's severe vibration and even worsens the wear of the motor. Therefore, in 2000, [44] presented an algorithm by considering motor torque rates, while [45] in 2012 considered jerk constraint (the derivative of acceleration) into clarification. Balkan proposed an approach by using a dynamic programming technique. The technique basis sets the limits on velocity, acceleration and jerk [46].

3.4. Energy consumption

Energy consumption in a mobile robot field refers to the minimum energy consumed, which means minimum actuator effort and related to saving [47]. According to [48], energy consumption in the industrial sector has been reducing since 1998. Their paper proposed a mobile robot approach and succeeded in reducing 12% work cycle energy consumption in pick-and-place operation cases. Shiller [49] introduced some techniques for optimal trajectory planning concerning time and energy. The integral of squared torques along the trajectories are considered in the objective function. In [46] also used the same technique by considering the function of total energy.

Saramago and Steffen considered trajectories parameterized by cubic spines subject to the kinematic constraint and dynamic constraint [50]. In [51], another idea is proposed by considering cubic B-splines as trajectories parameters. In [17], cubic splines is considered as the parameter subject to kinematic and dynamic constraints. The methodology considered both total energy consumption and total time execution devote along the entire trajectory. Another idea considered B-spline trajectory optimization using electrical energy sharing is presented in [52].

Then, simple motion trajectories, such as S-curve and trapezoidal, are presented in [53]. Gasparetto et al. [40] stressed that the term "energy" in trajectory planning is defined as the integral of squared torques (measured the effort of the robot actuators) instead of the physical quantity measured in Joules, assuming that torque is proportional to the current. Most of the algorithms related to collision-free trajectories planning aim to optimize a few operational parameters or objective functions.

Due to the development and improvement in robotics systems, energy consumption becomes one of the interest issues among researchers. Typically, the primary energy consumption is due to the motor. According to [54], most autonomous mobile robots (AMR) are battery-powered. Thus, energy-based planning algorithms are essential for effective operation. In [55-57], a study is conducted on lithium-ion battery degradation models, while [58] and [459] studied energy consumption for unmannered aerial vehicles (UAV) and car-like-robot. In 2020, [60] presented an industrial robotics system approach based on dynamic and electro-mechanical modeling. Author [54] proposed algorithms for energy consumption with task allocation and scheduling methods.

4. Discussion and conclusion

Mobile robot navigation consists of three main functions that help in the robot's movement and give information to the robot to move. Path planning has been practiced globally as the demand grows wide. An overview of path planning optimization multi-criteria for the autonomous mobile robot and navigation of mobile robots was presented and discussed briefly. The goal of research work is to find the mobile robot's optimal performance in the presence of obstacles, either in a static or dynamic environment.

Many mathematical criteria describe the success of the mission performed by the mobile robots: path length, probability of rescue, motion time on the trajectories from the first detection until final detection on the path, energy consumed during the movement and the smoothness of the path. As various method have been presented for mobile robot path planning problem, but most of the researchers only focus on at least two optimization performance criteria of the mobile robots. Among the recent studies, optimizing path length and path smoothness is the most preferred and has many possible techniques in mobile robot navigation in a known and dynamic environment. Path length and path smoothness have a simple algorithm to calculate rather than the trajectories problem since it can be calculated using distance in the cartesian plane while time consumption and energy consumption are using torque, velocity, acceleration and so on. On the other hand, time trajectories and energy consumption also contribute to their importance in mobile robots; however, the algorithms are more complex to build. With all the rise of demand in mobile robot path planning, more research should raise the target on considering all the possible criteria for a mobile robot instead of combining only two criteria in the future. Generally, this review paper gives better insights and study directions on mobile robot path planning in this field.

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